Phy 616: Problem Set 1
(Due: September 6, 2006)

1)a) A Lorentz transformation in the x^1 direction is given by

$$\begin{aligned} t' &= \gamma(t - vx^1) & (1)\\ x^{1\prime} &= \gamma(-vt + x^1) & (2)\\ x^{2\prime} &= x^2 & (3)\\ x^{3\prime} &= x^3\,, & (4) \end{aligned}$$

where $\gamma = (1-v^2)^{-1/2}$ and $c = 1$. Write down the inverse of this transformation (i.e., express (t, x^1) in terms of $(t', x^{1\prime})$, and use the chain rule of partial differentiation to show that ∂^μ and x^μ transform in the same manner under the Lorentz transformation.

b) Check that

$$\partial_\mu j^\mu = 0\,. \quad (5)$$

can be written as

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = \mathbf{0}\,. \quad (6)$$

2) Express each component of the field-strength tensor $F^{\mu\nu}$ in terms of electric and magnetic field components. How many independent components does $F^{\mu\nu}$ have? Verify that

$$\partial_\mu F^{\mu\nu} = j^\nu \quad (7)$$

reproduces

$$\begin{aligned} \nabla \cdot \mathbf{E} &= \rho & (8)\\ & & (9)\\ \nabla \times \mathbf{B} &= \mathbf{j} + \frac{\partial \mathbf{E}}{\partial \mathbf{t}}\,. & (10) \end{aligned}$$

3) Show that a vector in an inertial frame K', $C'^{\,\nu}$, and that in an inertial frame K, C^α, are related by

$$C'^{\,\nu} = \frac{\partial x'^\nu}{\partial x^\alpha} C^\alpha \quad (11)$$

and thus show that the field-strength tensors in the two frames are related by

$$F'^{\,\alpha\beta} = \frac{\partial x'^\alpha}{\partial x^\gamma}\frac{\partial x'^\beta}{\partial x^\delta} F^{\gamma\delta}\,. \quad (12)$$

Determine the explicit equations of transformation for **E**- and **B**-fields for the specific Lorentz transformation of Problem #1. In this context, consider a set of K' frames distinguished by the precise value of v and answer the following. Is it possible to have an electromagnetic field that appears as a purely electric field in one inertial frame and as a purely magnetic field in some other inertial frame? What are the criteria imposed on **E** and **B** such that there is an inertial frame in which there is no electric field?