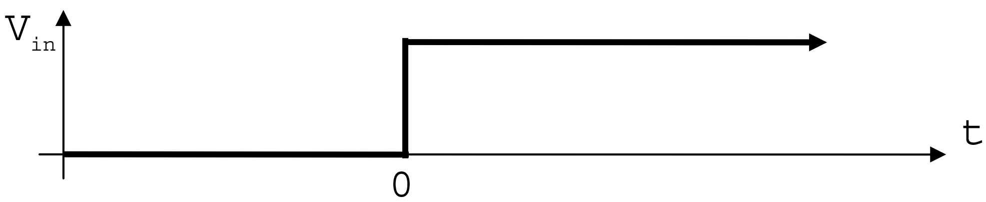
Robert E. Simpson
Introductory Electronics For Scientists and Engineers, 2nd edition
September 23, 2008

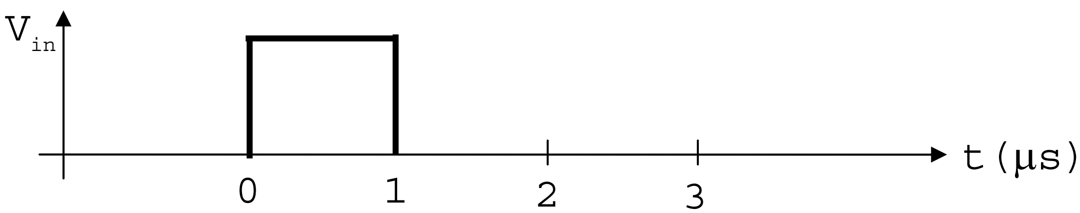
Problem 3-11

Sketch the approximate output from a differentiating circuit with R=10 kΩ, C=0.01μF for the following inputs:

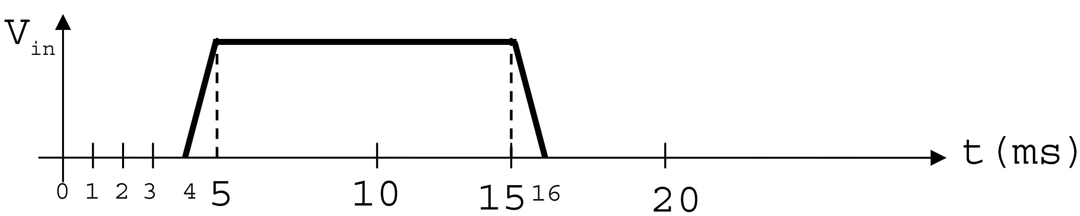
(a)



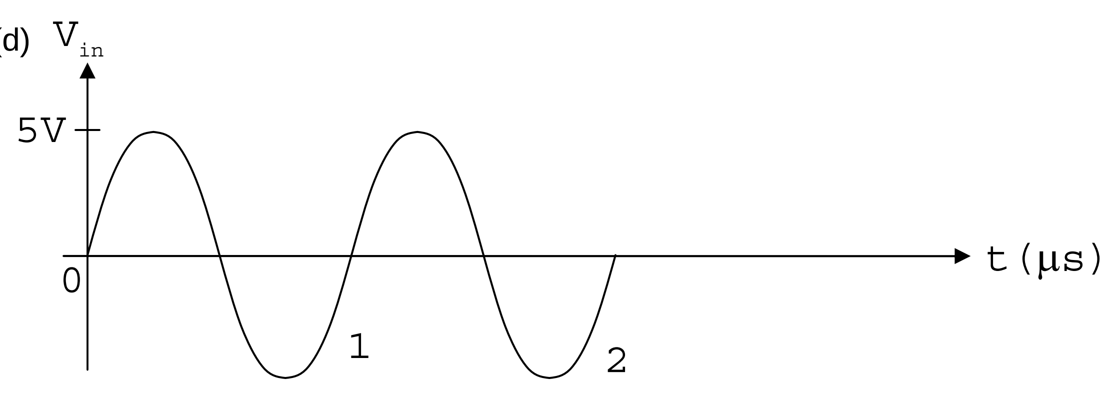
(b)



(c)



(d)

