University of Kentucky
Department of Physics and Astronomy
PHY 525: Solid State Physics II
Fall 2000
Test 2

Date: November 8, 2000 (Wednesday)

Time allowed: 50 minutes.

Answer all questions.

1. *Hall coefficient of a semiconductor*. (a) Starting from drift velocity approximation, show that the Hall coefficient of a semiconductor is given by

$$R_H = \frac{1}{n_i ec}\ \frac{1-b}{1+b}$$

where n_i is the intrinsic concentration, and b is the ratio of mobility in conduction and valance bands ($b=\mu_n/\mu_p$). (b) Consider two semiconductors A and B of with same energy gap. The following table gives the ratio of electron effective mass and relaxation time in the conduction and valance bands:

	m^*_n/m^*_p	τ_n/τ_p
Semiconductor A	0.3	30
Semiconductor B	0.8	15

Estimate the ratio of their Hall coefficients from Drude model. (c) For a semiconductor of energy gap equivalent to a temperature of 12500K, estimate the ratio of the Hall coefficient at 300K and 200K, $R_H(300K)/R_H(200K)$. Assume τ does not vary much in this temperature range.

Solution:

(a)
$$\hbar\left(\frac{d}{dt}+\frac{1}{\tau}\right)\vec{k} = q\left(\vec{E}+\frac{1}{c}\vec{v}\times\vec{B}\right) \qquad ---(0)$$

Assume steady case, $\frac{d\vec{v}}{dt}=0.$

Above equations become :

$$(0) \Rightarrow \quad v_x = \frac{q\tau}{m}E_x + \frac{q\tau B}{mc}v_y = \begin{cases} \mu_p E_x + \dfrac{\mu_p B}{c} v_{py} \\ -\mu_n E_x - \dfrac{\mu_n B}{c} v_{ny} \end{cases} \qquad ---(1)$$

$$(0) \Rightarrow \quad v_y = \frac{q\tau}{m}E_y - \frac{q\tau B}{mc}v_x = \begin{cases} \mu_p E_y - \dfrac{\mu_p B}{c} v_{px} \\ -\mu_n E_y + \dfrac{\mu_n B}{c} v_{nx} \end{cases} \qquad ---(2)$$

$v_y = 0 \Rightarrow n_i e v_{py} - n_i e v_{ny} = 0$ and $j = j_x = n_i e v_{px} - n_i e v_{nx}$:

$$(1) \Rightarrow \begin{cases} v_{px} = \mu_p E_x \\ v_{nx} = -\mu_n E_x \end{cases} \qquad \text{---}(3)$$

$$\Rightarrow \begin{cases} j_p = n_i e \mu_p E_x \\ j_n = n_i e \mu_n E_x \end{cases}$$

$$\Rightarrow j = n_i e(\mu_p + \mu_n) E_x \qquad \text{---}(4)$$

$$(3)\ \&\ (4) \quad \Rightarrow \begin{cases} v_{px} = \mu_p \dfrac{j}{n_i e(\mu_p + \mu_n)} \\ v_{nx} = -\mu_n \dfrac{j}{n_i e(\mu_p + \mu_n)} \end{cases} \qquad \text{---}(5)$$

$$(2) \Rightarrow n_i e\left(\mu_p E_y - \frac{\mu_p B}{c} v_{px}\right) - n_i e\left(-\mu_n E_y + \frac{\mu_n B}{c} v_{nx}\right) = 0 \qquad (\because n_i e v_{py} - n_i e v_{ny} = 0)$$

$$\Rightarrow n_i e(\mu_p + \mu_n) E_y - \frac{n_i e B}{c}\left(\mu_p v_{px} + \mu_n v_{nx}\right) = 0$$

$$\Rightarrow n_i e(\mu_p + \mu_n) E_y - \frac{n_i e B}{c}\left(\mu_p \mu_p \frac{j}{n_i e(\mu_p + \mu_n)} - \mu_n \mu_n \frac{j}{n_i e(\mu_p + \mu_n)}\right) = 0 \qquad (\text{from}(4))$$

