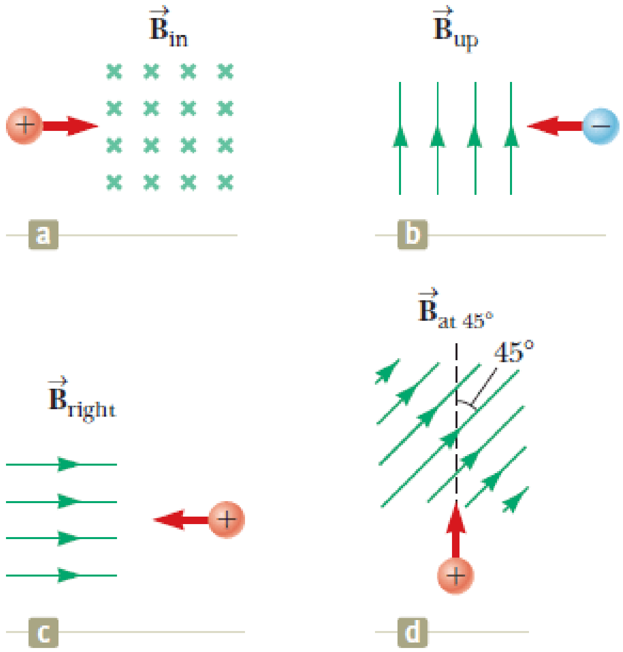
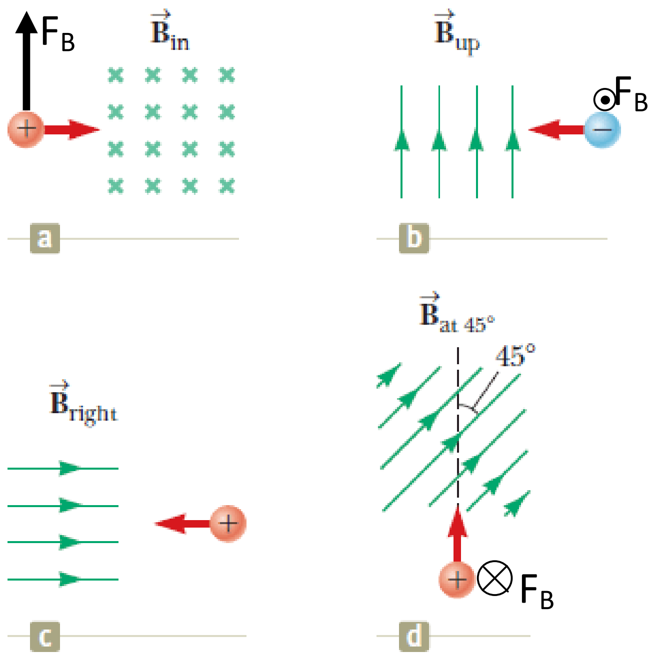
1. Determine the initial direction of the deflection of charged particles as they enter the magnetic fields shown in the figure below.



Solution:

Use right hand rule: $\vec{F}_B = q\vec{v} \times \vec{B}$

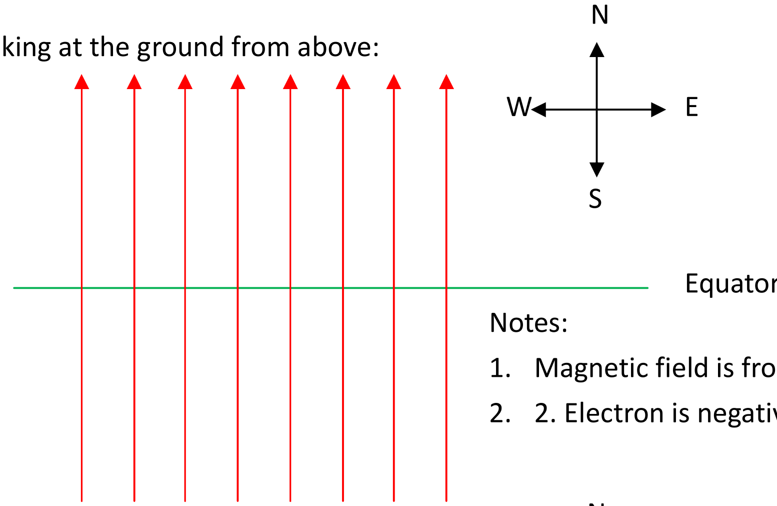


No magnetic force since $\sin 180^o = 0$

2. Consider an electron near the Earth's equator. In which direction does it tend to deflect if its velocity is directed in each of the following directions? (a) downward (b) northward (c) westward (d) southeastward

Solution:

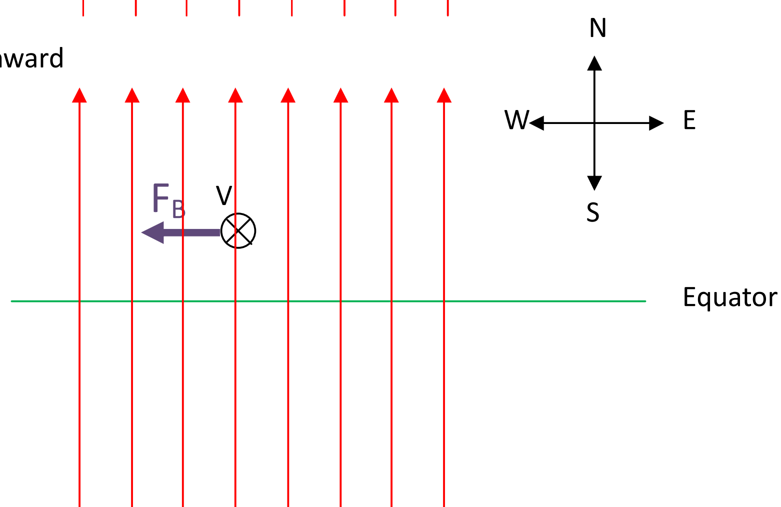
Top view looking at the ground from above:



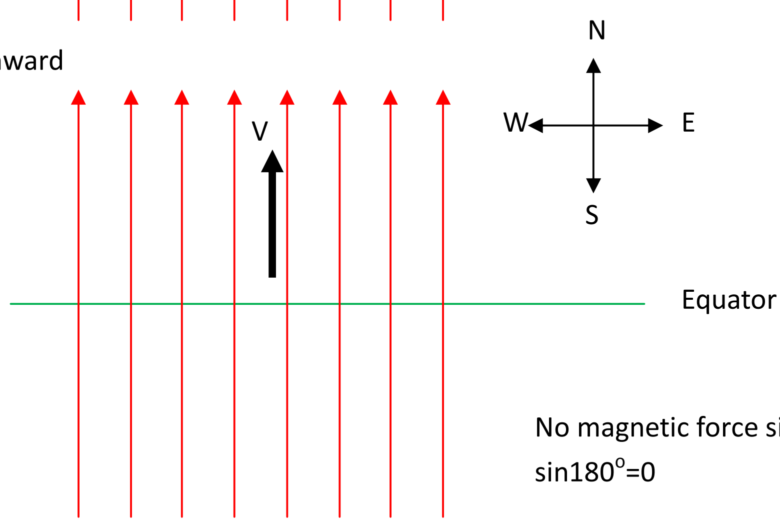
Notes:

1. Magnetic field is from South to North
2. 2. Electron is negative charge

(a) downward

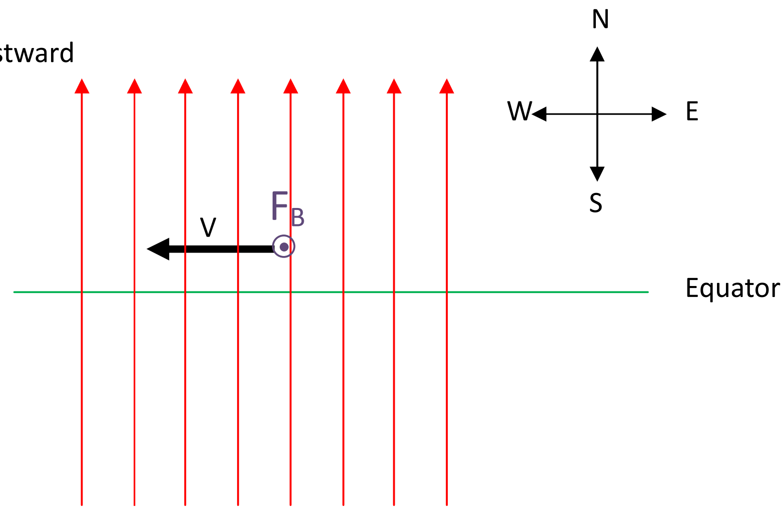


(b) northward

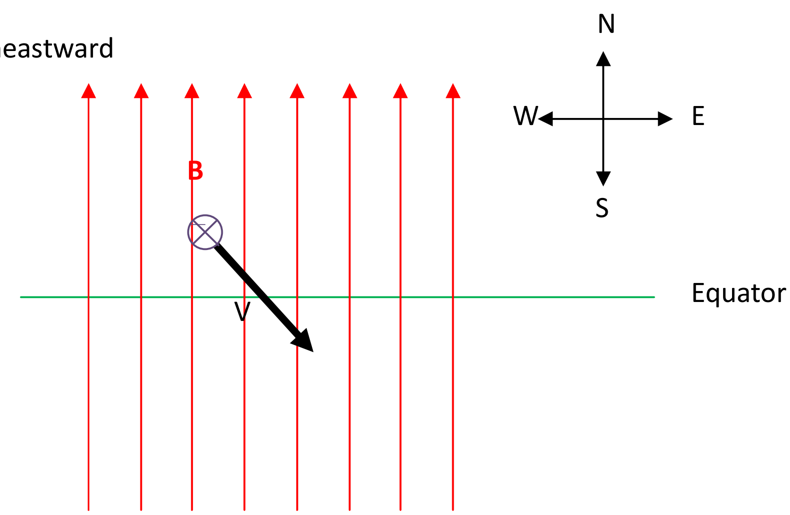


No magnetic force since $\sin 180^{o}=0$

(c) westward

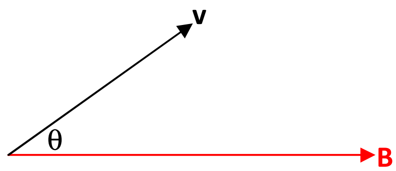


(d) southeastward



3. A proton travels with a speed of v at an angle of θ with the direction of a magnetic field of magnitude B in the positive *x*-direction. (a) What is the magnitude of the magnetic force on the proton? (b) What is the proton's acceleration?

Solution:



`

(a) Charge for proton = e

$$F_B = \underline{\underline{evB\sin\theta}}$$

(b) $F_B = ma$ (m = mass of proton)

$$\Rightarrow evB\sin\theta = ma$$

$$\Rightarrow a = \underline{\underline{\frac{evB}{m}\sin\theta}}$$

4. An electron is accelerated through a potential difference of V from rest and then enters a uniform magnetic field B. (a) What is the maximum magnitude of the magnetic force this particle can experience? (b) What is the proton's acceleration?

