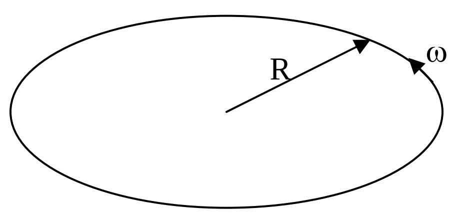
Chapter 11. Magnetism

Diamagnetism

1. Magnetisation **M** of diamagnetic material is *opposite* to the total magnetic field **B** (and applied field **H**), hence the magnetic susceptibility χ is *negative*. Magnetic susceptibility is defined as $\chi = \partial M/\partial B$.

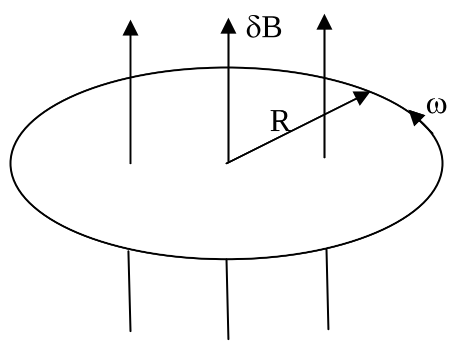
2. Diamagnetism is a characteristic of atoms with *closed shell*. Electrons will response to external field by Faraday's Law. Currents will arrange themselves to oppose the change of increasing field, and hence **M** is in the opposite direction of **H**.

3. Classical theory of diagmagnetism:
Consider an electron in a circular orbit of radius r and angular frequency ω.



Centripetal force $F = mR\omega_0^{\ 2}$

If a magnetic field δB is turned on, and assume it affect ω only.



$$mR\omega_0^{\ 2} + q\frac{v}{c}\delta B = \frac{mv^2}{R} \Rightarrow mR\omega_0^{\ 2} + q\frac{R\omega}{c}\delta B = mR\omega^2$$

$$\Rightarrow mR\omega_0^{\ 2} + q\frac{R(\omega_0 + \delta\omega)}{c}\delta B = mR(\omega_0 + \delta\omega)^2$$

$$\Rightarrow mR\omega_0^{\ 2} + q\frac{R(\omega_0 + \delta\omega)}{c}\delta B = mR(\omega_0^{\ 2} + 2\omega_0\delta\omega + \delta\omega^2)$$

Taking only first order term,

$$\therefore \quad q\frac{R\omega_0}{c}\delta B = 2mR\omega_0\delta\omega \quad \Rightarrow \quad \delta\omega = \frac{q\delta B}{2mc}$$

Classically, the B field will make the charge q revolve faster if q is positive. If the particle is an electron, it will revolve *slower*.

Current formed by the loop = I = Charge × revolution / second

$$\therefore\ I = q \cdot f = \frac{q\omega}{2\pi}$$

$$\delta I = \frac{q}{2\pi}\delta\omega$$

$$\therefore \text{For an electron in an atom,}\ \delta I = \frac{e}{2\pi}\cdot\frac{-e\delta B}{2mc}$$

$$= -\frac{e^2\delta B}{4\pi mc} \quad \text{(the negative sign gives rise to diamagnetism)}$$

$$\text{Magnetic moment of the atom} = \mu = \frac{1}{c}\sum_{i=1}^{Z} I_i A_i \qquad \text{(A = Area of the orbital loop)}$$

$$= \frac{\pi}{c}\sum_{i=1}^{Z} I_i {r_i}^2$$

$$= \frac{Z\pi}{c} I < R^2 >$$

$$\therefore \text{By appling a magnetic field}\ \delta B,$$

$$\delta\mu = \frac{Z\pi}{c}\delta I < R^2 > = \frac{Z\pi}{c}\left(-\frac{e^2\delta B}{4\pi mc}\right) < R^2 > = -\frac{Ze^2\delta B}{4mc^2} < R^2 >$$

R is the radius of the electron loops.

$$< R^2 > = < x^2 > + < y^2 >$$

If r = radius of the three dimensional electron shell,

$$< r^2 > = < x^2 > + < y^2 > + < z^2 >$$

If $< x^2 > = < y^2 > = < z^2 >$

$$\therefore < R^2 > = \frac{2}{3} < r^2 >$$

$$\delta\mu = \frac{2}{3}\cdot\left(-\frac{Ze^2\delta B}{4mc^2} < r^2 >\right) = -\frac{Ze^2\delta B}{6mc^2} < r^2 >$$

If N = Number of atoms per unit volume, and M = magnetization of the sample,

$$\therefore\ \delta M = N\,\delta\mu = -\frac{NZe^2\delta B}{6mc^2} < r^2 >$$

$$\chi = \frac{\delta M}{\delta B} = -\frac{NZe^2}{6mc^2} < r^2 > \qquad \text{- Langevin equation for diamagnetism}$$

Paramagnetism

1. When atoms possess their own magnetic moment, paramagnetism will occur.

2. Intrinsic magnetic moment if related to the total angular momentum (including orbital and spin) of the electrons in an atom.

$$\vec{\mu} = \gamma\hbar\vec{J} = -g\mu_B\vec{J}$$

γ is the gyromagnetic ratio, g is the g-factor, and μ_B is the Bohr magneton.

$$\mu_B = \frac{e\hbar}{2mc}$$

3. For the spin of a free electron, $\boxed{g=2}$. For the orbital momentum of an electron, $\boxed{g=1}$.

For a free atom, g is given by the Lande g-factor:

$$\vec{\mu} = -g\mu_B\vec{J}$$

In LS coupling,

$$\vec{\mu} = \sum_i -g_L\mu_B\vec{L}_i + \sum_i -g_s\mu_B\vec{S}_i$$

$$g_L = 1 \text{ and } g_s = 2$$

$$\therefore \vec{\mu} = -\mu_B\left(\sum_i \vec{L}_i + 2\sum_i \vec{S}_i\right)$$

On the other hand, $\vec{\mu} = -g\mu_B\vec{J}$

$$\therefore -\mu_B\left(\sum_i \vec{L}_i + 2\sum_i \vec{S}_i\right) = -g\mu_B\vec{J}$$

$$\Rightarrow \vec{L} + 2\vec{S} = g\vec{J} \qquad (\vec{L} = \sum_i \vec{L}_i, \vec{S} = \sum_i \vec{S}_i)$$

$$\Rightarrow \vec{L}\cdot\vec{J} + 2\vec{S}\cdot\vec{J} = g\vec{J}\cdot\vec{J}$$

$$\vec{S}^2 = (\vec{J} - \vec{L})^2 = \vec{J}^2 + \vec{L}^2 - 2\vec{L}\cdot\vec{J} \Rightarrow \vec{L}\cdot\vec{J} = \frac{1}{2}\left(\vec{J}^2 + \vec{L}^2 - \vec{S}^2\right)$$

Similarly,

$$\vec{L}^2 = (\vec{J} - \vec{S})^2 = \vec{J}^2 + \vec{S}^2 - 2\vec{S}\cdot\vec{J} \Rightarrow \vec{S}\cdot\vec{J} = \frac{1}{2}\left(\vec{J}^2 + \vec{S}^2 - \vec{L}^2\right)$$

Substitute these into the previous equation,

$$\therefore \frac{1}{2}\left(\vec{J}^2 + \vec{L}^2 - \vec{S}^2\right) + 2\cdot\frac{1}{2}\left(\vec{J}^2 + \vec{S}^2 - \vec{L}^2\right) = g\vec{J}\cdot\vec{J}$$

$$\Rightarrow \frac{3}{2}\vec{J}^2 + \frac{1}{2}\vec{S}^2 - \frac{1}{2}\vec{L}^2 = g\vec{J}^2$$

$$\Rightarrow \frac{3}{2}j(j+1) + \frac{1}{2}s(s+1) - \frac{1}{2}\ell(\ell+1) = gj(j+1)$$

$$\Rightarrow \boxed{g = \frac{3j(j+1) + s(s+1) - \ell(\ell+1)}{2j(j+1)} = 1 + \frac{j(j+1) + s(s+1) - \ell(\ell+1)}{2j(j+1)}}$$

4. Original quantum number: L,L_z, S, S_z.
Quantum number after LS coupling: L, S, J, J_z.
Atomic notation:

$$^{2S+1}L_j$$

L = S, P, D, F, G, H, …. for l = 0,1, 2, 3, 4, …. sespectively.

5. Quantum number j for the *total* angular momentum is determined by Hund's rule:
Let there be x electrons in the outer shell. Each shell can hold y electrons.
s shell can hold y=2 electrons.
p shell can hold y=6 electrons.
d shell can hold y=10 electrons.
f shell can hold y=14 electrons.
Draw y/2 boxes. Example, for d-shell:

(10/2=5 boxes)

Under each boxes, label L_z (according to the L of the shell) from *maximum to minimum*. Example, L=2 for d shell:

L_z:	+2	+1	0	-1	-2

Hund's rule #1 (how to fill up the boxes):
Always fill up boxes one by one *from left to right*. Do not double occupy the boxes until the shell is *half* full. Start to double occupy the boxes after the shell is half full, start from left to right again.

Hund's rule #2 (how to calculate L, S, and J):
S=ΣS_z, L=ΣL_z, J=|L-S| if shell is less than half full or half full, and J=L+S if shell is more than hall full or half full.

