Problem.

Transform the stress components of a screw dislocation:

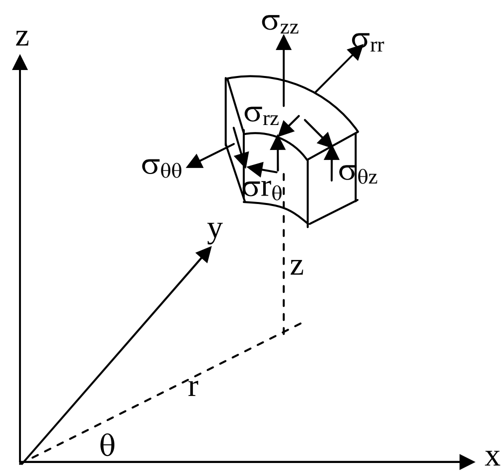
$$\sigma_{xz} = \frac{\mu b}{2\pi}\frac{y}{x^2 + y^2}$$

$$\sigma_{yz} = -\frac{\mu b}{2\pi}\frac{x}{x^2 + y^2}$$

$$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \sigma_{xy} = 0$$

from Cartesian to cylindrical coordinates ($r^2=x^2+y^2$, $\tan\theta = y/x$, $z=z$.).

Solution:



If the elastic displacement parallel to the r, θ, z directions are u_r, u_θ, u_z respectively.
Definition of strain with respect to the cylindrical element above:

$$e_{rr} = \frac{\partial u_r}{\partial r} \qquad e_{\theta\theta} = \frac{1}{r}\frac{\partial u_\theta}{\partial r} \qquad e_{zz} = \frac{\partial u_z}{\partial z}$$

$$e_{r\theta} = \frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r}$$

$$e_{rz} = \frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}$$

$$e_{z\theta} = \frac{1}{r}\frac{\partial u_z}{\partial \theta} + \frac{\partial u_\theta}{\partial z}$$

For screw dislocation, assume the dislocation line is along the z-axis, $u_r=r_\theta=0$ and $u_z= -\frac{b\theta}{2\pi}$. where b is the magnitude of Burgers vector in the z-direction.

Substitute these into the strain definition:

$$e_{rr} = \frac{\partial u_r}{\partial r} = 0 \quad e_{\theta\theta} = \frac{1}{r}\frac{\partial u_\theta}{\partial r} = 0 \quad e_{zz} = \frac{\partial u_z}{\partial z} = 0$$

$$e_{r\theta} = \frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} = 0$$

$$e_{rz} = \frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} = 0$$

$$e_{z\theta} = \frac{1}{r}\frac{\partial u_z}{\partial \theta} + \frac{\partial u_\theta}{\partial z} = \frac{1}{r}\frac{\partial}{\partial \theta}\left(-\frac{b\theta}{2\pi}\right) + 0 = -\frac{b}{2\pi r}$$

Following the notation we used in class, for isotropic materials:

$$\sigma_{rr} = (\lambda + 2\mu)e_{rr} + \lambda e_{\theta\theta} + \lambda e_{zz}$$

$$\sigma_{\theta\theta} = \lambda e_{rr} + (\lambda + 2\mu)e_{\theta\theta} + \lambda e_{zz}$$

$$\sigma_{zz} = \lambda e_{rr} + \lambda e_{\theta\theta} + (\lambda + 2\mu)e_{zz}$$

$$\sigma_{r\theta} = \mu e_{r\theta}$$

$$\sigma_{rz} = \mu e_{rz}$$

$$\sigma_{\theta z} = \mu e_{\theta z}$$

$\therefore$ For screw dislocation :

$$\sigma_{rr} = \sigma_{\theta\theta} = \sigma_{zz} = \underline{\underline{0}}$$

$$\sigma_{r\theta} = \sigma_{rz} = \underline{\underline{0}}$$

$$\sigma_{\theta z} = \mu e_{\theta z} = \underline{\underline{-\frac{b\mu}{2\pi r}}}$$