(a) Magnitude of charge of an electron is e. Let the mass of an electron be m.

$$\text{Conservation of energy :}$$
$$\frac{1}{2}mv^2 = eV \Rightarrow v = \sqrt{\frac{2eV}{m}}$$
$$F_B = evB\sin\theta$$
$$F_B \text{ is maximum when } \sin\theta = 1$$
$$\therefore F_{B\ max} = evB = \underline{\underline{eB\sqrt{\frac{2eV}{m}}}}$$

(b)

$$F_B = ma \Rightarrow ma = eB\sqrt{\frac{2eV}{m}}$$
$$\Rightarrow a = \frac{eB}{m}\sqrt{\frac{2eV}{m}}$$
$$\Rightarrow a = \underline{\underline{B\sqrt{\frac{2e^2V}{m^2}}}}$$

5. A proton moves with a velocity of $\vec{\mathbf{v}}$= ($v_x\hat{\mathbf{i}}$ + $v_y\hat{\mathbf{j}}$ + $v_z\hat{\mathbf{k}}$) m/s in a region in which the magnetic field is $\vec{\mathbf{B}}$= ($B_x\hat{\mathbf{i}}$ + $B_y\hat{\mathbf{i}}$ + $B_z\hat{\mathbf{k}}$) T. What is the magnitude of the magnetic force this particle experiences?

Solution:

Charge of a proton is e.

$$\vec{F}_B = e\vec{v} \times \vec{B} = e\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix}$$

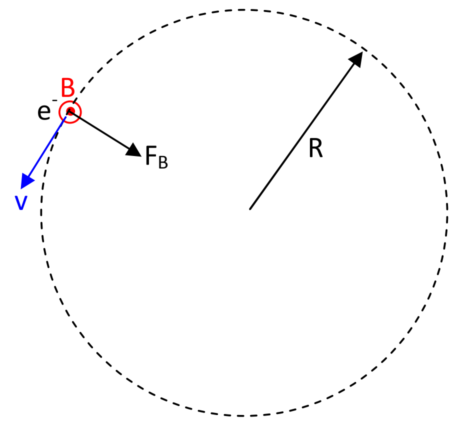
$$= e[\hat{i}(v_x B_y - v_y B_x) - \hat{j}(v_x B_z - v_z B_x) + \hat{k}(v_x B_z \text{ - } v_z B_x)]$$

$$\therefore |\vec{F}_B| = \underline{\underline{\sqrt{(v_x B_y - v_y B_x)^2 + (v_x B_z - v_z B_x)^2 + (v_x B_z \text{ - } v_z B_x)^2}}}$$

6. An electron moves in a circular path perpendicular to a uniform magnetic field with a magnitude of B. If the speed of the electron is v, determine the following. (a) the radius of the circular path. (b) the time interval required to complete one revolution.

Solution:

Magnitude of charge of an electron = e



(a)

Equation of motion :

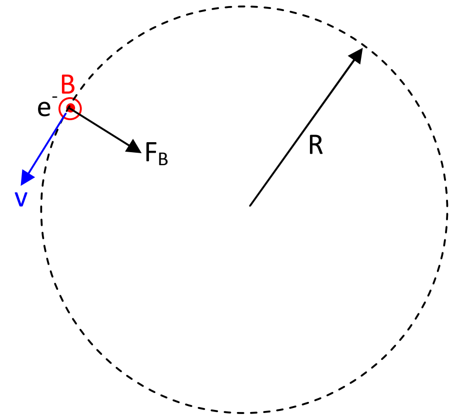
$$F = ma \Rightarrow evB = \frac{mv^2}{R}$$

$$\Rightarrow R = \frac{mv^2}{evB}$$

$$\Rightarrow R = \underline{\underline{\frac{mv}{eB}}}$$

$$\text{(b) } T = \frac{2\pi R}{v} = \frac{2\pi \cdot \frac{mv}{eB}}{v} = \underline{\underline{\frac{2\pi m}{eB}}}$$

7. An electron moves in a circular path perpendicular to a constant magnetic field with a magnitude of B. The angular momentum of the electron about the center of the circle is L. (a) Determine the radius of the circular path. (b) Determine the speed of the electron.