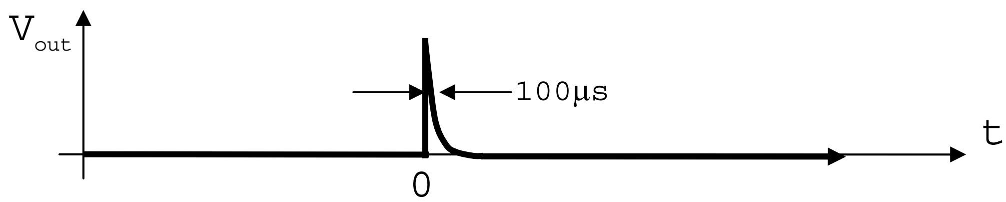


Solution:

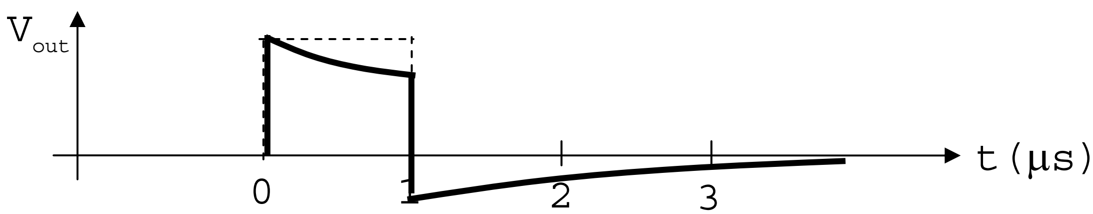
RC time constant:

$$RC = (10 \times 10^3)(0.01 \times 10^{-6}) = 1 \times 10^{-4}\,\text{s or } 100\mu\text{s or } 0.1\text{ms}$$

(a)



(b) Note that the width of the pulse is much shorter than the time constant (!1%).



(c)

At the leading edge,

$$V_{in} = \frac{Q}{C} + IR \Rightarrow \frac{dV_{in}}{dt} = \frac{1}{C}\frac{dQ}{dt} + R\frac{dI}{dt}$$

$$\Rightarrow K = \frac{I}{C} + R\frac{dI}{dt} \qquad \left(\frac{dV_{in}}{dt} = K\right)$$