Esample 1:

7 electrons in d-shell

	↑↓	↑↓	↑	↑	↑
L_z:	+2	+1	0	-1	-2

L=ΣL_z = +2+1+0-1-2+2+1 =3 (or F)
S=ΣS_z = 1/2×5-1/2×2=3/2 (2S+1=4)
J=L+S =9/2
Ground level of the atom: $^4F_{9/2}$

6. There are 2j+1 sub-levels with J_z = -J, -J+1,…,-1,0,1,…,J-1, J. If we define the energy for J_z =0 as 0, then the energy of each of this state is given by

$$E_{J_z} = -\vec{\mu}\cdot\vec{B} = g\mu_B J_B B$$

The relative population in level J_z can be calculated as:

$$\frac{N_{J_z}}{N} = \frac{e^{-E_{J_z}/k_BT}}{\sum_{J_z=-J}^{J} e^{-E_{J_z}/k_BT}} = \frac{e^{-g\mu_B J_z B/k_BT}}{\sum_{J_z=-J}^{J} e^{-g\mu_B J_z B/k_BT}}$$

$$<m> = \sum_{J_z=-J}^{J} -g\mu_B J_z \frac{N_{J_z}}{N}$$

$$= \frac{\sum_{J_z=-J}^{J} -g\mu_B J_z e^{-g\mu_B J_z B/k_BT}}{\sum_{J_z=-J}^{J} e^{-g\mu_B J_z B/k_BT}}$$

Let $\alpha = -g\mu_B, \beta = g\mu_B B/k_BT$

$$<m> = \frac{\alpha \sum_{J_z=-J}^{J} J_z e^{-\beta J_z}}{\sum_{J_z=-J}^{J} e^{-\beta J_z}}$$

$$= \frac{-\alpha \frac{\partial}{\partial\beta} \sum_{J_z=-J}^{J} e^{-\beta J_z}}{\sum_{J_z=-J}^{J} e^{-\beta J_z}}$$

$$= -\alpha \frac{\partial}{\partial\beta} \ell n \left[\frac{e^{\beta(J+\frac{1}{2})} - e^{-\beta(J+\frac{1}{2})}}{e^{\frac{\beta}{2}} - e^{-\frac{\beta}{2}}} \right]$$

$$(x + ax + a^2x + \cdots + a^{n-1}x = x \cdot \frac{1-a^n}{1-a}, x = e^{\beta J}, a = e^{-\beta}, n = 2J+1)$$

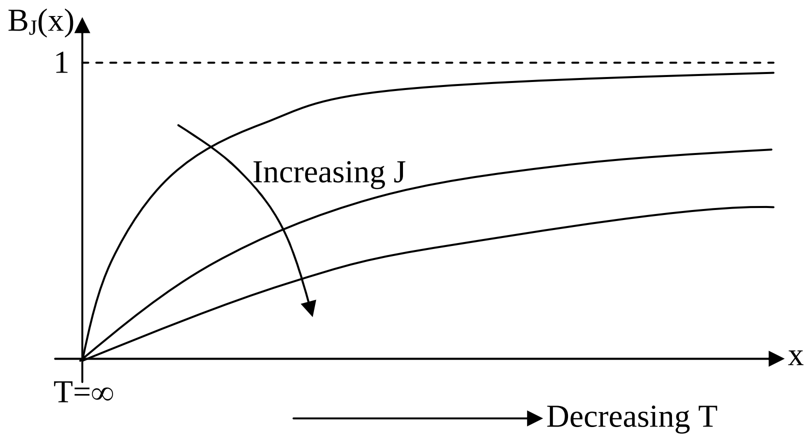
$$= -\alpha \frac{\partial}{\partial\beta} \left[\ell n \left(e^{\beta(J+\frac{1}{2})} - e^{-\beta(J+\frac{1}{2})} \right) - \ell n \left(e^{\frac{\beta}{2}} - e^{-\frac{\beta}{2}} \right) \right]$$

$$= -\alpha \frac{\partial}{\partial\beta} \left[\left(J + \frac{1}{2} \right) \left(\frac{e^{\beta(J+\frac{1}{2})} + e^{-\beta(J+\frac{1}{2})}}{e^{\beta(J+\frac{1}{2})} - e^{-\beta(J+\frac{1}{2})}} \right) - \frac{1}{2} \left(\frac{e^{\frac{\beta}{2}} + e^{-\frac{\beta}{2}}}{e^{\frac{\beta}{2}} - e^{-\frac{\beta}{2}}} \right) \right]$$

$$= -\alpha \left[\left(J + \frac{1}{2} \right) \coth \beta \frac{2J+1}{2} - \frac{1}{2} \coth \frac{\beta}{2} \right]$$

Define Brillouin function $B_J(x)$:

$$B_J(x) = \frac{2J+1}{2J} \coth \frac{2J+1}{2J} x - \frac{1}{2J} \coth \frac{1}{2J} x$$



We have

$$\begin{aligned} < m > &= -\alpha J\, B_J(\beta J) \\ &= g\mu_B J\, B_J(g\mu_B JB/k_B T) \end{aligned}$$

$$\therefore M = \frac{N}{V} < m > = \frac{N}{V} g\mu_B J\, B_J(g\mu_B JB/k_B T)$$

For small field, $g\mu_B JB << k_B T$ $\qquad$ $(T = 1\text{K for } B = 1\text{T})$

$$\coth x = \frac{e^x + e^{-x}}{e^x - e^{-x}} \approx \frac{1}{x} + \frac{1}{3} x + O(x^3) + \cdots$$

$$\begin{aligned} B_J(x) &= \frac{2J+1}{2J} \coth \frac{2J+1}{2J} x - \frac{1}{2J} \coth \frac{1}{2J} x \\ &\approx \left(\frac{1}{x} - \frac{1}{x} \right) + \left[\frac{1}{3} \frac{(2J+1)^2}{(2J)^2} x - \frac{1}{3} \frac{1}{(2J)^2} x \right] \\ &\approx \frac{(4J^2 + 4J)}{3(2J)^2} x \\ &\approx \frac{J+1}{3J} x \qquad \text{for small } x. \end{aligned}$$

$$\therefore M \approx \frac{NB}{3Vk_B T} (g\mu_B)^2 J(J+1) \qquad \text{for small field } B$$

$$\chi = \frac{\partial M}{\partial B} \approx \frac{N}{3Vk_B T}(g\mu_B)^2 J(J+1) \qquad \text{for small field B}$$

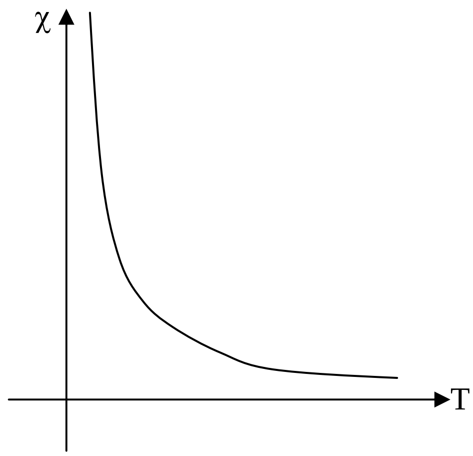
$$= \frac{N}{V}\frac{p^2 \mu_B{}^2}{3k_B T}$$

p = effective Bohr magnetron number

$$= g[J(J+1)]^{\frac{1}{2}}$$

Susceptibility of paramagnetism follows *Curie Law*:

$$\chi \approx \frac{C}{T}$$



C is the Curie constant,

$$C = \frac{N}{V}\frac{p^2 \mu_B{}^2}{3k_B}$$

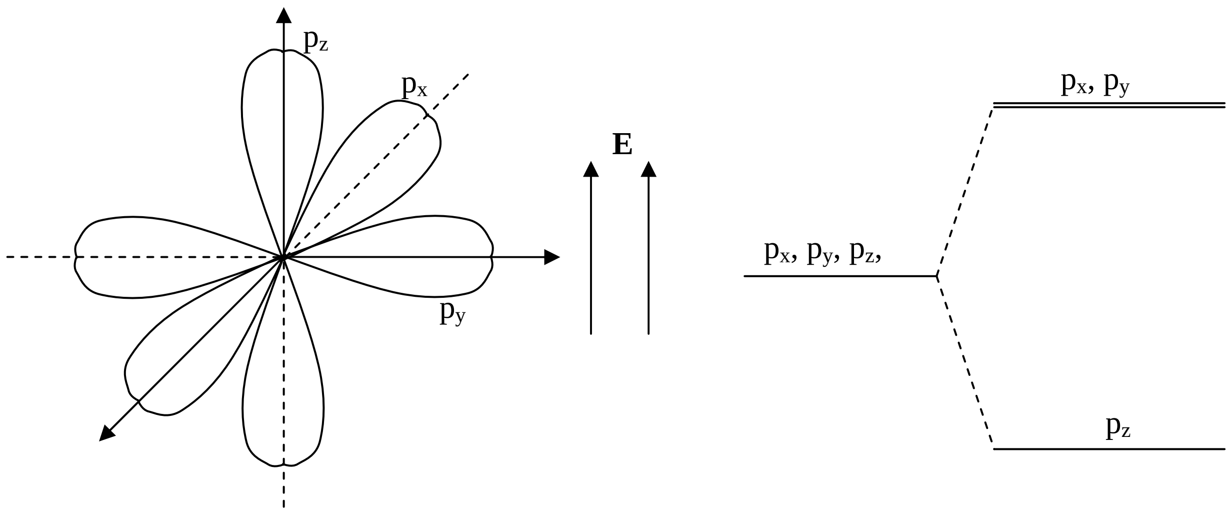
7. J, S and J can be estimated from Hund's rule. After J is estimated, g can be calculated with the Lande equation and the p (effective Bohr magnetron number) is known. The estimated value can be compared with experimental value. Notes:

(i) Hund's rule works well for most rare earth (4f electrons), and the calculated p is very close to the measured value. In some cases like Er^{3+} (Europium) and Sm^{3+} (Samarium), energy between the j-multiplets is too small and will cause problem in the 2nd order perturbation.

(ii) Hund's rule does not work fine for transition metals (d electrons). In case of rare earth, the 4f electrons are deep inside the ion and well covered by the 5s and 5p shells. This is not the case for the transition metals. The d-electrons are actually extended further out and exposed to the fields from the neighbors (crystal field). This crystal field will affect the LS coupling and modify Hund's rule in calculating J.

(iii) Crystal field will not couple with S, because spin hjas no real space variables in it. However, the crystal field potential will couple with L. It will split the original degenerate l-orbtals >> μB.