Solution:

Magnitude of electron charge = e

(a)

Equation of motion :

$$F = ma \Rightarrow evB = \frac{mv^2}{R}$$

$$\Rightarrow R = \frac{mv^2}{evB}$$

$$\Rightarrow R = \frac{mv}{eB} \qquad \text{---}(1)$$

$$\text{But } L = mvR \Rightarrow \ v = \frac{L}{mR} \qquad \text{---}(2)$$

$$\text{Substitute (2) into (1) } \Rightarrow R = \frac{m}{eB} \cdot \frac{L}{mR}$$

$$\Rightarrow R^2 = \frac{L}{eB}$$

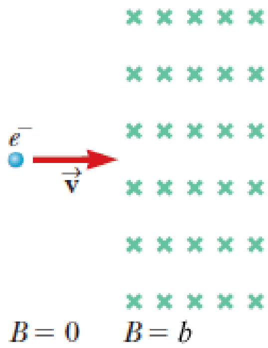
$$\Rightarrow R = \sqrt{\frac{L}{eB}}$$

(b)

$$(2) \Rightarrow \ v = \frac{L}{mR} \Rightarrow \ v = \frac{L}{m\sqrt{\frac{L}{eB}}}$$

$$\Rightarrow \ v = \frac{\sqrt{eBL}}{m}$$

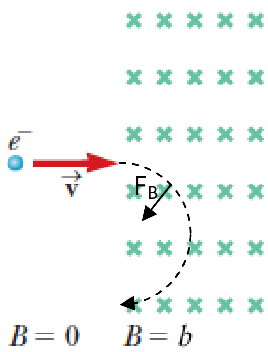
8. Assume the region to the right of a certain plane contains a uniform magnetic field of magnitude b and the field is zero in the region to the left of the plane as shown in the figure below. An electron, originally traveling perpendicular to the boundary plane, passes into the region of the field.



(a) Determine the time interval required for the electron to leave the "field-filled" region, noting that the electron's path is a semicircle.

(b) Assuming the maximum depth of penetration into the field is L, find the kinetic energy of the electron.

Solution:



(a) Radius of the semicircular path is L. Magnitude of electron charge is e.

Equation of motion :

$$F_B = ma \Rightarrow evb = \frac{mv^2}{L}$$

$$\Rightarrow L = \frac{mv^2}{evb}$$

$$\Rightarrow L = \frac{mv}{eb} \qquad \text{---}(1)$$

$\therefore$ Time interval required for the electron to leave the field region

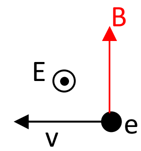
$$= \frac{\frac{1}{2}\cdot 2\pi L}{v} = \frac{\frac{1}{2}\cdot 2\pi \cdot \frac{mv}{eb}}{v} = \underline{\underline{\frac{m\pi}{eb}}}$$

(b) $(1) \Rightarrow v = \dfrac{Leb}{m}$

$$\Rightarrow KE = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{Leb}{m}\right)^2 = \underline{\underline{\frac{(Leb)^2}{2m}}}$$

9. A velocity selector consists of electric and magnetic fields described by the expressions $\vec{E} = E\ \hat{k}$ and $\vec{B} = B\ \hat{i}$. Find the value of E such that an electron of energy K moving in the negative *x* direction is undeflected.

Solution:



Charge of electron = -e

$$\vec{F}_E = -e\vec{E} = -eE\hat{k}$$

$$\vec{F}_B = q\vec{v}\times\vec{B} = -e\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -v & 0 & 0 \\ 0 & B & 0 \end{vmatrix} = evB\hat{k}$$

$$\therefore \vec{F} = \vec{F}_E + \vec{F}_B = -eE\hat{k} + evB\hat{k} = (evB - eE)\hat{k}$$

