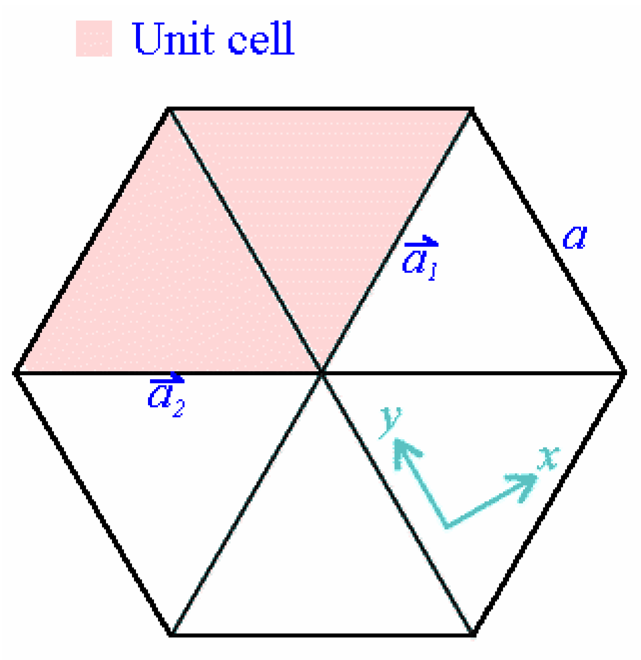
Kittel (7th Edition). Chapter 2. Problem 2.



(a) Area of the hexagonal base $= 2 \times \left(\dfrac{a \times \frac{\sqrt{3}}{2} a}{2} \right) = \dfrac{\sqrt{3}\, a^2}{2}$

$\therefore$ Volume of cell $=$ Area of base $\times$ height $= \dfrac{\sqrt{3}\, a^2 c}{2}$

$$(b)\ \vec{b}_1 = \frac{2\pi}{V}\vec{a}_2 \times \vec{a}_3 = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -\frac{\sqrt{3}a}{2} & \frac{a}{2} & 0 \\ 0 & 0 & c \end{vmatrix} = \frac{2\pi \times 2}{\sqrt{3}a^2 c}\left(\frac{ac}{2}\hat{x} + \frac{ac\sqrt{3}}{2}\hat{y}\right)$$

$$= \frac{2\pi}{\sqrt{3}a}\hat{x} + \frac{2\pi}{a}\hat{y}$$

$$\vec{b}_2 = \frac{2\pi}{V}\vec{a}_3 \times \vec{a}_1 = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & c \\ \frac{\sqrt{3}a}{2} & \frac{a}{2} & 0 \end{vmatrix} = \frac{2\pi \times 2}{\sqrt{3}a^2 c}\left(-\frac{ac}{2}\hat{x} + \frac{ac\sqrt{3}}{2}\hat{y}\right)$$

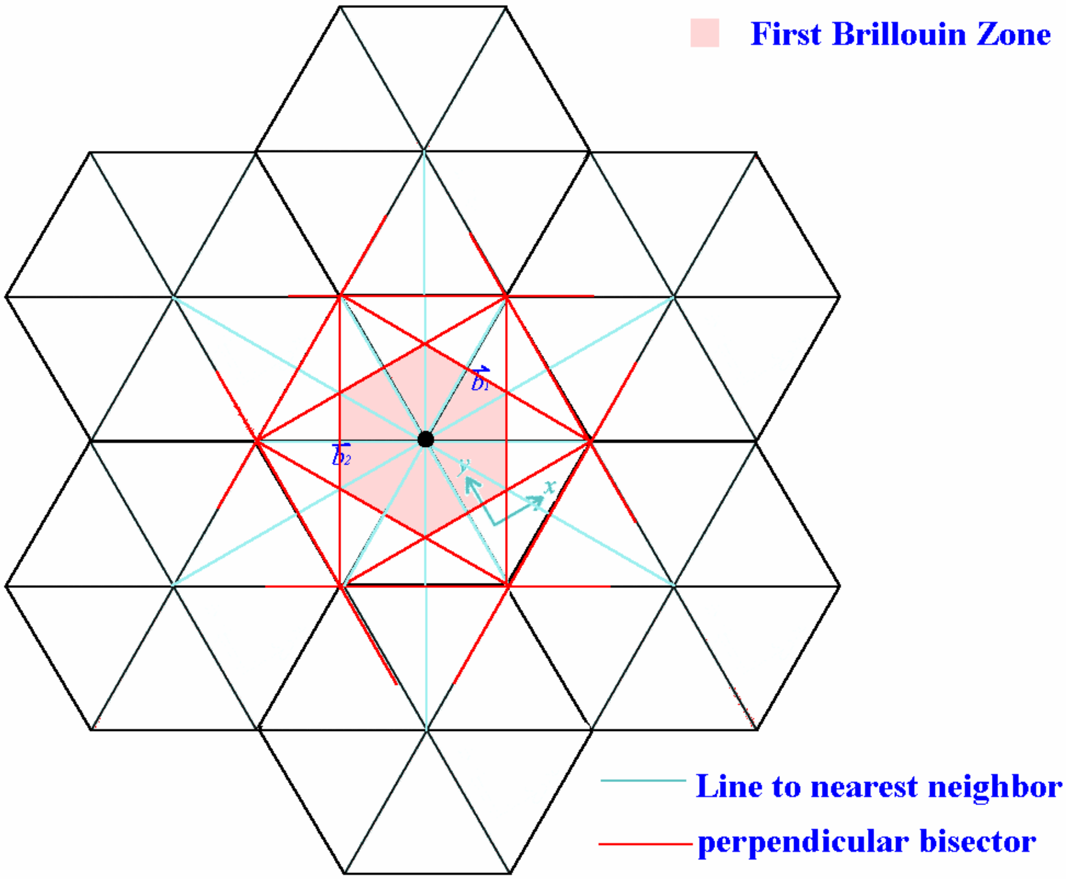
$$= -\frac{2\pi}{\sqrt{3}a}\hat{x} + \frac{2\pi}{a}\hat{y}$$

$$\vec{b}_3 = \frac{2\pi}{V}\vec{a}_1 \times \vec{a}_2 = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\sqrt{3}a}{2} & \frac{a}{2} & 0 \\ -\frac{\sqrt{3}a}{2} & \frac{a}{2} & 0 \end{vmatrix} = \frac{2\pi \times 2}{\sqrt{3}a^2 c}\left(\frac{\sqrt{3}a^2}{4} + \frac{\sqrt{3}a^2}{4}\right)\hat{z}$$

$$= \frac{2\pi}{c}\hat{z}$$

The reciprocal lattice is the same as the original lattice, with lattice parameter $\frac{4\pi}{a}, \frac{4\pi}{a}, \frac{2\pi}{c}$.

(c)


First Brillouin Zone
$\vec{b}_1$
$\vec{b}_2$
y
x
Line to nearest neighbor
perpendicular bisector