Lorentz Transformation (Text Appendix I)

Galilean Transformation:

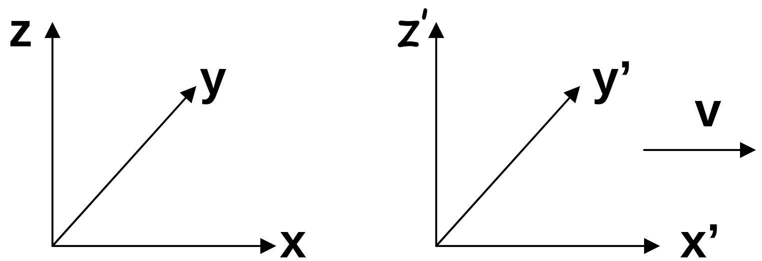
If the origins of two inertial frames coincide at $t=t'=0$, then:

$x'=x-vt$

$y'=y$

$z'=z$

$t'=t$



1. Newton's law of motion F=ma is invariant (appear in the same form) under Galilean Transformation.

2. Wave equation is not invariant under Galilean Transformation. For this reason, we have to assume the common wave equation we know only works in the inertial frame in which the medium is at rest. This leads to the necessity to have a medium for any type of wave. In the case of electromagnetic wave, it was ether. Non-existence of ether means the break down of the whole classical wave theory. One way to save the situation is to discover another transformation in which the wave equation (actually equations of all physical laws) remains invariant!

Lorentz Transformation (Text Appendix I)

Lorentz Transformation:

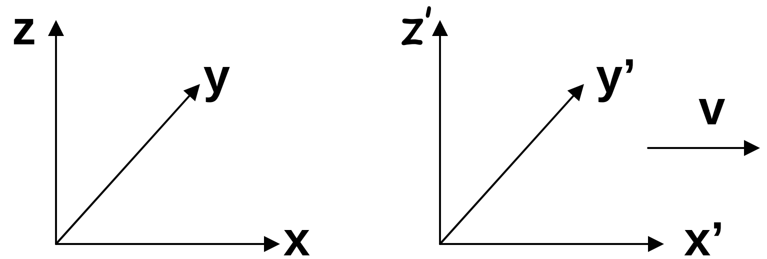
If the origins of two inertial frames coincide at t=t'=0, the

$$x' = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma(t - \frac{vx}{c^2})$$



1. Lorentz Transformation fills in the mathematical rigor of special relativity.

2. Both the Newton's law of motion and wave equation are invariant under Lorentz Transformation.

3. Lorentz transformation reduces to Galilean Transformation when v is small ($v \rightarrow 0$, i.e. $v/c \ll 1$)

Lorentz Transformation (Text Appendix I)

Time dilation from Lorentz transformation:

For the same x,

$$t' = \gamma(t - \frac{vx}{c^2})$$

$$\Rightarrow \begin{cases} t_1' = \gamma(t_1 - \frac{vx}{c^2}) \\ t_2' = \gamma(t_2 - \frac{vx}{c^2}) \end{cases}$$

$$\Rightarrow t = t_2' - t_1' = \gamma(t_2 - t_1) = \gamma t_0$$

Note that the two times have to be measured at the same place (x) in the un-prime frame (proper time).

Length contraction from Lorentz transformation:

For the same t,

$$x' = \gamma(x - vt)$$

$$\Rightarrow \begin{cases} x_1' = \gamma(x_1 - vt) \\ x_2' = \gamma(x_2 - vt) \end{cases}$$

$$\Rightarrow L_0 = x_2' - x_1' = \gamma(x_2 - x_1) = \gamma L$$

Note that the object is at rest and it has the proper length in the prime frame. This is the only frame that length can be measured even if $t_1' \neq t_2'$. For any other frame (unprime), the two positions have to be measured at the same time (t, i.e. simultaneously) $\Rightarrow$ meaning of length of an object..