Example of crystal field splitting (p-orbitals):



(iv) Under the crystal field slitting, L_z is not a good quantum number any more. On average over time, $\langle L_z \rangle = 0$. Therefore, for transition metal, p (with 2 electrons in shell) should be calculated as $g[s(s+1)]^{1/2} = 2[s(s+1)]^{1/2}$ instead of $g[j(j+1)]^{1/2}$, since L does not contribute to magnetic properties.

(v) For splitting of *all* degenerate orbitals, the crystal field cannot be symmetric, Very often, if the crystal is high symmetric (e.g. cubic), the ions will displaced themselves to produce a non-symmetric crystal potential to quench the angular momentum. This is called Jahn-Teller effect,

Pauli paramagnetism

1. Electron has spin, so free electrons demonstrate paramagnetic property, This is known as *Pauli paramagnetism*.

2. The effect of Pauli paramagnetism is very small, becayse electrons inside the Fermi sphere cannot flip their spins easily when nearly all states are occupied. Only electrons near the Fermi surface can contribute to Pauli paramagnetism. According to Curie Law:

$$\chi = \frac{C}{T} \qquad \text{for small field}$$

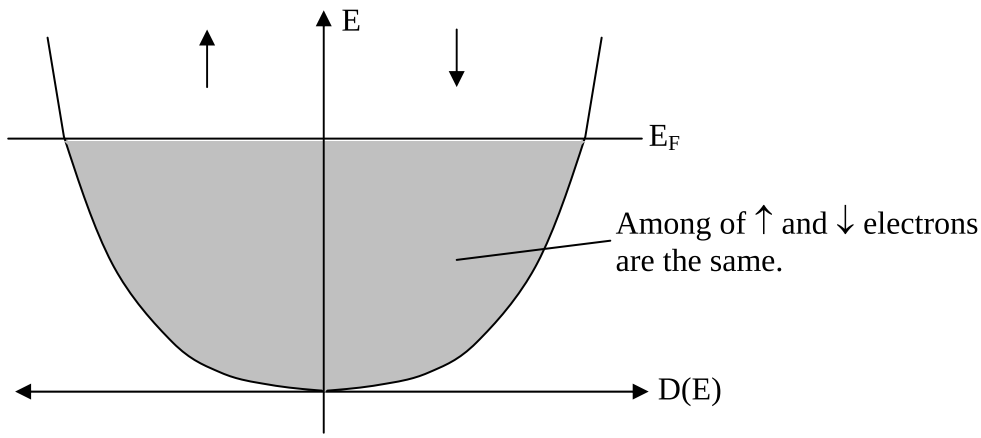
Percentage of electrons that have the freedom to flip spin $=T/T_F$.

$$\therefore \chi \text{ for metal} = \frac{C}{T} \cdot \frac{T}{T_F} = \frac{C}{T_F}$$

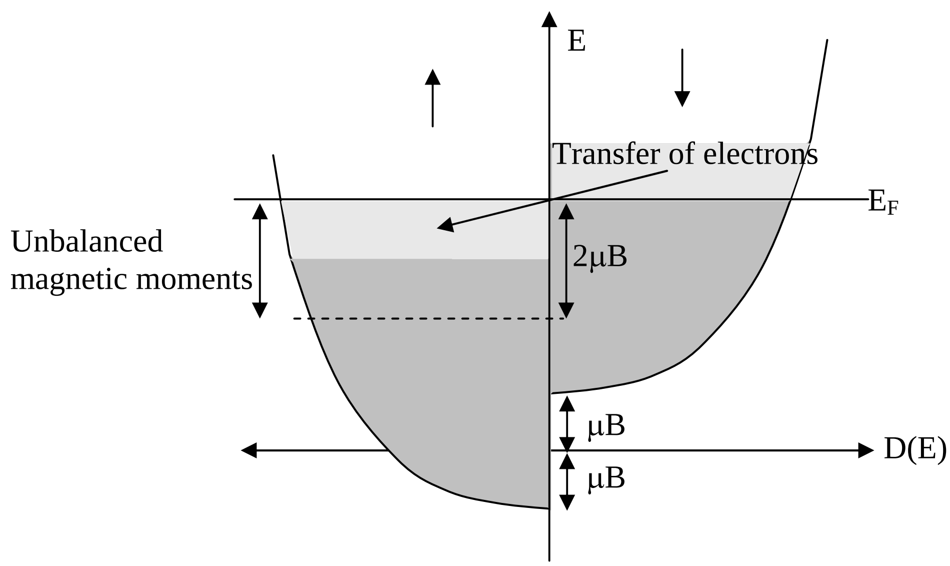
Pauli paramagnetism is *independent of temperature*.

3. More quantitative treatment:

When there is no field:



When an external field B is applied, say, in the ↑ direction, it will lower the energy of the ↑ electrons by μB and raise the energy of the ↓ electrons by μB



$$N_{\uparrow} = \frac{1}{2}\int_{-\mu B}^{E_F} dE\, D(E+\mu B) \approx \frac{1}{2}\int_{0}^{E_F} dE D(E) + \frac{1}{2}\mu B D(E_F)$$

$$N_{\downarrow} = \frac{1}{2}\int_{\mu B}^{E_F} dE\, D(E-\mu B) \approx \frac{1}{2}\int_{0}^{E_F} dE D(E) - \frac{1}{2}\mu B D(E_F)$$

$$\therefore M_{Pauli} = \mu\,(N_{\uparrow} - N_{\downarrow}) = \mu^2 B D(E_F)$$

$$= \mu^2 B \cdot \frac{3N}{2E_F} = \frac{3N\mu^2}{2k_B T_F} B$$

This treatment has ignored the spatial effect of magnetic field. In fact, the magnetic field can modify the electron wave function and produces diamagnetism. This diamagnetism is about 1/3 of the above estimated paramagnetism in magnitude:

$$M_{dia} = -\frac{1}{3} M_{Pauli} = -\frac{N\mu^2}{2k_B T_F} B$$

$$\therefore \text{Total magnetism} = M_{Pauli} + M_{dia} = \frac{N\mu^2}{k_B T_F} B$$

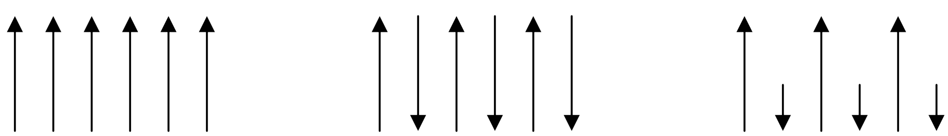
$$\boxed{\chi_{freeelectron} = \frac{\partial M}{\partial B} = \frac{N\mu^2}{k_B T_F}}$$

Long range magnetic ordering

1. Long range magnetic ordering is due to *exchange field* $\mathbf{B}_E$ from neighbors. In other words, we assume an exchange field between neighbors that gives rise to long range ordering.

2. Magnetic ordered states has higher symmetry and it occurs only at low temperatures when $T < T_c$. T_c is the critical temperature of the magnetic transition.

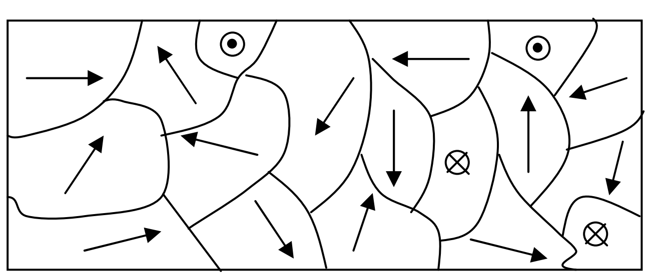
3. Three common types of magnetic ordering:



Ferromagnetic ordering   Antiferromagnetic ordering   Ferrimagnetic ordering

4. It is clear from above schematic drawings that ferromagnetism and ferrimagnetism will give rise to *spontaneous* magnetisation then ordering occurs at $T<T_c$. The antiferromagnetism will not produce any magneisation because of the two opposing spin components. When $T>T_c$ there will be no ordering and the material has to be paramagnetic (i.e. the ions should have their own spin at the beginning).

Example: T_c for iron (Fe) is 1043K. Iron is actually ferromagnetic possessing ordering and spontaneous magnetisation at room temperature. It is not a magnet because of domain formation.



Total magnetic moment = 0

Ferromagnetism

1. The exchange field $\mathbf{B}_E$ is approximated by the average magnetization field $\mathbf{M}$ within the sample:

$$\mathbf{B}_E = \lambda\, \mathbf{M}$$

where λ is a temperature independent constant. This is known as *the mean field approximation*. Note that now the exchange field will become stronger as temperature is lowered, because that is what $\mathbf{M}$ does according to Curie Law.

2. When $T>T_c$ – Curie Weiss Law and relationship between λ and T_c:

If $\mathbf{B}_a$ = applied field and χ_P= paramagnetic susceptibility.

$$\vec{M} = \chi_P(\vec{B}_a + \vec{B}_E)$$

$$= \frac{C}{T}(\vec{B}_a + \vec{B}_E) \qquad (\chi_P = \frac{C}{T})$$

$$\text{Mean field approximation} \Rightarrow \ = \frac{C}{T}(\vec{B}_a + \lambda\vec{M})$$

$$\Rightarrow\ (1 - \frac{\lambda C}{T})\vec{M} = \frac{C}{T}\vec{B}_a$$

$$\Rightarrow \chi = \frac{\vec{M}}{\vec{B}_a} = \frac{C}{T - C\lambda}$$

As $T \to T_c$, χ has to $\to \infty$ so that $\vec{M}$ is finite (spon tan eous magnetisation) when $\vec{B}_a = 0$.

$$\text{i.e.} \quad T_c - C\lambda = 0 \qquad \Rightarrow\ \lambda = \frac{T_c}{C}$$

$$\boxed{\chi = \frac{C}{T - T_c}} \qquad \text{Curie - Weiss Law for } T > T_c$$

$$\text{More accurate renormalization group theoury gives } \chi \propto \frac{C}{(T - T_c)^{1.33}}$$

$$\text{For } j = \frac{1}{2} \text{ and } g = 2 \text{ (hence } p = \sqrt{3}),\ C = \frac{N}{V}\frac{p^2\mu_B{}^2}{3k_B} = \frac{N}{V}\frac{\mu_B{}^2}{k_B} \Rightarrow T_c = \lambda C = \frac{N\mu_B{}^2\lambda}{k_B V}$$

3. When $T>T_c$- calculation of spontaneous magnetisation:

$\mathbf{B}_E$ is so strong that $\mathbf{B}_a$ can be ignored. i.e., $\mathbf{B}= \mathbf{B}_a + \mathbf{B}_E \sim \mathbf{B}_E$. For simplicity, lwt us consider j=1/2 (2 levels), and g=2.

