Kittel (8th Edition). Chapter 3. Problem 8.
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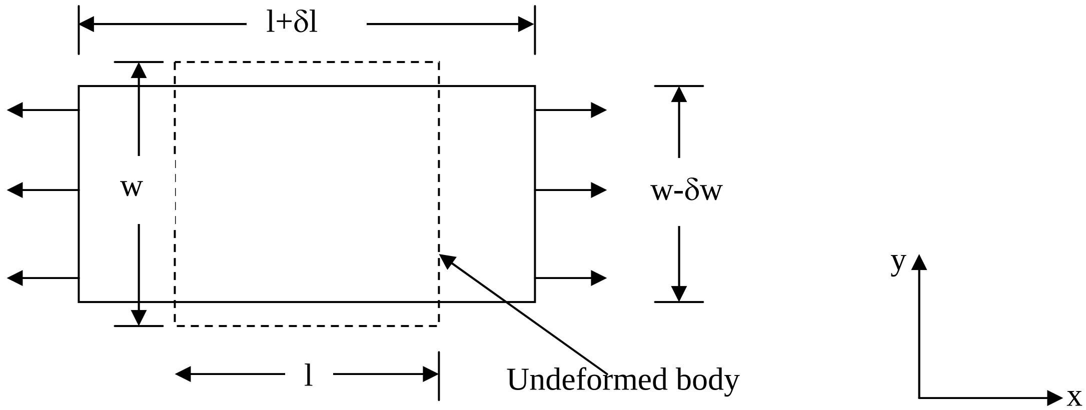

For cubic system,
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$$\begin{pmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \sigma_{yz}\\ \sigma_{zx}\\ \sigma_{xy}\end{pmatrix}=\begin{pmatrix}C_{11} & C_{12} & C_{12} & 0 & 0 & 0\\ C_{12} & C_{11} & C_{12} & 0 & 0 & 0\\ C_{12} & C_{12} & C_{11} & 0 & 0 & 0\\ 0 & 0 & 0 & C_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & C_{44} & 0\\ 0 & 0 & 0 & 0 & 0 & C_{44}\end{pmatrix}\begin{pmatrix}e_{xx}\\ e_{yy}\\ e_{zz}\\ e_{yz}\\ e_{zx}\\ e_{xy}\end{pmatrix}$$

For stress along [100] direction,

$$\begin{pmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \sigma_{yz}\\ \sigma_{zx}\\ \sigma_{xy}\end{pmatrix}=\begin{pmatrix}\sigma_{xx}\\ 0\\ 0\\ 0\\ 0\\ 0\end{pmatrix}\Rightarrow\begin{pmatrix}\sigma_{xx}\\ 0\\ 0\\ 0\\ 0\\ 0\end{pmatrix}=\begin{pmatrix}C_{11} & C_{12} & C_{12} & 0 & 0 & 0\\ C_{12} & C_{11} & C_{12} & 0 & 0 & 0\\ C_{12} & C_{12} & C_{11} & 0 & 0 & 0\\ 0 & 0 & 0 & C_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & C_{44} & 0\\ 0 & 0 & 0 & 0 & 0 & C_{44}\end{pmatrix}\begin{pmatrix}e_{xx}\\ e_{yy}\\ e_{zz}\\ e_{yz}\\ e_{zx}\\ e_{xy}\end{pmatrix}$$

$$\Rightarrow\begin{cases}C_{11}e_{xx}+C_{12}e_{yy}+C_{12}e_{zz}=\sigma_{xx}\\ C_{12}e_{xx}+C_{11}e_{yy}+C_{12}e_{zz}=0\\ C_{12}e_{xx}+C_{12}e_{yy}+C_{11}e_{zz}=0\\ e_{yz}=e_{zx}=e_{xy}=0\end{cases}$$

$$\begin{cases} C_{11}e_{xx} + C_{12}e_{yy} + C_{12}e_{zz} = \sigma_{xx} \\ C_{12}e_{xx} + C_{11}e_{yy} + C_{12}e_{zz} = 0 \\ C_{12}e_{xx} + C_{12}e_{yy} + C_{11}e_{zz} = 0 \\ e_{yz} = e_{zx} = e_{xy} = 0 \end{cases} \Rightarrow \begin{cases} C_{11} + C_{12}\dfrac{e_{yy}}{e_{xx}} + C_{12}\dfrac{e_{zz}}{e_{xx}} = \dfrac{\sigma_{xx}}{e_{xx}} \\ C_{12} + C_{11}\dfrac{e_{yy}}{e_{xx}} + C_{12}\dfrac{e_{zz}}{e_{xx}} = 0 \\ C_{12} + C_{12}\dfrac{e_{yy}}{e_{xx}} + C_{11}\dfrac{e_{zz}}{e_{xx}} = 0 \\ e_{yz} = e_{zx} = e_{xy} = 0 \end{cases}$$

$$\Rightarrow \begin{cases} C_{11} + C_{12}\dfrac{e_{yy}}{e_{xx}} + C_{12}\dfrac{e_{zz}}{e_{xx}} = \dfrac{\sigma_{xx}}{e_{xx}} \\ (C_{12}C_{11} - C_{12}{}^{2}) + (C_{11}{}^{2} - C_{12}{}^{2})\dfrac{e_{yy}}{e_{xx}} = 0 \\ (C_{12}{}^{2} - C_{12}C_{11}) + (C_{12}{}^{2} - C_{11}{}^{2})\dfrac{e_{zz}}{e_{xx}} = 0 \\ e_{yz} = e_{zx} = e_{xy} = 0 \end{cases}$$

$$\Rightarrow C_{11} - C_{12}\frac{(C_{12}C_{11} - C_{12}{}^{2})}{(C_{11}{}^{2} - C_{12}{}^{2})} - C_{12}\frac{(C_{12}{}^{2} - C_{12}C_{11})}{(C_{12}{}^{2} - C_{11}{}^{2})} = \frac{\sigma_{xx}}{e_{xx}}$$

$$\Rightarrow Y = \frac{\sigma_{xx}}{e_{xx}} = \frac{C_{11}(C_{11}{}^{2} - C_{12}{}^{2}) - C_{12}(C_{12}C_{11} - C_{12}{}^{2}) + C_{12}(C_{12}{}^{2} - C_{12}C_{11})}{(C_{11}{}^{2} - C_{12}{}^{2})}$$

$$\Rightarrow Y = \frac{C_{11}(C_{11}{}^{2} - C_{12}{}^{2}) - 2C_{12}(C_{12}C_{11} - C_{12}{}^{2})}{(C_{11}{}^{2} - C_{12}{}^{2})}$$

$$\Rightarrow Y = \frac{C_{11}{}^{3} - 3C_{11}C_{12}{}^{2} + 2C_{12}{}^{3}}{(C_{11}{}^{2} - C_{12}{}^{2})}$$

$$\Rightarrow Y = \frac{(C_{11} - C_{12})(C_{11}{}^{2} + C_{11}C_{12} - 2C_{12}{}^{2})}{(C_{11} - C_{12})(C_{11} + C_{12})}$$

$$\Rightarrow Y = \frac{C_{11}{}^{2} + C_{11}C_{12} - 2C_{12}{}^{2}}{C_{11} + C_{12}}$$

$$\text{Poisson ratio} = \frac{e_{yy}}{e_{xx}} = -\frac{C_{12}C_{11} - C_{12}{}^{2}}{C_{11}{}^{2} - C_{12}{}^{2}} = -\frac{C_{12}(C_{11} - C_{12})}{(C_{11} - C_{12})(C_{11} + C_{12})} = -\frac{C_{12}}{C_{11} + C_{12}}$$