Equation of motion for undeflected electron $(a = 0)$:

$$F = ma \Rightarrow evB - eE = m(0)$$

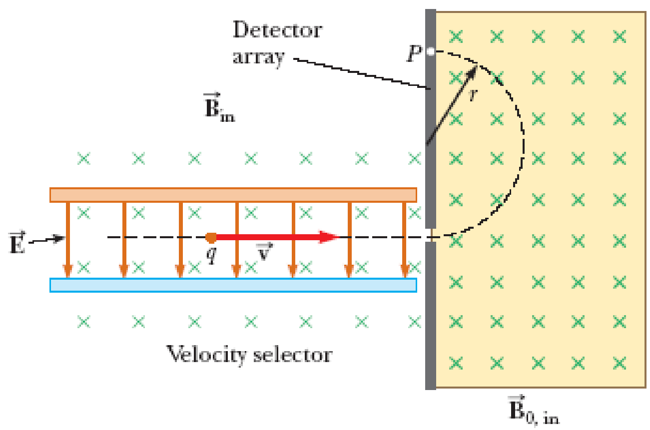
$$\Rightarrow evB - eE = 0$$

$$\Rightarrow E = vB$$

$$\text{But } K = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2K}{m}}$$

$$\therefore\ E = \underline{\underline{B\sqrt{\frac{2K}{m}}}}$$

10. Consider the mass spectrometer shown schematically in the figure below. The magnitude of the electric field between the plates of the velocity selector is E, and the magnetic field in both the velocity selector and the deflection chamber has a magnitude of B. Calculate the radius of the path for a singly charged ion having a mass m.



First calculate the velocity of the particle leaving the velocity selector:

Equation of motion for undeflected particle :

$$F = m(0) \Rightarrow qE - qvB = 0$$

$$\Rightarrow v = \frac{E}{B}$$

We can now calculate the radius of the circular path in the mass spectrometer:

Equation of motion :

$$F = ma \Rightarrow qvB = \frac{mv^2}{r}$$

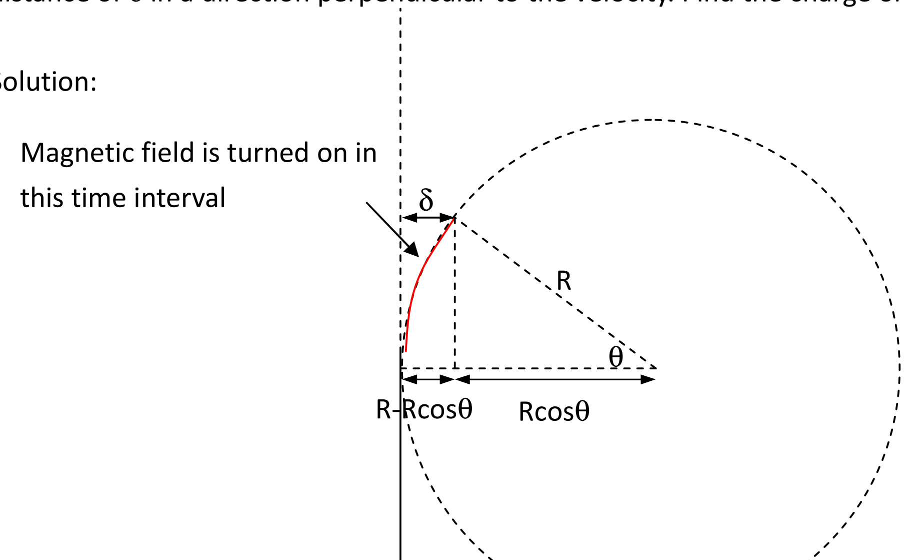
$$\Rightarrow r = \frac{mv}{qB}$$

$$\Rightarrow r = \frac{m}{qB}\frac{E}{B}$$

$$\Rightarrow r = \underline{\underline{\frac{mE}{qB^2}}}$$

11. A charged particle of mass m is moving at a speed of v. Suddenly, a uniform magnetic field of magnitude B in a direction perpendicular to the particle's velocity is turned on and then turned off in a time interval of t. During this time interval, the magnitude and direction of the velocity of the particle undergo a negligible change, but the particle moves by a distance of δ in a direction perpendicular to the velocity. Find the charge on the particle.