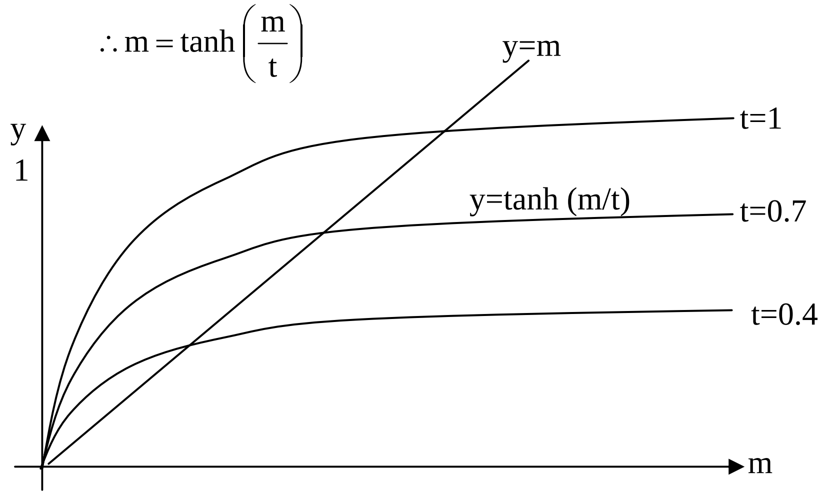
$$M\ =\ -\alpha\left[\left(J + \frac{1}{2}\right)\coth\beta\frac{2J+1}{2} - \frac{1}{2}\coth\frac{\beta}{2}\right] = g\mu_B\left[\coth\beta - \frac{1}{2}\coth\frac{\beta}{2}\right]$$

$$= \mu_B\left[2\coth\beta - \coth\frac{\beta}{2}\right]$$

$$= \mu_B \left[2\frac{e^{\beta} + e^{-\beta}}{e^{\beta} - e^{-\beta}} - \frac{e^{\beta/2} + e^{-\beta/2}}{e^{\beta/2} - e^{-\beta/2}} \right]$$
$$= \mu_B \left[2\frac{e^{\beta} + e^{-\beta}}{e^{\beta} - e^{-\beta}} - \frac{(e^{\beta/2} + e^{-\beta/2})^2}{e^{\beta} - e^{-\beta}} \right]$$
$$= \mu_B \left[\frac{e^{\beta} + e^{-\beta} - 2}{e^{\beta} - e^{-\beta}} \right]$$
$$= \mu_B \left[\frac{e^{\beta/2} - e^{-\beta/2}}{e^{\beta/2} + e^{-\beta/2}} \right]$$
$$= \mu_B \tanh \beta/2$$
$$= \mu_B \tanh \mu_B B/k_B T \quad (\beta = g\mu_B B/k_B T = 2\mu_B B/k_B T)$$
$$= \mu_B \tanh \lambda\mu_B M/k_B T \qquad (B \sim \lambda M)$$

Let $m = \dfrac{MV}{N\mu_B}$ and $t = \dfrac{k_B TV}{N{\mu_B}^2\lambda}$ $\qquad \left(\dfrac{m}{t} = \dfrac{MV}{N\mu_B} \cdot \dfrac{N{\mu_B}^2\lambda}{k_B TV} = \dfrac{M\mu_B\lambda}{k_B T}\right)$

$$\therefore m = \tanh\left(\frac{m}{t}\right)$$



At T=T$_c$, or $t = \dfrac{k_B T_c V}{N{\mu_B}^2\lambda}$, there is no solution because spontaneous magnetization ceased to exist.

At small m,

$$\tanh\left(\frac{m}{t}\right) \sim \left(\frac{m}{t}\right)$$

This curve y ~ m/t will have no solution with y=m (except at m=0) when

$$\left(\frac{m}{t}\right) = m \Rightarrow t = 1$$

$$\therefore \quad \frac{k_B T_c V}{N{\mu_B}^2\lambda} = 1 \Rightarrow T_c = \frac{N{\mu_B}^2\lambda}{k_B V}$$

This is consistent with the result we derived from the side $T>T_c$.

4. Spontaneous magnetization neat T_c:
As $T \to T_c$, m is small. Expansion of tanh x for small x:

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{2(x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots)}{2(1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots)}$$

$$\approx (x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots)(1 - \frac{x^2}{2!} + \cdots)$$

$$\approx x + \frac{x^3}{3!} - \frac{x^3}{2!} + O(x^5)$$

$$\approx x - \frac{x^3}{3}$$

$$m = \tanh\left(\frac{m}{t}\right) \Rightarrow m = \frac{m}{t} - \frac{1}{3}\left(\frac{m}{t}\right)^3$$

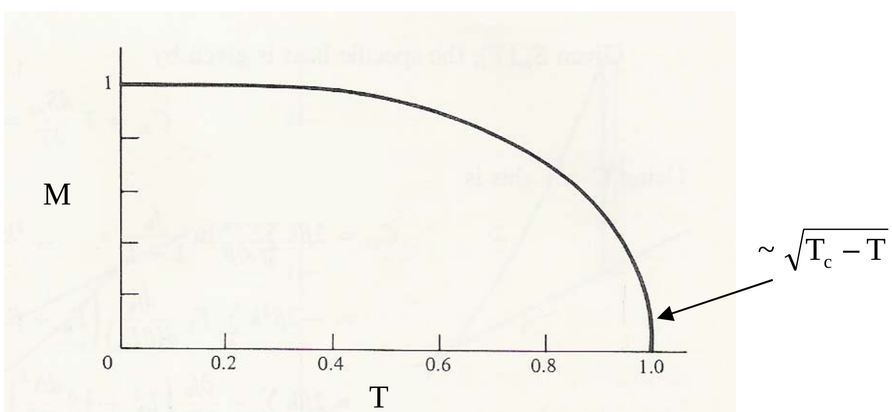
$$\Rightarrow m^2 = 3t^2(1-t)$$

$$\Rightarrow m^2 = 3\frac{T^2}{T_c^3}(T_c - T)$$

$$\therefore \quad M = \frac{Nm}{V}\mu_B \propto \sqrt{T_c - T} \qquad (T \to T_c)$$

More accurate renormalization theoy gives

$$M \propto (T_c - T)^{0.33}$$



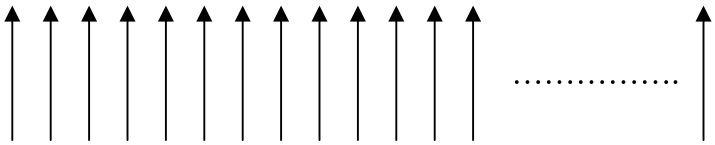
Note that the behavior of M is similar to that of Δ in the case of syuperconductivity at $T \sim T_c$.

5. Low temperature excitations – magnons:

Consider N spins coupled to their neighbors:

$$U = -2J\sum_{i=1}^{N} \vec{S}_i \cdot \vec{S}_{i+1} \qquad \text{(Heisenberg interaction)}$$

U is minimum when all spins are parallel (ground state) at T=0:



Ground state

$$U = -2J\,NS$$

If we consider the j-th spin in antiparallel to the others,

$$U = -2J\left[\sum_{i=1}^{j-2} \vec{S}_i \cdot \vec{S}_{i+1} + \sum_{i=j+1}^{N} \vec{S}_i \cdot \vec{S}_{i+1} - \vec{S}_{j-1}\cdot\vec{S}_j - \vec{S}_j \cdot \vec{S}_{j+1}\right]$$

$$= -2JS^2\left[(N-2)-2\right]$$

$$= -2NJS^2 + 8JS^2$$

$$= U_0 + 8JS^2$$

This will raise the system energy by an amount of $8JS^2$. This excitation energy will be smaller if we allow this antiparallel spin to be shared by all members of the system – formation of *magnons*.