Lorentz Transformation (Text Appendix I)

Suppose both S and S' frames are observing a *moving* particle. The velocity of this particle defined in this two frames are:

$$V_x = \frac{dx}{dt} \qquad V_y = \frac{dy}{dt} \qquad V_z = \frac{dz}{dt}$$

$$V_x' = \frac{dx'}{dt'} \qquad V_y = \frac{dy'}{dt'} \qquad V_z = \frac{dz'}{dt'}$$

Lorentz transformation

$$\left.\begin{aligned} x' &= \gamma(x - vt) \\ y' &= y \\ z' &= z \\ t' &= \gamma\left(t - \frac{vx}{c^2}\right) \end{aligned}\right\} \Rightarrow \begin{aligned} dx' &= \gamma(dx - vdt) \\ dy' &= dy \\ dz' &= dz \\ dt' &= \gamma\left(dt - \frac{vx}{c^2}\right) \end{aligned} \Rightarrow$$

$$V_x' = \frac{dx'}{dt'} = \frac{V_x - v}{1 - \frac{vV_x}{c^2}}$$

$$V_y' = \frac{V_y}{\gamma\left(1 - \frac{vV_x'}{c^2}\right)}$$

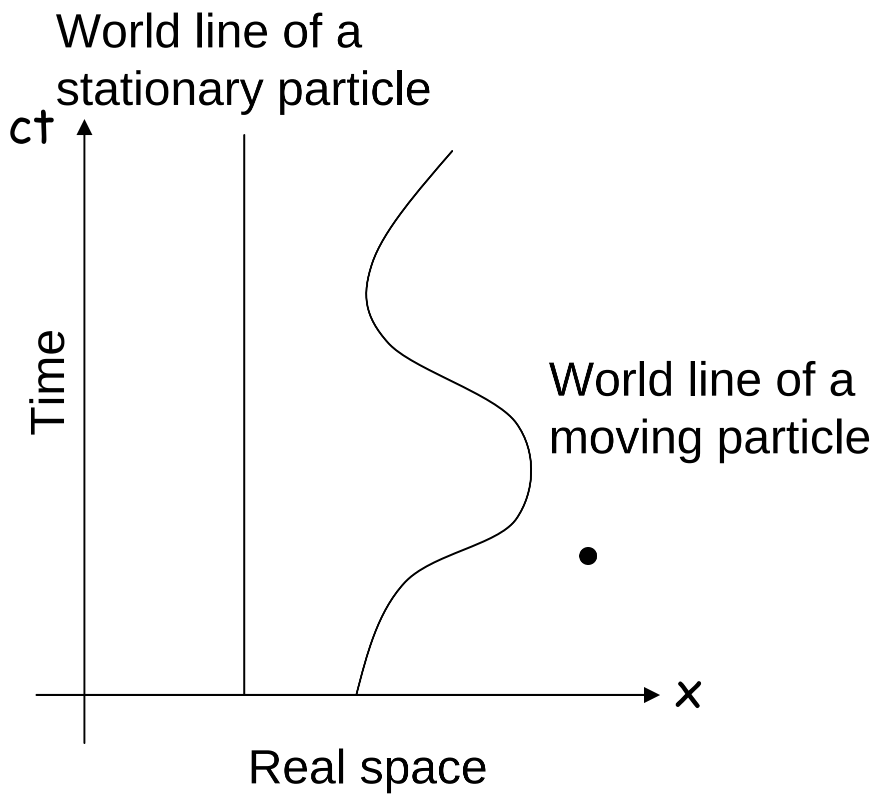
$$V_z' = \frac{V_z}{\gamma\left(1 - \frac{vV_x'}{c^2}\right)}$$

Spacetime (Text Appendix II)

1. Lorentz transformation is just a mathematical generalization of what we have learnt so far, and it makes sense.

2. This generalization allows us to conceptualize special relativity and apply it easily to physics problems broadly through mathematical formulation.