Solution:



Let the charge of the particle be q.
When the field is turned on,

$$\text{Equation of motion}: qvB = \frac{mv^2}{R} \Rightarrow v = \frac{qBR}{m} \qquad ---(1)$$

$$\omega = \frac{v}{R} = \frac{qB}{m}$$

But according to the figure, $\delta = R - R\cos\theta \Rightarrow \delta = R(1-\cos\theta) \qquad ---(2)$

For small angle (hinted by the statement "direction of the velocity of particle undergo a negligible change",

$$\cos\theta \approx 1 - \frac{\theta^2}{2}$$

$$(2) \Rightarrow \delta = R[1-(1-\frac{\theta^2}{2})] \Rightarrow \delta = R\frac{\theta^2}{2} \qquad ---(3)$$

$$\theta = \omega t = \frac{qBt}{m}$$

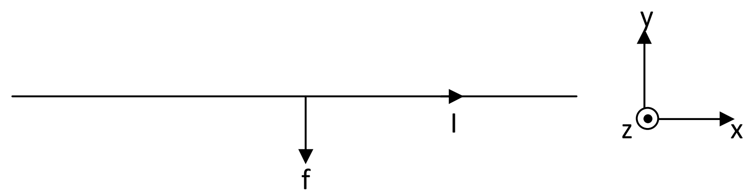
$$\text{Substitute this into (2)} \Rightarrow \delta = \frac{R}{2}\left(\frac{qBt}{m}\right)^2 \Rightarrow R = 2\delta\left(\frac{m}{qBt}\right)^2$$

$$\text{Substitute this into (1)} \Rightarrow v = \frac{qB}{m}\cdot 2\delta\left(\frac{m}{qBt}\right)^2 \Rightarrow v = 2\delta\frac{m}{qBt^2}$$

$$\Rightarrow q = \underline{\underline{2\delta\frac{m}{Bvt^2}}}$$

12. A conductor carrying a current I is directed along the positive *x* axis and perpendicular to a uniform magnetic field. A magnetic force per unit length of f acts on the conductor in the negative *y* direction. (a) Determine the magnitude of the magnetic field in the region through which the current passes. (b) Determine the direction of the magnetic field in the region through which the current passes.

Solution:



(a)

$dF_B = Id\vec{s} \times \vec{B}$

For a straigh wire of length L perpendicular to $\vec{B}$,

$dF_B = IdsB\sin 90^o \Rightarrow dF_B = IBds$

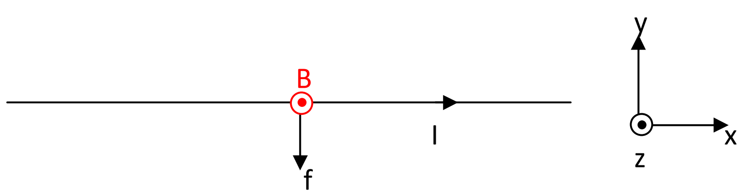
$$\Rightarrow \int dF_B = IB\int ds$$

$$\Rightarrow F_B = ILB$$

$$\Rightarrow f = \frac{F_B}{L} = IB$$

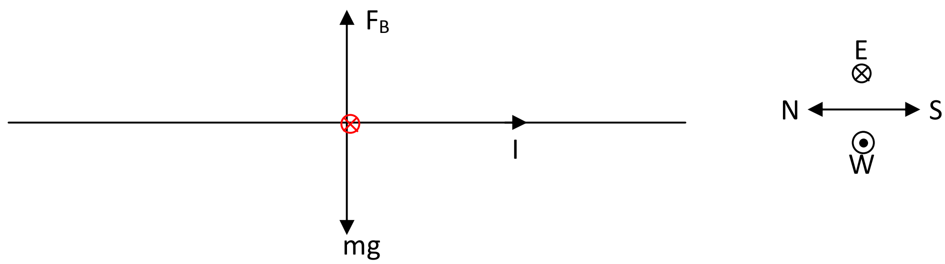
$$\Rightarrow B = \underline{\underline{\frac{f}{I}}}$$

(b)



13. A wire having a mass per unit length λ carries a current I horizontally to the south. (a) What is the direction of the minimum magnetic field needed to lift this wire vertically upward? (b) What is the magnitude of the minimum magnetic field needed to lift this wire vertically upward?