Consider the j-th spin in the system:

$$U_j = -2J\left[\vec{S}_{j-1}\cdot\vec{S}_j + \vec{S}_j \cdot \vec{S}_{j+1}\right]$$

$$= -2J\vec{S}_j \cdot \left[\vec{S}_{j-1} + \vec{S}_{j+1}\right]$$

$\mathbf{S}_j$ is related to its magnetic moment as

$$\vec{\mu}_j = -g\mu_B \vec{S}_j$$

$$\therefore \quad U_j = -\vec{\mu}_j \left\{-\frac{2J}{g\mu_B}\cdot\left[\vec{S}_{j-1} + \vec{S}_{j+1}\right]\right\}$$

The term in { } can be identified as tne exchange field acting on the jth - spin as

$$\vec{B}_{Ej} = -\frac{2J}{g\mu_B}\cdot\left[\vec{S}_{j-1} + \vec{S}_{j+1}\right]$$

Torque acting on the j - th spin $= \vec{\mu}_j \times \vec{B}_{Ej}$

$$\therefore \quad \frac{d}{dt}(\hbar\vec{S}_j) = \vec{\mu}_j \times \vec{B}_{Ej} \;\Rightarrow\; \frac{d}{dt}(\vec{S}_j) = -\frac{g\mu_B}{\hbar}\vec{S}_j \times \vec{B}_{Ej}$$

$$\Rightarrow \frac{d}{dt}(\vec{S}_j) = \frac{2J}{\hbar}\vec{S}_j \times (\vec{S}_{j-1} + \vec{S}_{j+1})$$

$$\therefore \frac{d}{dt}(S_j^{\ x}) = \frac{2J}{\hbar}[S_j^{\ y}(S_{j-1}^{\ \ z} + S_{j+1}^{\ \ z}) - S_j^{\ z}(S_{j-1}^{\ \ y} + S_{j+1}^{\ \ y})]$$

Assume $\mathbf{S}_j$ is not off aligned with the other spins so that S_j^x, $S_j^y << S_j^z$. We can ignor second order terms like $S_j^xS_j^y$ and approximate S_j^z as S. The equations of the j-th spin can the be written as:

$$\frac{d}{dt}(S_j^{\ x}) = \frac{2JS}{\hbar}[2S_j^{\ y} - (S_{j-1}^{\ \ y} + S_{j+1}^{\ \ y})]$$

$$\frac{d}{dt}(S_j^{\ y}) = -\frac{2JS}{\hbar}[2S_j^{\ x} - (S_{j-1}^{\ \ x} + S_{j+1}^{\ \ x})]$$

$$\frac{d}{dt}(S_j^{\ z}) = 0$$

Trial solution :

$$S_j^{\ x} = ue^{i(jka-\omega t)}$$

$$S_j^{\ y} = ve^{i(jka-\omega t)}$$

u, v are constants, amplitude that measure the maximum deviation of the spin. Substitute these trial solutions into the differential equations:

$$-i\omega ue^{i(jka-\omega t)} = \frac{2Js}{\hbar}[2e^{i(jka-\omega t)} - e^{i[(j-1)ka-\omega t]} - e^{i[(j+1)ka-\omega t]}]$$

$$\Rightarrow \ -i\omega u = \frac{2Js}{\hbar}v[2 - e^{-ika} - e^{ika}]$$

$$\Rightarrow \ -i\omega u = \frac{4Js}{\hbar}v[1 - \cos ka]$$

Similarly, from the equation for $\frac{d}{dt}(S_j^{\ y})$:

$$-i\omega v = -\frac{4Js}{\hbar}u[1 - \cos ka]$$

u, v have non – trivial solution only if {

$$\begin{vmatrix} i\omega & \frac{4Js}{\hbar}[1-\cos ka] \\ -\frac{4Js}{\hbar}[1-\cos ka] & i\omega \end{vmatrix} = 0$$

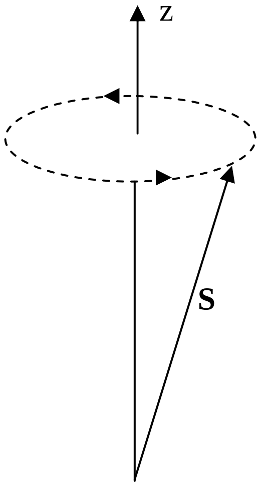
$$\Rightarrow \ \ \omega^2 = \left[\frac{4Js}{\hbar}[1-\cos ka]\right]^2$$

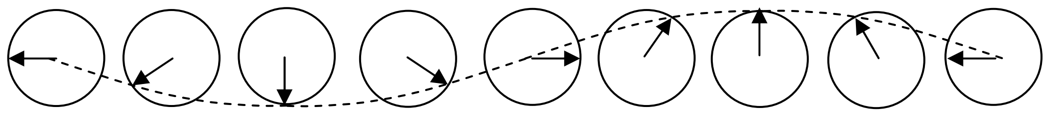
$$\Rightarrow \ \hbar\omega = 4Js(1-\cos ka)$$

If this condition is satisfy, solving for u and v :

$$\left.\begin{matrix} -i\omega u = \omega v \\ -i\omega v = -\omega u \end{matrix}\right\} \Rightarrow \ v = -iu$$

$\therefore$ S_j^x and S_j^y are 90o out of phase with equal amplitude. i.e. The spin is precessing circularly about the z-axis:





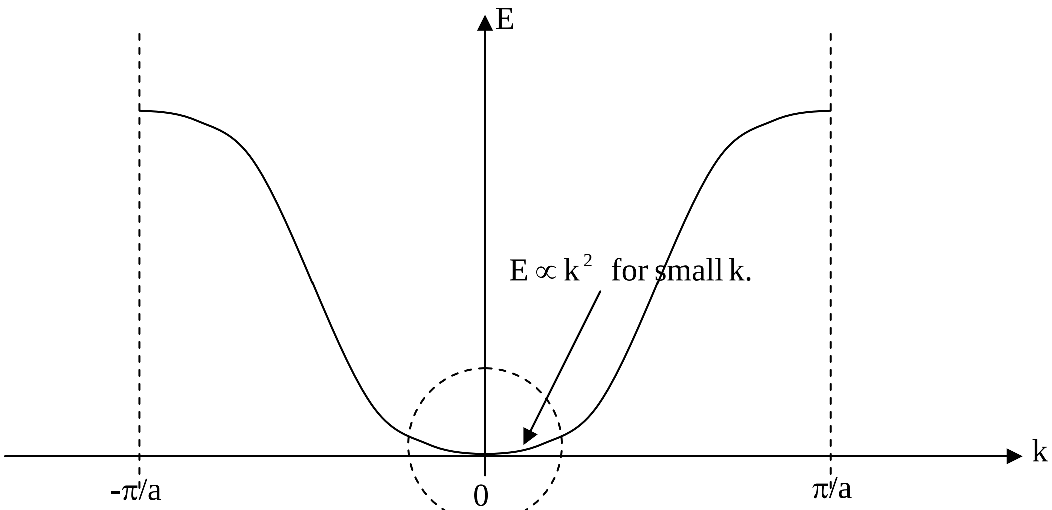
"Normal mode" of spin dynamics

Spin dynamics is "quantized" into magnons, each of energy $\hbar\omega$. Any spin configuration can be expressed as combination of these magnons.
Dispersion relation of magnons:

$$\hbar\omega = 4Js(1-\cos ka)$$
$$= 8Js\sin^2 \tfrac{1}{2}ka$$

For small k,

$$\hbar\omega \sim 8Js\cdot(\tfrac{1}{2}ka)^2 = 2Jsk^2a^2$$
$$\therefore \boxed{E \propto k^2 \text{ for small k.}}$$



6. Thermal excitations of magnons:
We can derive the thermal properties of magnons from the dispersion relationship. Similar to the case of phonons, magnons are bosons:

$$< n_{\bar{k}} > = \frac{1}{\exp(\hbar\omega_{\bar{k}} / k_B T) - 1}$$

Each magnon has a spin of s. 1 magnon corresponds to 1 spin flip out of Ns.

$$\therefore M(T) = M(0)[1 - \frac{\text{Number of magnons}}{Ns}]$$

$$= M(0)[1 - \frac{\sum n_{\bar{k}}}{Ns}]$$

$$\frac{\Delta M}{M(0)} = \frac{M(0) - M(T)}{M(0)} = \frac{\sum n_{\bar{k}}}{Ns}$$

$$\sum n_{\bar{k}} = \int d\omega D(\omega) < n(\omega) > = \int d\omega D(\omega) \frac{1}{\exp(\hbar\omega_{\bar{k}} / k_B T) - 1}$$

Approximate the whole band by that of small k :

$$\hbar\omega = 2Jsk^2a^2 \Rightarrow \frac{d\omega}{dk} = \frac{4Jska^2}{\hbar} = \frac{4Jsa^2}{\hbar}\sqrt{\frac{\hbar\omega}{2Jsa^2}} = 2\sqrt{\frac{2Jsa^2\omega}{\hbar}}$$

$$\therefore D(\omega)d\omega = \frac{4\pi k^2 dk}{\frac{(2\pi)^3}{V}} \Rightarrow D(\omega) = \frac{4\pi k^2}{\frac{(2\pi)^3}{V}} \frac{dk}{d\omega} = \frac{Vk^2}{2\pi^2}\frac{dk}{d\omega}$$

$$\Rightarrow D(\omega) = \frac{Vk^2}{2\pi^2} \cdot \frac{1}{2}\sqrt{\frac{\hbar}{2Jsa^2\omega}}$$

$$\Rightarrow D(\omega) = \frac{V}{2\pi^2} \cdot \frac{\hbar\omega}{2Jsa^2} \cdot \frac{1}{2}\sqrt{\frac{\hbar}{2Jsa^2\omega}}$$

$$\Rightarrow D(\omega) = \frac{V}{4\pi^2} \cdot \left(\frac{\hbar}{2Jsa^2}\right)^{\frac{3}{2}} \sqrt{\omega}$$

$$\therefore \sum n_{\bar{k}} = \int d\omega \left[\frac{V}{4\pi^2} \cdot \left(\frac{\hbar}{2Jsa^2}\right)^{\frac{3}{2}} \sqrt{\omega}\right] \frac{1}{\exp(\hbar\omega_{\bar{k}} / k_B T) - 1}$$

$$= \frac{V}{4\pi^2} \cdot \left(\frac{\hbar}{2Jsa^2}\right)^{\frac{3}{2}} \int_0^\infty d\omega \frac{\sqrt{\omega}}{\exp(\hbar\omega_{\bar{k}} / k_B T) - 1}$$

