University of Kentucky
Department of Physics and Astronomy

PHY 525: Solid State Physics II

Fall 2000

Test 1

Date: October 9, 2000 (Monday)

Time allowed: 50 minutes.

Answer all questions.

1. ***Specific heat of a d-dimensional insulator.*** Consider a d-dimensional crystal with the dispersion relation given as $\omega = Ak^{\lambda}$ where A and λ are constants. Let N be the number of lattice points in the sample. (a) Calculate group velocity in terms of k. (b) If the Debye temperature θ_D is proportional to N^{α}. Calculate α in terms of λ and d. (c) If the phonon density of states $D(\omega)$ is proportional to ω^{β}. Calculate β in terms of λ and d. (d) If the heat capacity C at low temperatures is proportional to T^{δ}. Calculate δ in terms of λ and d. Discuss your results for the particular case of linear dispersion relation, with d=1, 2, and 3.

   Solution:

(a) $\omega = Ak^{\lambda} \Rightarrow v = \dfrac{d\omega}{dk} = A\lambda k^{\lambda-1}$

(b) $N = \dfrac{dBk_D{}^d}{\dfrac{(2\pi)^d}{V}}$ $\quad$ V = dimensional "volume" of sample. $B = \begin{cases} 1 & d = 1 \\ 4\pi & d = 1 \\ \frac{4}{3}\pi & d = 3 \end{cases}$

$$\Rightarrow k_D = 2\pi\left(\frac{N}{VdB}\right)^{1/d} = 2\pi\left(\frac{n}{dB}\right)^{1/d}$$

$$\omega_D = Ak_D{}^{\lambda} = 2A\pi\left(\frac{N}{VdB}\right)^{\lambda/d}$$

$$\therefore \theta_D = \frac{\hbar\omega_D}{k_B} = \frac{2\pi A\hbar}{k_B}\left(\frac{N}{VdB}\right)^{1/d} = \frac{Ah}{k_B}\left(\frac{N}{VdB}\right)^{1/d}$$

$$\therefore \alpha = \frac{\lambda}{d}$$

(c) $D(\omega)\delta\omega = Ck^{d-1}\delta k \cdot \dfrac{V}{(2\pi)^d}$ $\quad$ $C = \begin{cases} 1 & d = 1 \\ 2\pi & d = 2 \\ 4\pi & d = 3 \end{cases}$

$$\Rightarrow D(\omega) = \frac{VCk^{d-1}}{(2\pi)^d} \cdot \frac{1}{\dfrac{\delta\omega}{\delta k}}$$

$$\Rightarrow D(\omega) = \frac{VCk^{d-1}}{(2\pi)^d} \cdot \frac{1}{A\lambda k^{\lambda-1}}$$

$$\Rightarrow D(\omega) = \frac{VC}{(2\pi)^d A\lambda} k^{d-\lambda}$$

$$\Rightarrow D(\omega) = \frac{VC}{(2\pi)^d A\lambda}\left(\frac{\omega}{A}\right)^{\frac{d-\lambda}{\lambda}}$$

$$\therefore \beta = \frac{d-\lambda}{\lambda} = \underline{\underline{\frac{d}{\lambda} - 1}}$$

$$\text{(d)}\quad U = d\int_0^{\omega_D} \frac{\hbar\omega D(}{e^{\frac{\hbar\omega}{k_BT}}-1} \propto \int_0^{\omega_D} \frac{\omega\omega^{\frac{d}{\lambda}-1}d\omega}{e^{\frac{\hbar\omega}{k_BT}}-1}$$

$$\propto \int_0^{\omega_D} \frac{\omega^{\frac{d}{\lambda}}d\omega}{e^{\frac{\hbar\omega}{k_BT}}-1}$$

$$\frac{\hbar\omega}{k_BT} = x \Rightarrow \quad x_D = \frac{\hbar\omega}{k_BT} = \frac{\theta_D}{T} \qquad \theta_D = \frac{\hbar\omega_D}{k_B}$$

$$\therefore\ U \propto \int_0^{x_D} \frac{\left(\frac{k_BTx}{\hbar}\right)^{\frac{d}{\lambda}} \frac{k_BT}{\hbar}dx}{e^x - 1}$$

$$\propto T^{\frac{d}{\lambda}+1}\int_0^{x_D} \frac{x^{\frac{d}{\lambda}}dx}{e^x-1}$$

At low temperatures, $T \to 0, x_D \to \infty$.

$$U \propto T^{\frac{d}{\lambda}+1}$$

$$C = \frac{\partial U}{\partial T} \propto T^{\frac{d}{\lambda}}$$

For $\lambda = 1, C \propto T^d$.