Solution:

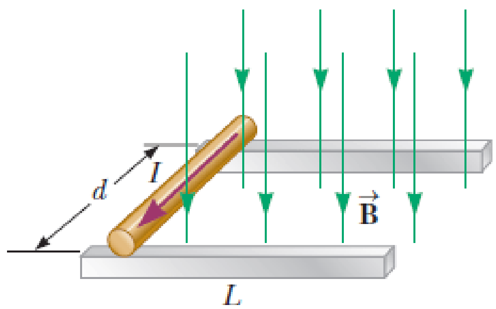


(a) To produce an upward magnetic force, it is necessary for the magnetic field pointing towards east (see figure above).

(b) Equation of motion :

$$F = ma \Rightarrow F_B - mg = m(0)$$
$$\Rightarrow IBL - \lambda Lg = 0$$
$$\Rightarrow IB - \lambda g = 0$$
$$\Rightarrow B = \frac{\lambda g}{I}$$

14. A rod of mass M and radius R rests on two parallel rails (see figure below) that are d apart and L long. The rod carries a current of I in the direction shown and rolls along the rails without slipping. A uniform magnetic field of magnitude B is directed perpendicular to the rod and the rails. If it starts from rest, what is the speed of the rod as it leaves the rails? (Assume that the rod is of uniform density.)



Side view

Solution:

Equation of motion (horizontal direction):

$$F_B = Ma \Rightarrow IBd = Ma \Rightarrow a = \frac{IBd}{M} \quad \text{(constant)}$$

$$v_0 = 0$$

$$a = \frac{IBd}{M}$$

$$x - x_0 = L$$

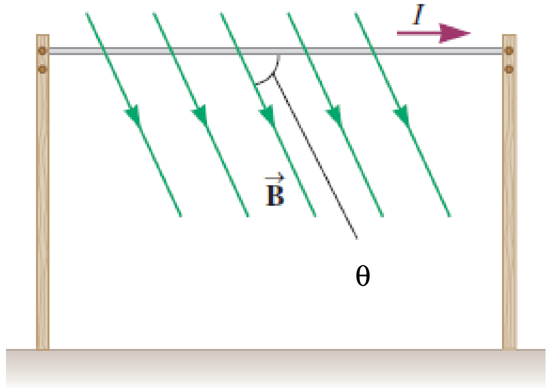
$$v = ?$$

$$v^2 = v_0^2 + 2a(x - x_0) \Rightarrow v^2 = 0 + 2 \cdot \frac{IBd}{M} \cdot L$$

$$\Rightarrow v^2 = \frac{2IBdL}{M}$$

$$\Rightarrow v = \sqrt{\frac{2IBdL}{M}}$$

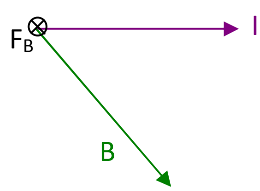
15. A horizontal power line of length L carries a current of I northward as shown in the figure below. The Earth's magnetic field at this location has a magnitude of B. The field at this location is directed toward the north at an angle θ below the power line.



(a) Find the magnitude of the magnetic force on the power line. (b) Find the direction of the magnetic force on the power line.

Solution:

Do part (b) first, using right hand rule:



(a) $\vec{F}_B = I\vec{L} \times \vec{B} \quad \Rightarrow |\vec{F}_B| = \underline{\underline{ILB\sin\theta}}$