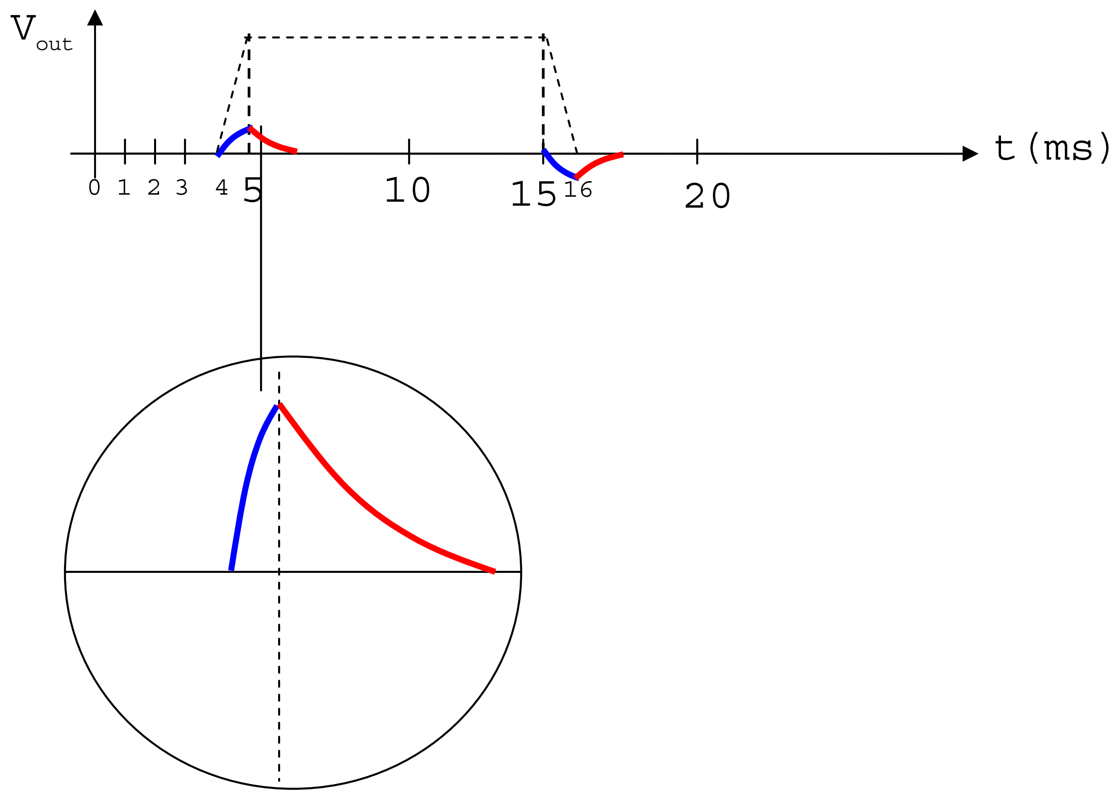
Integrating,

$$I = KC(1 - e^{-t/RC})$$

$$\therefore V_{out} = V_R = IR$$

$$= KCR(1 - e^{-t/RC})$$

The time scale (~1ms) is much longer than the RC time constant (0.1mS). At 1ms, $V_{out} \sim KRC = 0.1\ V_{in,\,max}$.



(d)

$$X_C = \frac{1}{i\omega C}$$

$$I = \frac{V_{in}}{R + X_C} = \frac{V_{in}}{R + \frac{1}{i\omega C}} = \frac{i\omega C V_{in}}{1 + i\omega CR}$$

$$= V_{in}\left(\frac{i\omega C}{1 + i\omega CR}\right)$$

$$V_{out} = IR = V_{in}\left(\frac{i\omega RC}{1 + i\omega CR}\right)$$

$$= V_{in}\left(\frac{i\omega RC}{1 + i\omega CR}\right)\left(\frac{1 - i\omega CR}{1 - i\omega CR}\right)$$

$$= V_{in}\left(\frac{\omega^2 C^2 R^2 + i\omega CR}{1 + \omega^2 C^2 R^2}\right)$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \left|\frac{\omega CR\sqrt{1+\omega^2C^2R^2}}{1+\omega^2C^2R^2}\right|$$

$$= \frac{\omega CR}{\sqrt{1+\omega^2C^2R^2}}$$

V_{out} leads V_{in} by a phase angle of $\tan^{-1}\left(\frac{1}{\omega RC}\right)$

$RC = 10^{-4}\,s$

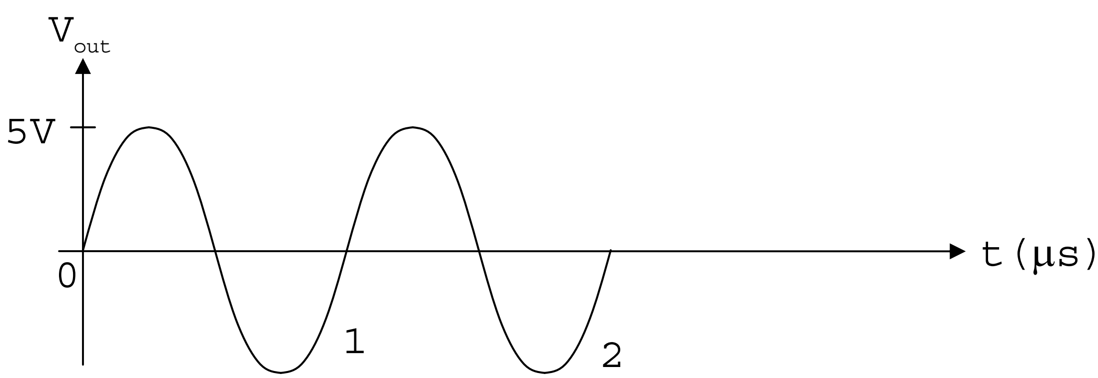
Input wave has a periof of $1\mu s \Rightarrow f = 1\,MHz$