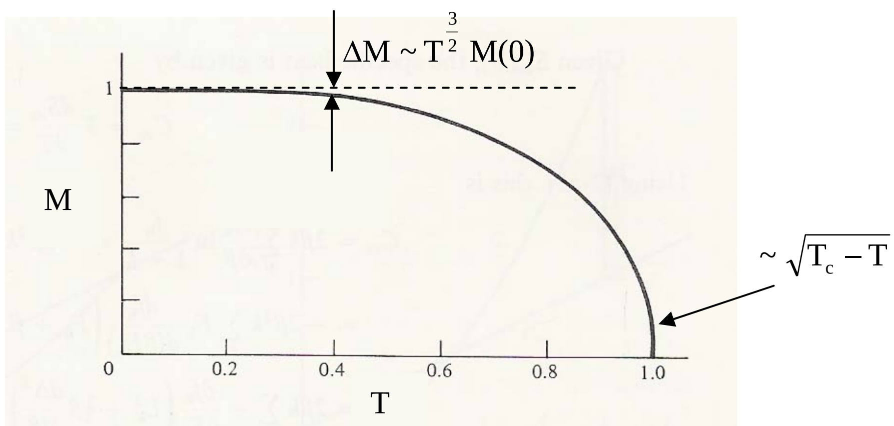
Let $\frac{\hbar\omega}{k_B T} = x$ and $\sqrt{\omega} = \sqrt{\frac{k_B T x}{\hbar}}$, $d\omega = \frac{k_B T}{\hbar} dx$

$$\sum n_{\bar{k}} = \frac{V}{4\pi^2} \cdot \left(\frac{k_B T}{2Jsa^2}\right)^{\frac{3}{2}} \underbrace{\int_0^\infty dx \frac{\sqrt{x}}{e^x - 1}}_{=(0.0587)(4\pi^2) = A\,4\pi^2}$$

$$\therefore \sum n_{\vec{k}} = AV\left(\frac{k_B T}{2Jsa^2}\right)^{\frac{3}{2}}$$

$$\frac{\Delta M}{M(0)} = \frac{\sum n_{\vec{k}}}{Ns} = \frac{0.0587V}{Ns}\left(\frac{k_B T}{2Jsa^2}\right)^{\frac{3}{2}} = \frac{A}{ns}\left(\frac{k_B T}{2Jsa^2}\right)^{\frac{3}{2}} \qquad (n = \frac{N}{V})$$

$$\therefore \boxed{\frac{\Delta M}{M(0)} \sim T^{\frac{3}{2}}}$$ at low temperature.

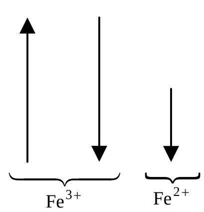


Ferrimagnetic order

1. Example:
   Magnetite

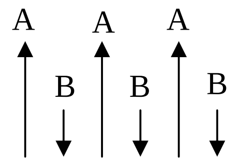
   Fe_3O_4 — FeO, Fe_2O_3

   At low temperatures:



N(0) is much smaller than that by considering Fe_3O_4 as ferromagnetic.

In genetal:



Ferrimagnetic
ordering

Exchange field on site A:

$$\vec{B}_A = \underbrace{\mu \vec{M}_A}_{\text{Due to sublattice A}} - \underbrace{\lambda \vec{M}_B}_{\text{Due to sublattice B}}$$

Exchange field on site B:

$$\vec{B}_B = \underbrace{-\lambda \vec{M}_A}_{\text{Due to sublattice A}} + \underbrace{\nu \vec{M}_B}_{\text{Due to sublattice B}}$$

For ferromagnetic order to occur, $\lambda >> \mu, \nu$:

$$\therefore \ \vec{B}_A = \underbrace{-\lambda \vec{M}_B}_{\text{Due to sublattice B}}$$

$$\vec{B}_B = \underbrace{-\lambda \vec{M}_A}_{\text{Due to sublattice A}}$$

Let the Curie constants of sublattice A and B be C_A and C_B respectively. Mean field theory, when $T>T_c$:

$$\vec{M}_A = \frac{C}{T}(\underbrace{\vec{B}_a}_{\text{Applied field}} + \vec{B}_A) = \frac{C_A}{T}(\vec{B}_a - \lambda \vec{M}_B)$$

$$\vec{M}_B = \frac{C}{T}(\underbrace{\vec{B}_a}_{\text{Applied field}} + \vec{B}_A) = \frac{C_B}{T}(\vec{B}_a - \lambda \vec{M}_A)$$

At $T = T_c$, ignoring $\vec{B}_A$, for non-trivial solution of $\vec{M}_A$ and $\vec{M}_B$:

$$\begin{vmatrix} T_c & \lambda C_A \\ \lambda C_B & T_c \end{vmatrix} = 0$$

$$\Rightarrow T_c{}^2 - \lambda^2 C_A C_B = 0$$

$$\Rightarrow T_c = \lambda\sqrt{C_A C_B}$$

$$\left.\begin{aligned} \vec{M}_A &= \frac{C_A}{T}(\vec{B}_a - \lambda \vec{M}_B) \\ \vec{M}_B &= \frac{C_B}{T}(\vec{B}_a - \lambda \vec{M}_A) \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} \vec{M}_A + \frac{\lambda C_A}{T}\vec{M}_B &= \frac{C_A}{T}\vec{B}_a \\ \vec{M}_B + \frac{\lambda C_B}{T}\vec{M}_A &= \frac{C_B}{T}\vec{B}_a \end{aligned}\right.$$

$$\Rightarrow \begin{cases} (1-\dfrac{\lambda^2 C_A C_B}{T^2})\vec{M}_A = (\dfrac{C_A}{T} - \dfrac{\lambda C_A C_B}{T^2})\,\vec{B}_a \\ (1-\dfrac{\lambda^2 C_A C_B}{T^2})\vec{M}_B = (\dfrac{C_B}{T} - \dfrac{\lambda C_A C_B}{T^2})\,\vec{B}_a \end{cases}$$

$$\Rightarrow (1-\frac{\lambda^2 C_A C_B}{T^2})(\vec{M}_A + \vec{M}_B) = (\frac{C_A + C_B}{T} - \frac{2\lambda C_A C_B}{T^2})\,\vec{B}_a$$

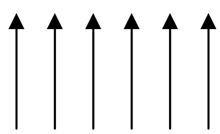
$$\Rightarrow \chi = \frac{\vec{M}_A + \vec{M}_B}{\vec{B}_a} = \frac{\dfrac{C_A + C_B}{T} - \dfrac{2\lambda C_A C_B}{T^2}}{1-\dfrac{\lambda^2 C_A C_B}{T^2}}$$

$$= \frac{(C_A + C_B)T - 2\lambda C_A C_B}{T^2 - \lambda^2 C_A C_B}$$

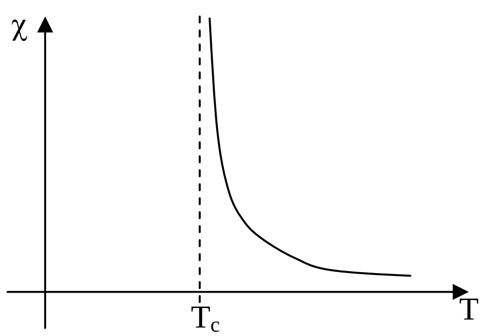
$$= \frac{(C_A + C_B)T - 2T_c\sqrt{C_A C_B}}{T^2 - T_c{}^2}$$

Antiferromangetism

1. Ferromagnetic ordering

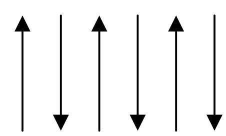


Exchange field: J>0

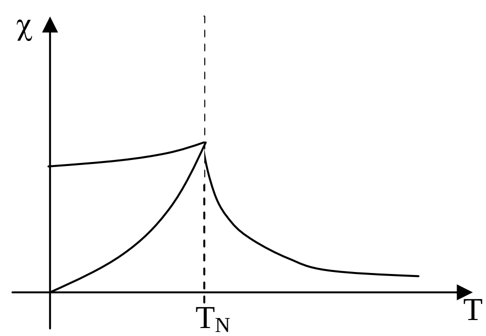


Critical temperature:
Curie temperature T_c

Antiferromagnetic ordering



Exchange field: J<0



Critical temperature:
Neel temperature T_N

2. Antiferromagnetism is a special case of ferrimagnetism with $C_A=C_B$, i.e.

$$T_c \to T_N = \lambda\sqrt{C^2} = \lambda C$$

$$\chi = \frac{(C_A + C_B)T - 2T_c\sqrt{C_A C_B}}{T^2 - T_c^{\ 2}} = \frac{2CT - 2T_N\sqrt{C^2}}{T^2 - T_N^{\ 2}}$$

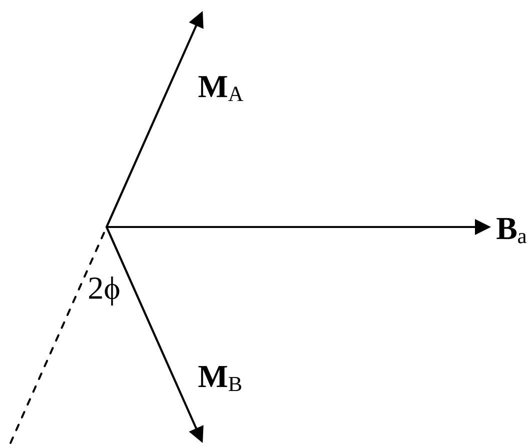
$$\Rightarrow \ \chi = \frac{2C}{T + T_N}$$

Experimentally,

$$\chi = \frac{2C}{T-\theta} \qquad (\theta \text{ is not exactly } T_N \text{ because of next - nearest neighbor interaction})$$