$$\Rightarrow E_y - \frac{Bj}{c n_i e(\mu_p + \mu_n)^2}\left(\mu_p{}^2 - \mu_n{}^2\right) = 0$$

$$\Rightarrow \quad E_y = \frac{Bj(\mu_p - \mu_n)}{c n_i e(\mu_p + \mu_n)}$$

$$\Rightarrow \quad E_y = \frac{Bj(1-b)}{c n_i e(1+b)} \qquad (b = \frac{\mu_n}{\mu_p})$$

$$\therefore \quad R_H = \frac{E_y}{Bj} = \frac{1}{n_i e c}\frac{1-b}{1+b}$$

(b)

Since the energy gap is the same for both semiconductors, $\therefore n_i$ is the same for both A and B.

$$\therefore \frac{(R_H)_A}{(R_H)_B} = \frac{\left(\frac{1-b_A}{1+b_A}\right)}{\left(\frac{1-b_B}{1+b_B}\right)}$$

$$\mu = \left|\frac{e\tau}{m}\right| \propto \frac{\tau}{m} \Rightarrow b = \frac{\mu_n}{\mu_p} = \frac{\tau_n}{m_n} \Big/ \frac{\tau_p}{m_p} = \frac{(\tau_n/\tau_p)}{(m_n/m_p)}$$

$$\therefore \left.\begin{aligned} b_A &= \frac{30}{0.3} = 100 \\ b_B &= \frac{15}{0.8} = 18.75 \end{aligned}\right\} \Rightarrow \frac{(R_H)_A}{(R_H)_B} = \frac{\left(\frac{1-100}{1+100}\right)}{\left(\frac{1-18.75}{1+18.75}\right)} = \frac{0.9802}{0.8987} = \underline{\underline{1.09}}$$

(c)

$$R_H = \frac{1}{n_i ec}\frac{1-b}{1+b} \propto n_i^{-1} \qquad \text{(assuming b constant)}$$

$$n_i^2 = np = N_C N_V \exp\left[-\frac{E_g}{k_B T}\right]$$

$$n_i = \sqrt{N_C N_V}\exp\left[-\frac{E_g}{2k_B T}\right]$$

Note that N_C and $N_V \propto T^{3/2}$

$$\therefore \quad n_i \propto T^{3/2}\exp\left[-\frac{E_g}{2k_B T}\right]$$

$$\frac{R_H(300K)}{R_H(200K)} = \frac{n_i(200K)}{n_i(300K)} = \left(\frac{300}{200}\right)^{3/2} \frac{\exp\left[-\frac{12500k_B}{2k_B \times 200}\right]}{\exp\left[-\frac{12500k_B}{2k_B \times 300}\right]}$$

$$= \left(\frac{300}{200}\right)^{3/2} \frac{2.681\times10^{-14}}{8.958\times10^{-10}}$$

$$= \underline{\underline{5.5\times10^{-5}}}$$

2. ***Graded diode.*** In the depletion layer (width 2w) of a graded p-n junction, the doping level varies linearly with position: N_D-N_A = kx for $-w \le x \le +w$. If the p- and n-semiconductors have doping level of N_A and N_D respectively. Let the effective density of state of the conduction and valance band be N_C and N_V respectively. Also let the energy gap of the intrinsic semiconductor be E_g. Assume all impurities are fully ionized. (a) Calculate chemical potential μn and μp in terms of N_D, N_A, N_C, N_V, and E_g, from this calculate the potential difference Δφ across the depletion layer. (b) What should be the electric field E at the two sides of the depletion layer? Find the electric field E(x) within the depletion layer. (c) Find the electric potential φ(x) within the depletion layer. (d) Determine the layer width 2w in term of Δφ.