$\omega = 2\pi f = 6.283\times10^{6}\,Hz$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{\omega CR}{\sqrt{1+\omega^2C^2R^2}} = \frac{6.283\times10^{6}\times10^{-4}}{\sqrt{1+(6.283\times10^{6}\times10^{-4})^2}} = 1.000$$

With $|V_{in}| = 5V, |V_{out}| = 5V$

V_{out} leads V_{in} by a phase angle of $\tan^{-1}\left(\frac{1}{6.283\times10^{6}\times10^{-4}}\right) = 0.092^{o}$

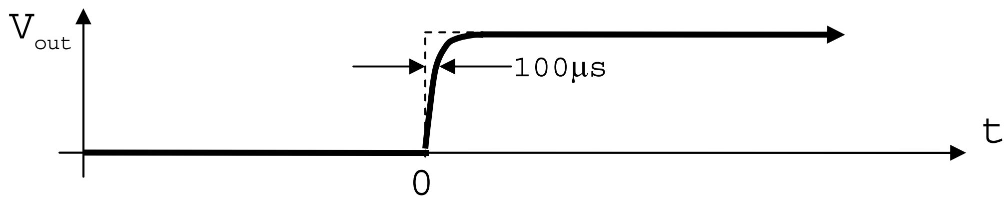


In the case for an integrator (not asked by the question):

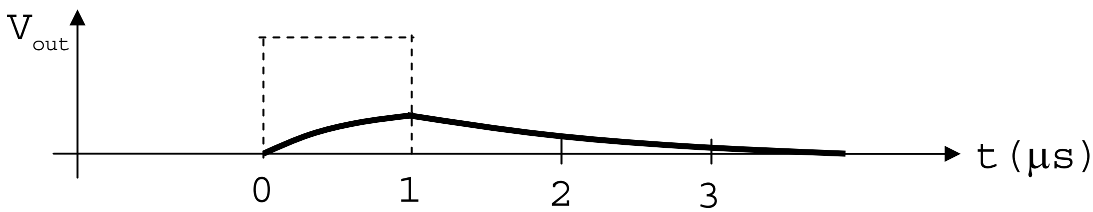
RC time constant :

$$RC = (10\times10^{3})(0.01\times10^{-6}) = 1\times10^{-4}\,\text{s or } 100\mu\text{s or } 0.1\text{ms}$$

(a)



(b) Note that the width of the pulse is much shorter than the time constant (!1%).



(c)

At the leading edge,

$$V_{in} = \frac{Q}{C} + IR \Rightarrow \frac{dV_{in}}{dt} = \frac{1}{C}\frac{dQ}{dt} + R\frac{dI}{dt}$$

$$\Rightarrow K = \frac{I}{C} + R\frac{dI}{dt} \qquad\qquad (\frac{dV_{in}}{dt} = K)$$