3. When $T<T_N$:

Case 1. If $\mathbf{B}_a \perp$ axis of spin
Let $M = |\,\mathbf{M}_A\,| = |\,\mathbf{M}_B\,|$



$$U = -\frac{1}{2}(\vec{B}_A \cdot \vec{M}_A + \vec{B}_B \cdot \vec{M}_B) - \vec{B}_a \cdot (\vec{M}_A + \vec{M}_B) \qquad (\vec{B}_A \text{ and } \vec{B}_B \text{ are exchange fields})$$

$$= -\frac{1}{2}(-\lambda\vec{M}_B \cdot \vec{M}_A - -\lambda\vec{M}_A \cdot \vec{M}_B) - \vec{B}_a \cdot (\vec{M}_A + \vec{M}_B)$$

$$= \lambda\vec{M}_B \cdot \vec{M}_A - \vec{B}_a \cdot (\vec{M}_A + \vec{M}_B)$$

$$= \lambda M^2 \cos(180^o - 2\phi) - 2MB_a \cdot \cos(90^o - \phi)$$

$$= -\lambda M^2 \cos 2\phi - 2MB_a \sin\phi$$

$$\approx -\lambda M^2 [1 - \frac{1}{2}(2\phi)^2] - 2MB_a\phi$$

To minimize U:

$$\frac{dU}{d\phi} = 0 \ \Rightarrow \ 4\lambda M^2\phi - 2MB_a = 0$$

$$U = -\frac{1}{2}(\vec{B}_A \cdot \vec{M}_A + \vec{B}_B \cdot \vec{M}_B) - \vec{B}_a \cdot (\vec{M}_A + \vec{M}_B) \qquad (\vec{B}_A \text{ and } \vec{B}_B \text{ are exchange fields})$$

$$= -\frac{1}{2}(-\lambda\vec{M}_B \cdot \vec{M}_A - -\lambda\vec{M}_A \cdot \vec{M}_B) - \vec{B}_a \cdot (\vec{M}_A + \vec{M}_B)$$

$$= \lambda\vec{M}_B \cdot \vec{M}_A - \vec{B}_a \cdot (\vec{M}_A + \vec{M}_B)$$

$$= \lambda M^2 \cos(180^o - 2\phi) - 2MB_a \cdot \cos(90^o - \phi)$$

$$= -\lambda M^2 \cos 2\phi - 2MB_a \sin\phi$$

$$\approx -\lambda M^2 [1 - \frac{1}{2}(2\phi)^2] - 2MB_a\phi$$

To minimize U :

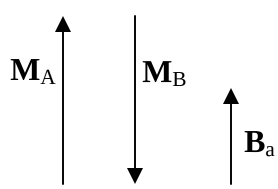
$$\frac{dU}{d\phi} = 0 \Rightarrow 4\lambda M^2\phi - 2MB_a = 0$$

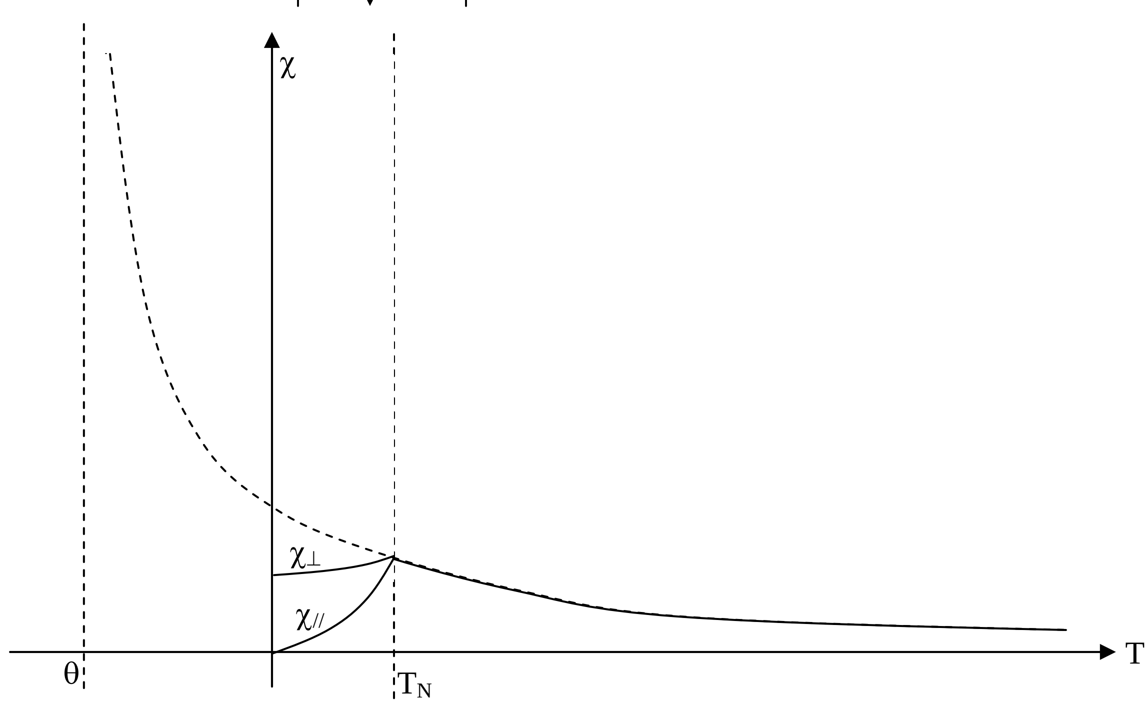
$$\Rightarrow \phi = \frac{B_a}{2\lambda M}$$

$$\therefore \quad \chi_\perp = \frac{2M\sin\phi}{B_a} \approx 2M \cdot \frac{B_a}{2\lambda M} \cdot \frac{1}{B_a} = \frac{1}{\lambda} \qquad (= \frac{C}{T_N})$$

Case 2. If $\mathbf{B}_a$ // axis of spin

There is no change in U. $\therefore$ $\chi_{//}(0) = 0$





4. Antiferromagnetic magnons:

$$-i\omega u e^{i(jka-\omega t)} = \frac{2Js}{\hbar}[2e^{i(jka-\omega t)} - e^{i[(j-1)ka-\omega t]} - e^{i[(j+1)ka-\omega t]}]$$

$$\Rightarrow -i\omega u = \frac{2Js}{\hbar} v[2 - e^{-ika} - e^{ika}]$$

$$\Rightarrow -i\omega u = \frac{4Js}{\hbar} v[1 - \cos ka]$$

Similarly, from the equation for $\frac{d}{dt}(S_j{}^y)$:

$$-i\omega v = -\frac{4Js}{\hbar} u[1 - \cos ka]$$

u, v have non – trivial solution only if {

$$\begin{vmatrix} i\omega & \frac{4Js}{\hbar}[1-\cos ka] \\ -\frac{4Js}{\hbar}[1-\cos ka] & i\omega \end{vmatrix} = 0$$

$$\Rightarrow \omega^2 = \left[\frac{4Js}{\hbar}[1-\cos ka]\right]^2$$

$$\Rightarrow \hbar\omega = 4Js(1-\cos ka)$$

If this condition is satisfy, solving for u and v:

$$\left.\begin{aligned} -i\omega u &= \omega v \\ -i\omega v &= -\omega u \end{aligned}\right\} \Rightarrow v = -iu$$

From case of ferromagnetism:

Now, if lattice A corresponds to even indices (2j) and lattice B to odd indices (2j+1), then $S_{2j}{}^x$ should have opposite sign with $S_{2j-1}{}^x$ and $S_{2j+1}{}^x$. Rewriting the equation for ferromagnetism:

$$\frac{d}{dt}(S_j{}^x) = \frac{2J}{\hbar}[S_j{}^y(S_{j-1}{}^z + S_{j+1}{}^z) - S_j{}^z(S_{j-1}{}^y + S_{j+1}{}^y) \qquad \text{(feromagnetic)}$$

$$\xrightarrow{\text{rewritten}} \frac{d}{dt}(S_{2j}{}^x) = \frac{2J}{\hbar}[S_{2j}{}^y(-S_{2j-1}{}^z - S_{2j+1}{}^z) - S_{2j}{}^z(S_{2j-1}{}^y + S_{2j+1}{}^y)] \qquad \text{(antiferromagnetic)}$$

$$\Rightarrow \frac{d}{dt}(S_{2j}{}^x) = \frac{2JS}{\hbar}[-2S_{2j}{}^y - S_{2j-1}{}^y - S_{2j+1}{}^y)] \qquad (S_{2j}{}^z = S)$$

and similarly,

$$\frac{d}{dt}(S_{2j}{}^y) = -\frac{2JS}{\hbar}[2S_{2j}{}^x - S_{2j-1}{}^x - S_{2j+1}{}^x)]$$

Let $S^+ = S^x + iS^y$

$$\therefore \quad \frac{d}{dt}(S_{2j}{}^+) = \frac{2iJS}{\hbar}[2S_{2j}{}^+ + S_{2j-1}{}^+ + S_{2j+1}{}^+)]$$

Corresponding equations for lattice B:

$$\frac{d}{dt}(S_{2j+1}{}^{x}) = \frac{2JS}{\hbar}[2S_{2j+1}{}^{y} - S_{2j}{}^{y} - S_{2j+2}{}^{y}] \qquad (S_{2j}{}^{z} = S)$$

$$\frac{d}{dt}(S_{2j+1}{}^{y}) = -\frac{2JS}{\hbar}[2S_{2j+1}{}^{x} - S_{2j}{}^{x} - S_{2j+2}{}^{x}]$$

Let $S^{+} = S^{x} + iS^{y}$

$$\therefore \frac{d}{dt}(S_{2j+1}{}^{+}) = -\frac{2iJS}{\hbar}[2S_{2j+1}{}^{+} + S_{2j}{}^{+} + S_{2j+2}{}^{+})]$$

Trial solution :

$S_{2j}{}^{+} = u^{\,i[(2j)ka-\omega t]}$ and $S_{2j+1}{}^{+} = u^{\,i[(2j+1)ka-\omega t]}$

Above defferential equations become

$$-i\omega u = \frac{2iJS}{\hbar}[2u + ve^{-ika} + ve^{ika}]$$

$$\Rightarrow \omega u = -\frac{4JS}{\hbar}[u + v\cos ka]$$

$$-i\omega u = -\frac{2iJS}{\hbar}[2v + ue^{-ika} + ue^{ika}]$$

$$\Rightarrow \omega v = [v + u\cos ka]$$

For non - trivial solution :