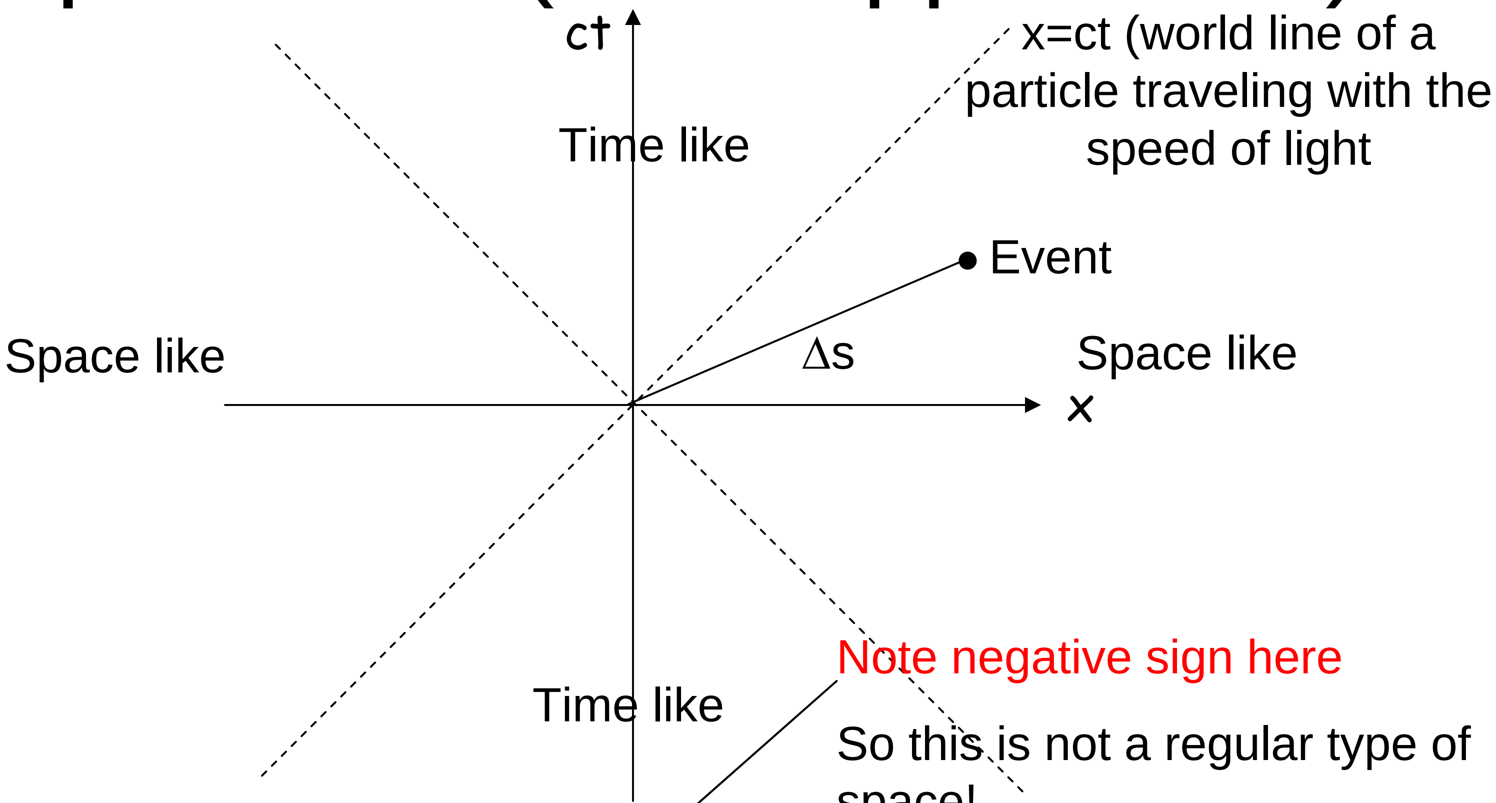
Spacetime (Text Appendix II)



In relativity, time is an independent dimension of the four dimensional space.

In Newtonian mechanics, time is a snap shot parameter.

Spacetime (Text Appendix II)



Note negative sign here

So this is not a regular type of space!

Pythagoras theorem:

$$\Delta s^2 = (c\Delta t)^2 - [(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2]$$

An event in the space like region cannot be correlated the origin by light signal. $(\Delta s)^2 > 0$ for time like event, and $(\Delta s)2 < 0$ for space like event.