Integrating,

$$I = KC(1 - e^{-t/RC})$$

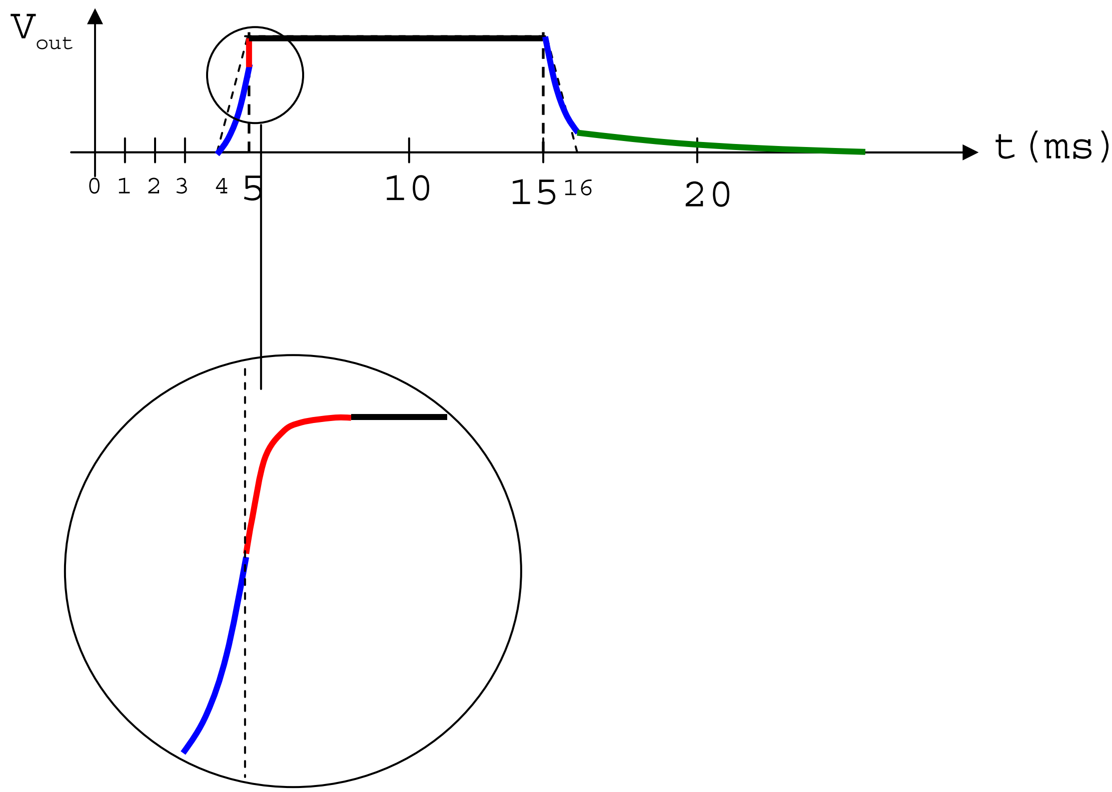
$$\therefore V_{out} = V_c = V_{in} - IR$$

$$= Kt - KCR(1 - e^{-t/RC})$$

At the discharging edge, similar mathematics show

$$V_{out} = (V_{max} - Kt) - KCR(e^{-t/RC} - 1)$$

The time scale (~1ms) is much longer than the RC time constant (0.1mS). At 1ms, Vout ~ K(1ms)-K(0.1ms) = K(0.9mS), or 90% of the maximum voltage.



(d)

$$X_C = \frac{1}{i\omega C}$$

$$I = \frac{V_{in}}{R + X_C} = \frac{V_{in}}{R + \frac{1}{i\omega C}} = \frac{i\omega C V_{in}}{1 + i\omega CR}$$

$$= V_{in}\left(\frac{i\omega C}{1 + i\omega CR}\right)$$

$$V_{out} = IX_C = V_{in}\left(\frac{i\omega C}{1 + i\omega CR}\right)\left(\frac{1}{i\omega C}\right)$$

$$= V_{in}\left(\frac{1}{1 + i\omega CR}\right)$$

$$= V_{in}\left(\frac{1}{1 + i\omega CR}\right)\left(\frac{1 - i\omega CR}{1 - i\omega CR}\right)$$

$$= V_{in}\left(\frac{1 - i\omega CR}{1 + \omega^2 C^2 R^2}\right)$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \left|\frac{1 - i\omega CR}{1 + \omega^2 C^2 R^2}\right|$$

$$= \left|\frac{\sqrt{1 + \omega^2 C^2 R^2}}{1 + \omega^2 C^2 R^2}\right|$$

$$= \frac{1}{\sqrt{1 + \omega^2 C^2 R^2}}$$

V_{out} lags V_{in} by a phase angle of $\tan^{-1}(\omega RC)$

$RC = 10^{-4}\,s$

Input wave has a periof of $1\mu s \Rightarrow f = 1\,MHz$

$\omega = 2\pi f = 6.283 \times 10^6\,Hz$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{1 + \omega^2 C^2 R^2}} = \frac{1}{\sqrt{1 + (6.283 \times 10^6 \times 10^{-4})^2}} = 1.591 \times 10^{-3}$$

With $|V_{in}| = 5V, |V_{out}| = 7.96mV$

V_{out} lags V_{in} by a phase angle of $\tan^{-1}\left(6.283 \times 10^6 \times 10^{-4}\right) = 89.9^o$