Kittel (8th Edition). Chapter 3. Problem 11.

$$U = \frac{1}{2} C_{11} (e_{xx}{}^2 + e_{yy}{}^2 + e_{zz}{}^2) + \frac{1}{2} C_{44} (e_{xy}{}^2 + e_{yz}{}^2 + e_{zx}{}^2) + C_{12} (e_{xx} e_{yy} + e_{yy} e_{zz} + e_{zz} e_{xx})$$

If $e_{xx} = - e_{yy} = \frac{1}{2} e,\ e_{xy} = e_{yz} = e_{zx} = e_{zz} = 0$

$$U = \frac{1}{2} C_{11} (\frac{1}{4} e^2 + \frac{1}{4} e^2) + C_{12} (-\frac{1}{4} e^2) \Rightarrow U = \frac{1}{2}\left(\frac{1}{2} C_{11} - \frac{1}{2} C_{12}\right) e^2$$

$$\Rightarrow U = \frac{1}{2} C' e^2 \qquad \text{with } C' = \frac{1}{2}\left(C_{11} - C_{12}\right)$$

Note that the corresponding stress (to produce such a special strain) is given by:or cubic `

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{zx} \\ \sigma_{xy} \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{12} & C_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{44} \end{pmatrix} \begin{pmatrix} \frac{1}{2} e \\ -\frac{1}{2} e \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} e(C_{11} - C_{12}) \\ \frac{1}{2} e(C_{12} - C_{11}) \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} C' e \\ -C' e \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$