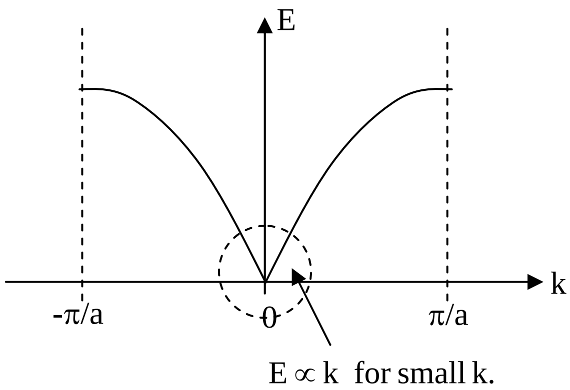
$$\begin{vmatrix} \omega + \frac{4JS}{\hbar} & \frac{4JS}{\hbar}\cos ka \\ -\frac{4JS}{\hbar}\cos ka & \omega - \frac{4JS}{\hbar} \end{vmatrix} = 0 \Rightarrow \left(\omega + \frac{4JS}{\hbar}\right)\left(\omega - \frac{4JS}{\hbar}\right) + \left(\frac{4JS}{\hbar}\cos ka\right)^2 = 0$$

$$\Rightarrow \omega^2 = \left(\frac{4JS}{\hbar}\right)^2 - \left(\frac{4JS}{\hbar}\cos ka\right)^2$$

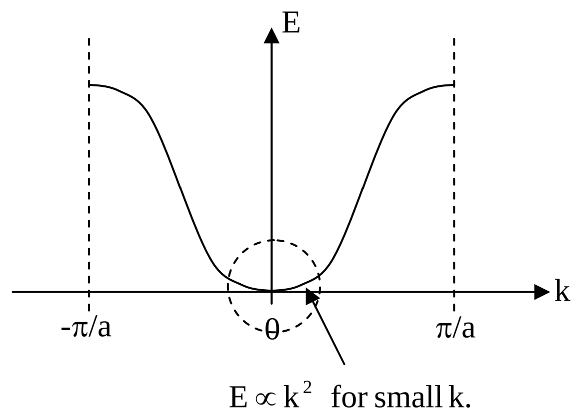
$$\Rightarrow \omega^2 = \left(\frac{4JS}{\hbar}\right)^2 (1 - \cos ka)^2$$

$$\Rightarrow \omega^2 = \left(\frac{4JS}{\hbar}\right)^2 \sin^2 ka$$

$$\Rightarrow \omega = \left(\frac{4JS}{\hbar}\right) |\sin ka|$$



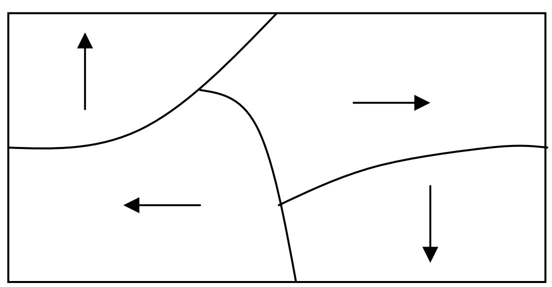
Antiferromagnetism



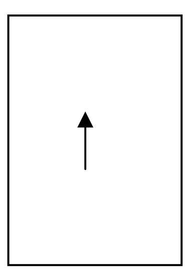
Ferromagnetism

Domains

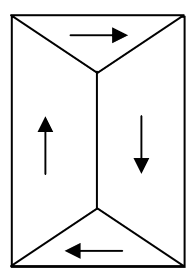
1. Spin in a material with long range magnetic ordering (ferromagnetic, antiferromagnetic etc.) form domains.



2. Reason for domain formation:

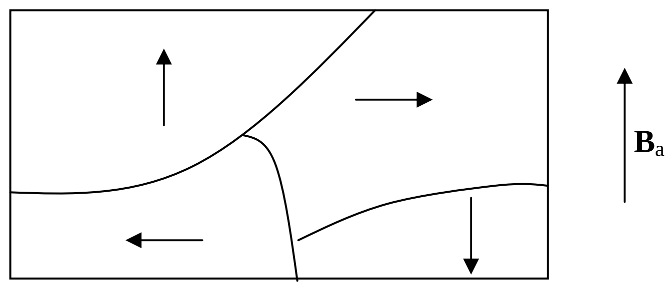


Higher energy

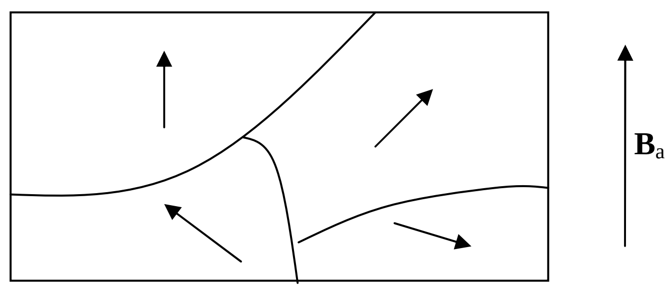


Lower energy

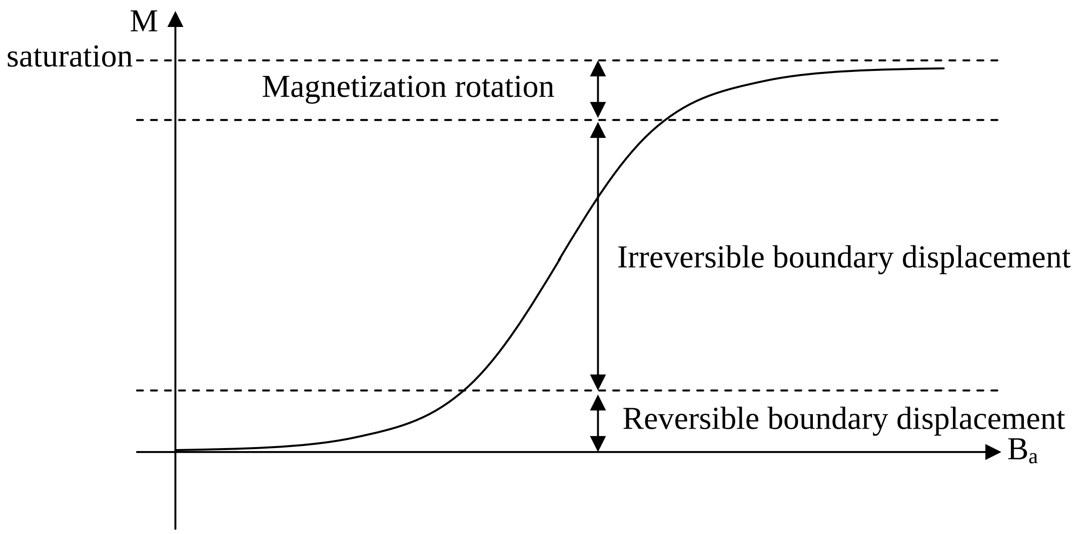
3. For small field: domain size will change in accordance to the direction of the magnetic field. Change in domain size can be reversible or irreversible.



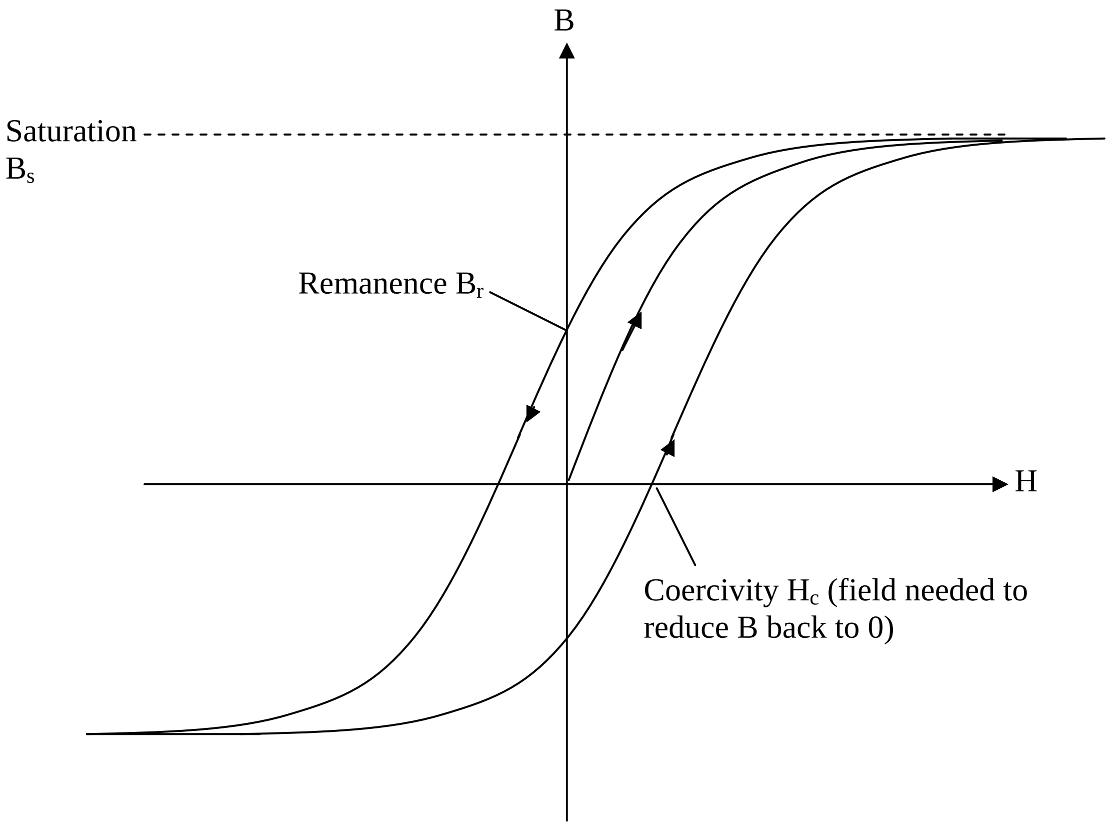
4. For large field: domain magnetization will re-align with the external field.



5.



6. Irreversible boundary displacement and magnetization rotation are the causes of hysteresis:



B
Saturation
B_s
Remanence B_r
H
Coercivity H_c (field needed to
reduce B back to 0)