Phy 554: Problem Set 1
(Due: January 31, 2001)

1). Gasiorowicz, Chap. 14, Problem 4.

2). Gasiorowicz, Chap. 14, Problem 5.

3). Gasiorowicz, Chap. 14, Problem 7.

4). Gasiorowicz, Chap. 14, Problem 9.

5). Gasiorowicz, Chap. 14, Problem 10.

6). Gasiorowicz, Chap. 14, Problem 12.

7.) Consider, to start, an orbital angular momentum of $l = 3/2$ and $m = 3/2$. (We will show in this problem that this is not possible.)

a) Construct the coordinate space wave function of an eigenstate of orbital angular momentum with $l = 3/2$ and $m = 3/2$ — that is, construct $Y_{3/2,3/2}(\theta, \phi)$. You need not normalize it. (*Hint:* In analogy to lecture, obtain a differential equation from the condition $\hat{L}_+|3/2, 3/2\rangle = 0$.)

b) Apply the lowering operator $\hat{L}_-$ three times to this wave function to obtain the wave function of a state which should have $l = 3/2$ and $m = -3/2$. Is it proportional to Y^*_{lm}? Is it normalizable? Is it possible, then, to have $l = 3/2$?

8.) (EXTRA CREDIT)

a) Show that the eigenvalues of the Pauli matrices are ± 1.

b) Verify that the Pauli matrices satisfy the identity $\sigma_i \sigma_j = \delta_{ij} + i\varepsilon_{ijk}\sigma_k$.

c) Use the result of part b) to show that the matrices $s_k = \hbar\sigma_k/2$ satisfy the commutation rules of angular momentum operators.

d) Let $\mathbf{a}$ and $\mathbf{b}$ be any two vectors, and let σ denote the triplet of matrices $(\sigma_1, \sigma_2, \sigma_3)$. Prove the identity

$$\mathbf{a} \cdot \sigma \mathbf{b} \cdot \sigma = \mathbf{a} \cdot \mathbf{b} + i\sigma \cdot \mathbf{a} \times \mathbf{b}. \tag{1}$$

Recall that $(\mathbf{a} \times \mathbf{b})_i = \varepsilon_{ijk} a_j b_k$ and that ε_{ijk} is the Levi-Civita tensor (a totally antisymmetric tensor such that $\varepsilon_{ijk} \neq 0$ only if $i \neq j \neq k$ and $\varepsilon_{123} = 1$).