Kittel (7th Edition). Chapter 1. Problem 3.

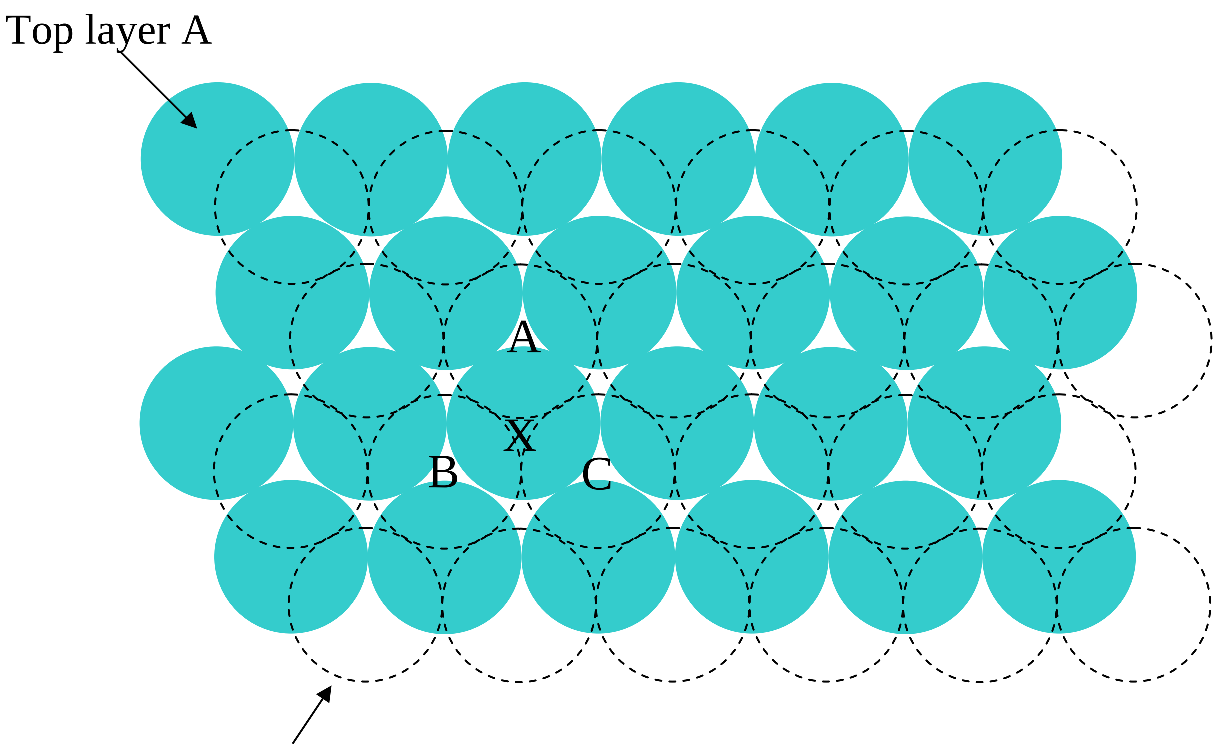
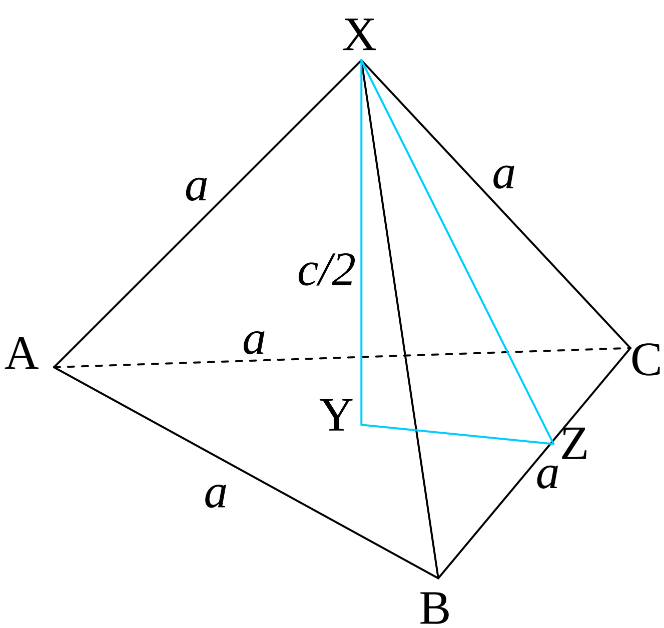


Figure 1.



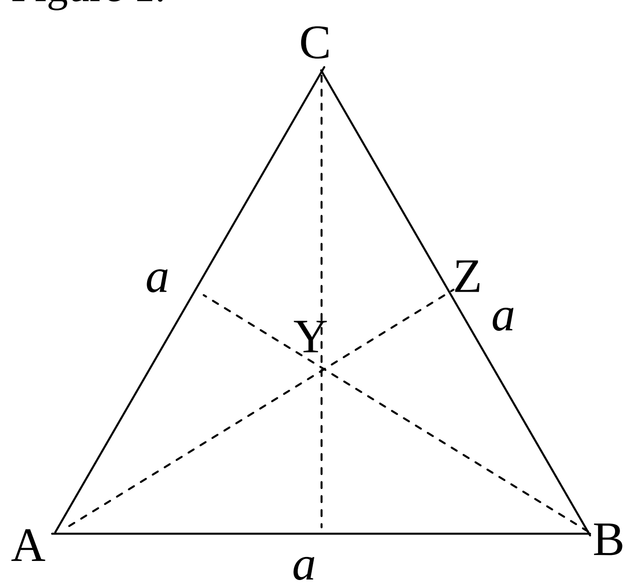


Figure 3.

Figure 2.

The top two layers of hcp are shown in figure 1. Center of the spheres locating at sites A, B, C, and X are shown in figure 2. For a perfect hcp, the spheres are in contact with each other, hence the tetrahedron in figure 2 is regular with each side equal to the lattice parameter *a*, which is two times the radius of the sphere. Figure 3 show the base of a tetrahedron.

Look at triangle XYZ in figure 2,

$$(\mathrm{XZ})^2 = (\mathrm{XY})^2 + (\mathrm{YZ})^2 \qquad \text{----(1)}$$

$$\mathrm{XY} = \frac{\mathrm{c}}{2} \qquad \text{----(2)}$$

$$\mathrm{XZ} = a\cos 30^{\mathrm{o}} = \frac{a\sqrt{3}}{2} \qquad \text{----(3)}$$

From figure 3,

$$\mathrm{YZ} = \frac{a}{2}\tan 30^{\mathrm{o}} = \frac{a}{2\sqrt{3}} \qquad \text{----(4)}$$

Substitute (2), (3), and (4) into (1):

$$\left(\frac{a\sqrt{3}}{2}\right)^2 = \left(\frac{\mathrm{c}}{2}\right)^2 + \left(\frac{a}{2\sqrt{3}}\right)^2 \Rightarrow \frac{3a^2}{4} = \frac{\mathrm{c}^2}{4} + \frac{a^2}{12}$$

$$\Rightarrow \frac{\mathrm{c}^2}{4} = \frac{3a^2}{4} - \frac{a^2}{12}$$

$$\Rightarrow 3\mathrm{c}^2 = 9a^2 - a^2$$

$$\Rightarrow 3\mathrm{c}^2 = 8a^2$$

$$\Rightarrow \mathrm{c}^2 = \frac{8}{3}a^2$$

$$\Rightarrow \mathrm{c} = \sqrt{\frac{8}{3}}a$$