University of Kentucky
Department of Physics and Astronomy

PHY 525. Introduction to Solid State Physics II

Test 2.

Date: Oct 12, 2001
Time: 9:00-9:50
Answer all questions.

1. (25 points)
The electron energy near the top of the valance band in a semiconductor is given by

$$E_v = -10^{-37} k^2 \qquad (E_v \text{ in Joules}, k \text{ in m}^{-1})$$

where **k** is the wavevector. An electron is removed from the state

$$k = 10^9 \hat{k}_x \quad \text{m}^{-1}$$

where $\hat{k}_x$ is a unit vector along the x axis. Calculate the following quantities of the resulting hole:
(i) The effective mass.
(ii) The energy.
(iii) The momentum.
(iv) The velocity.
Each quantity must include the sign (or direction).

2. (25 points)
Consider the close orbits of an electron in real space and k space when an external magnetic field B is applied. Let the area be A and S respectively. Note that the magnet flux BA is quantized in unit of Φ_0=h/e.
(i) Write down the relationship between A and S and hence the relationship between S and B.
(ii) Calculate S for a metal X of valance 1 (i.e. one conducting electron per atom). The atomic density of the metal is 8.5×10^{28} m^{-3}. Assume free electron model.
(iii) In a de Haas-van Alphen experiment of metal X, the magnetic susceptibility is oscillating periodically with $\delta(1/B)$. Calculate the periodicity. How many oscillations are there as B is changed from 10.70 T to 10.93T?

Solution:

1.(i) $$m_h^{-1} = \frac{1}{\hbar^2}\frac{\partial^2 E_h}{\partial k^2} = -\frac{1}{\hbar^2}\frac{\partial^2 E_v}{\partial k^2}$$

$$= -\frac{1}{\hbar^2}\frac{\partial^2}{\partial k^2}\left[-10^{-37}k^2\right]$$

$$= -\frac{1}{(1.055\times10^{-34})^2}\left[2\times(-10^{-37})\right]$$

$$= \underline{\underline{5.57\times10^{-32}\,\text{kg, or } 0.061 m_{\text{free electron}}}}$$

Note that the mass is positive.

(ii) $$E_h = -E_v = -\left[-10^{-37}k^2\right]$$

$$= 10^{-37}\times(10^9)^2$$

$$= \underline{\underline{1\times10^{-19}\text{ J, or } 0.624\text{ eV}}}$$

Note that the energy is positive.

(iii) $$\vec{p}_h = -\hbar\vec{k}_e = -1.055\times10^{-34}\times(10^9\ \hat{x})$$

$$= \underline{\underline{-1.055\times10^{-25}\text{ kgm/s }\hat{x}}}$$

Note that it is in the $-\hat{x}$ direction.

(iv) $$\vec{p}_h = m_h\vec{v}_h \Rightarrow \vec{v}_h = \frac{\vec{p}_h}{m_h} = \frac{-1.055\times10^{-25}\ \hat{x}}{5.57\times10^{-32}}$$

$$= \underline{\underline{-1.896\times10^{-6}\text{ m/s }\hat{x}}}$$

Note that it is in the $-\hat{x}$ direction.

2. (i) $\hbar\dot{\vec{k}} = e\vec{v}\times\vec{B} \Rightarrow \hbar\Delta k = e\Delta r B \Rightarrow \hbar^2(\Delta k)^2 = e^2(\Delta r)^2 B^2$

$$\Rightarrow \underline{\underline{\hbar^2 S = e^2 A B^2}} \quad \text{----(1)}$$

With $AB = \Phi$, (1) $\Rightarrow \underline{\underline{\hbar^2 S = e^2 \Phi B}}$ ----(2)

(ii) $$2\times \frac{\frac{4}{3}\pi k_F^{\,3}}{\frac{(2\pi)^3}{V}} = N \Rightarrow k_F^{\,3} = \frac{N}{V}(2\pi)^3 \frac{3}{4\pi}\cdot\frac{1}{2}$$

$$\Rightarrow k_F^{\,3} = 8.5\times 10^{28}\times(2\pi)^3 \frac{3}{4\pi}\cdot\frac{1}{2}$$

$$\Rightarrow k_F^{\,3} = 2.5167\times 10^{30}$$

$$\Rightarrow k_F = 1.3602\times 10^{10}\ \text{m}^{-1}$$

$$\therefore S = \pi k_F^{\,2} = \pi(1.3602\times 10^{10})^2 = \underline{\underline{5.813\times 10^{20}\ \text{m}^{-2}}}$$

(iii) (2) $$\Rightarrow \frac{1}{B} = \frac{e^2\Phi}{\hbar^2 S} \Rightarrow \delta\left(\frac{1}{B}\right) = \frac{e^2}{\hbar^2 S}\delta\Phi$$

$$\Rightarrow \delta\left(\frac{1}{B}\right) = \frac{e^2}{\hbar^2 S}\Phi_0 \qquad (\Phi_0 = \frac{h}{e})$$

$$\Rightarrow \delta\left(\frac{1}{B}\right) = \frac{e^2}{\hbar^2 S}\frac{h}{e} = \frac{he}{\hbar^2 S}$$

$$\Rightarrow \delta\left(\frac{1}{B}\right) = \frac{6.626\times 10^{-34}\times 1.6\times 10^{-19}}{(1.055\times 10^{-34})^2\times 5.813\times 10^{20}}$$

$$\Rightarrow \delta\left(\frac{1}{B}\right) = 1.639\times 10^{-5}\ \text{T}^{-1}$$

For the given magnetic fields, $B_1 = 10.7\text{T} \Rightarrow \frac{1}{B_1} = \frac{1}{10.7} = 0.09346\ \text{T}^{-1}$

$$B_2 = 10.93\text{T} \Rightarrow \frac{1}{B_2} = \frac{1}{10.93} = 0.09149\ \text{T}^{-1}$$

$$\therefore \Delta\left(\frac{1}{B}\right) = \frac{1}{B_1} - \frac{1}{B_2} = 0.09346 - 0.09149 = 1.969\times 10^{-3}\,\text{T}^{-1}$$

$$\therefore \text{Number of oscillations} = \frac{1.969\times 10^{-3}}{1.639\times 10^{-5}} = \underline{\underline{120.2\ \text{oscillations}}}$$