University of Kentucky
Department of Physics and Astronomy

PHY 525: Solid State Physics II

Fall 2000

Final Examination

Date: December 11, 2000 (Monday)

Time allowed: 120 minutes.

Answer all questions.

1. *Heat capacity of ferromagnets.* (a) Use the approximation magnon dispersion relation $\omega = Ak^2$ to find the leading term in the heat capacity of a three-dimensional ferromagnet at low temperatures $k_B T << J$. The result is $0.113k_B(k_BT/\leftarrow A)3/2$, per unit volume. The definite intergral $\int_0^\infty \frac{x^{\frac{3}{2}}dx}{[\exp(x)-1]}$ is given as $\frac{3}{4}\sqrt{\pi}\,\zeta\left(\frac{5}{2}\right)$ with $\zeta\left(\frac{5}{2}\right) \approx 1.3415$ approximately. (b) How does the heat capacity of a metal depend on temperature at low T (just write down the result, you do not need to do any derivation)? Free electron has a similar dispersion relation as E ;k^2, why the heat capacities of metal and ferromagnet depend differently on temperature at low T?

2. *Magnetic field penetration in a plate.* The penetration equation may be written as $\lambda^2 \nabla^2 B = B$, where λ is the penetration depth. (a) Show that B(x) inside a superconding plate perpendicular to the x axis and of thickness δ is given by
$$B(x) = B_a \frac{\cosh(x/\lambda)}{\cosh(\delta/2\lambda)} ,$$
where B_a is the field outside the plate and parallel to it; here x=0 is at the center of the plate. (b) The effective magnetization M(x) in the plane is defined by $B(x)-B_a = 4\pi M(x)$. Show that, in CGS, $4\pi M(x) = -B_a(1/8\lambda^2)(\delta^2 - 4x^2)$, for $\delta << \lambda$. In SI, we replace the 4π by μ_0.

3. *Susceptibility of d- and f- electrons.* (a) Apply Hund rule to determine the ground state of (i) Dy^{3+} in the configuration of $4f^9 5s^2 p^6$, and (ii) Cu^{2+} in the configuration of $3d^9$. Express your answer in standard atomic notation. (b) The Curie constant C is given as
$$C = \frac{Ng^2[J\{J+1)]^2 \mu_B{}^2}{3k_B}$$
for f-electrons. Estimate the Curie constants for Dy^{3+} and Cu^{2+}. Avogadro's number is given as 6.022×10^{23} mol^{-1}. Note that the g-factor is given by the Landé equation:
$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

(c) What is the ratio $\chi(300K)/\chi(4.2K)$ of the same paramagnetic material, for small field?

4. *Molecular rotation in solid.* We can model the polarization vector **P** always relaxes towards an equilibrium value $\mathbf{P}_E$ at a rate called relaxation time τ. $\mathbf{P}_E$ depends on the electric field **E** as $\mathbf{P}_E = \chi(0)\mathbf{E}$. The relaxation equation can then be written as $\frac{d\mathbf{P}}{dt} = \frac{\mathbf{P}_E - \mathbf{P}}{\tau}$. (a) By considering $\mathbf{E} = \mathbf{E}_0 e^{i\omega t}$, show that

$$\varepsilon'(\omega) = 1 + \frac{4\pi\chi(0)}{1+\omega^2\tau^2} \text{ and } \varepsilon''(\omega) = -\frac{i4\pi\omega\chi(0)}{1+\omega^2\tau^2}$$

(b) At what ω is the dielectric loss maximum? What are the relaxation times at frequencies 300Hz, 1000Hz, and 5000Hz? (c) Real and imaginary parts of the dielectric constant of ice as a function of temperature at different frequencies are given in the figure. Show that the temperature dependence is consistent with $\tau = \tau_0 \exp(T_0/T)$ and find values of τ_0 and T_0. (d) What is the physical basis for a temperature dependence of the kind given in part (c)?