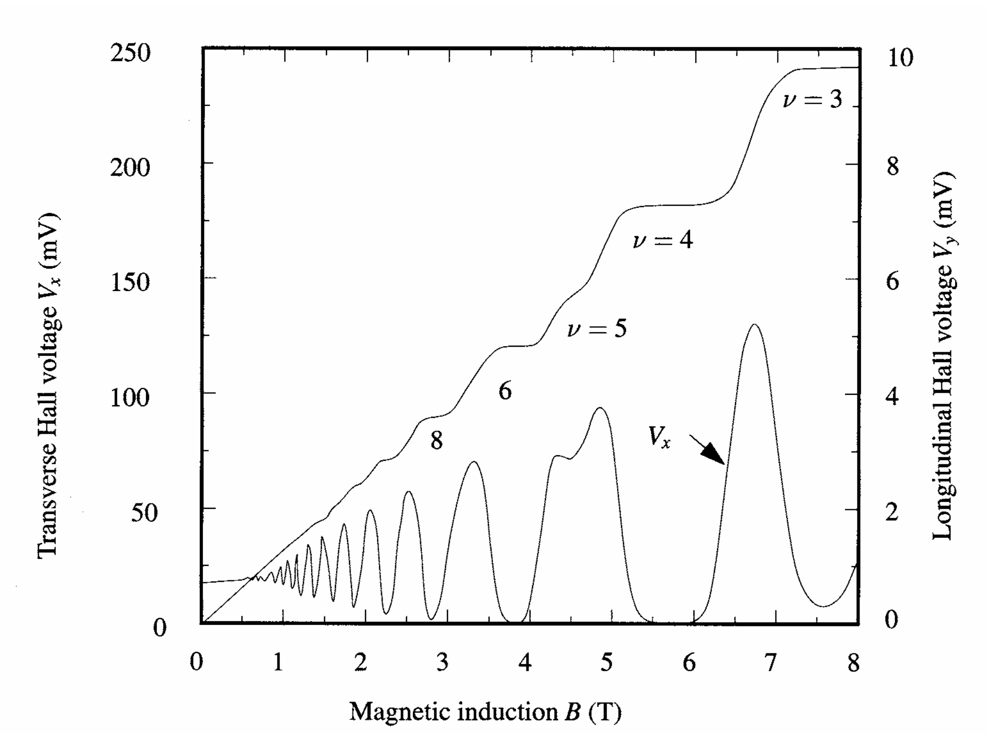
For example, for 3 dimensional insulator, $C \propto T^3$.

2. ***Quantum Hall Effect.*** Consider a two dimensional electron gas in a magnetic field B. Note that one flux quanta $\Phi_0 = \frac{2\pi\hbar c}{e}$

(a) What is the separation (in energy) between the Landau levels? (b) Calculate the number of states in one Landau level, in terms of sample area and other physical constants. (c) From your result in (b) derive the quantum of conductance. (d) Consider the experimental data provided in the following figure. Estimate the current through the strip with the step indexed as given. (e) Estimate the electron surface density from the figure.

Useful constants: $h = 6.626\times10^{-27}$ erg s $= 6.626\times10^{-34}$ Js

$e = 4.803\times10^{-10}$ esu $= 1.602\times10^{-19}$ C



Solution:

(a) Separation between Landau levels $= \hbar\omega_c = \dfrac{eB}{m^* c}$

(b) Changing Landau level by 1 corresponds to adding one flux quanta to the cyclrotron orbit.

$\therefore B\Delta A_c = \Phi_0 = \dfrac{2\pi\hbar c}{e}$ where A_c = area of cyclotron orbit

$$\therefore \text{ Area between Landau levels in k-space } = \Delta S = \frac{(2\pi)^2}{\Delta A_c}$$
$$= 4\pi^2 \cdot \frac{eB}{2\pi\hbar c}$$
$$= \frac{2\pi eB}{\hbar c}$$

$$\therefore \text{ Number of states in one level} = \frac{\frac{2\pi eB}{\hbar c}}{\text{Area per state}}$$
$$= \frac{\frac{2\pi eB}{\hbar c}}{\frac{(2\pi)^2}{A}} \qquad \text{where } A = \text{area of sample}$$
$$= \frac{eBA}{2\pi\hbar c} = \underline{\underline{\frac{eBA}{hc}}}$$

(c) If n = electron surface concentration

$\therefore$ Total number of electrons $= nA$

If these electrons fill up to the ν-th level corresponding to B_i

$$\therefore nA = \frac{\nu e B_\nu A}{hc} \Rightarrow B_\nu = \frac{nhc}{\nu e}$$

$$\text{Transverse conductance} = \frac{nec}{B_\nu} = \nu\frac{e^2}{h}$$

$\therefore$ One quantum of conductance is $\dfrac{e^2}{h}$.

(d) $\rho_T = \dfrac{1}{\sigma_T} = \dfrac{h}{\nu e^2} = \dfrac{6.626\times10^{-34}}{1.602\times10^{-19}\nu} = \dfrac{25818.2\Omega}{\nu}$

Look at the $\nu = 4$ step, $V \approx 170$ mV.

$$\therefore \frac{170\times10^{-3}}{I} = \frac{25818.2\Omega}{4}$$

$$\Rightarrow I = \frac{170\times10^{-3}\times4}{25818.2} = 2.63\times10^{-5}\,A = 26.3\mu A$$

(e) $B_\nu = \dfrac{nhc}{\nu e} \Rightarrow \nu = \dfrac{nhc}{Be}$

For two consecutive levels, $\Delta\nu = 1$

$$1 = \Delta\left(\frac{1}{B}\right)\frac{nhc}{e} \Rightarrow n = \frac{e}{hc\Delta\left(\frac{1}{B}\right)}$$

Look at the differce in $\dfrac{1}{B}$ between any two peeks in ρ_L, say, at 2.5T and 3.3T

$$\therefore n = \frac{4.8\times10^{-10}}{6.626\times10^{-27}\times3\times10^{10}\times\left(\dfrac{1}{2.5\times10^4} - \dfrac{1}{3.3\times10^4}\right)}$$

$$= \underline{\underline{2.5\times10^{11}\ \text{cm}^{-2}}}$$