Solution:
(a)

Let us first conside the n - type semiconductor. Since the impurities are fully ionized.

$$\therefore n \approx N_D = N_c \exp[-\frac{E_g - \mu_n}{k_B T}] \;\Rightarrow\; \mu_n - E_g = k_B T \ln\left[\frac{N_D}{N_c}\right]$$

$$\Rightarrow\; \mu_n = E_g - k_B T \ln\left[\frac{N_C}{N_D}\right]$$

Similarly, for the p - type semiconductor :

$$p \approx N_A = N_V \exp[-\frac{\mu_p}{k_B T}] \;\Rightarrow\; -\mu_p = k_B T \ln\left[\frac{N_A}{N_V}\right]$$

$$\Rightarrow\; \mu_p = k_B T \ln\left[\frac{N_V}{N_A}\right]$$

$$e\Delta\varphi \approx \mu_n - \mu_p = E_g - k_B T \ln\left[\frac{N_C}{N_D}\right] - k_B T \ln\left[\frac{N_V}{N_A}\right]$$

$$= E_g - k_B T \ln\left[\frac{N_C N_V}{N_D N_A}\right]$$

(b)

Electric field at $x = \pm w$ should be zero.

Applied Gauss' law to the static charges at the depletion layer.

$$\varepsilon\oint \vec{E}\cdot d\vec{S} = 4\pi Q_{enclosed} \Rightarrow \varepsilon dEA = 4\pi\rho A dx$$

$$\Rightarrow \quad dE = \frac{4\pi ekxdx}{\varepsilon} \qquad [\rho = e(N_D - N_A)]$$

$$\Rightarrow \quad E = \frac{2\pi ekx^2}{\varepsilon} + C$$

$$\text{E at } x = \pm w \text{ is } 0 \Rightarrow \frac{2\pi ekw^2}{\varepsilon} + C = 0$$

$$\Rightarrow C = -\frac{2\pi ekw^2}{\varepsilon}$$

$$\therefore \ E = \frac{2\pi ekx^2}{\varepsilon} - \frac{2\pi ekw^2}{\varepsilon} = \frac{2e\pi k}{\varepsilon}(x^2 - w^2)$$

(c)

$$d\varphi = -Edx \Rightarrow d\varphi = -\frac{2e\pi k}{\varepsilon}(x^2 - w^2)x$$

$$\Rightarrow \varphi = \frac{2e\pi k}{\varepsilon}(-\tfrac{1}{3}x^3 + w^2x) + A$$

$$\text{Let } \varphi = 0 \text{ at } x = -w \Rightarrow \frac{2e\pi k}{\varepsilon}(\tfrac{1}{3}w^3 - w^3) + A = 0$$

$$\Rightarrow A = \frac{4e\pi k}{3\varepsilon}w^3$$

$$\therefore \varphi = \frac{2e\pi k}{\varepsilon}(-\tfrac{1}{3}x^3 + w^2x) + \frac{4e\pi k}{3\varepsilon}w^3$$

$$= \frac{2e\pi k}{\varepsilon}x(w^2 - \tfrac{1}{3}x^2) + \frac{4e\pi k}{3\varepsilon}w^3$$

(d)

At $x = +w$, $\varphi = \Delta\varphi$.

$$\therefore \frac{2e\pi k}{\varepsilon} w(w^2 - \frac{1}{3}w^2) + \frac{4e\pi k}{3\,\varepsilon} w^3 = \Delta\varphi \Rightarrow \frac{4e\pi k}{3\varepsilon} w^3 + \frac{4e\pi k}{3\,\varepsilon} w^3 = \Delta\varphi$$

$$\Rightarrow \frac{8e\pi k}{3\,\varepsilon} w^3 = \Delta\varphi$$

$$\Rightarrow w = \left[\frac{3\varepsilon\Delta\varphi}{8e\pi k}\right]^{\frac{1}{3}}$$