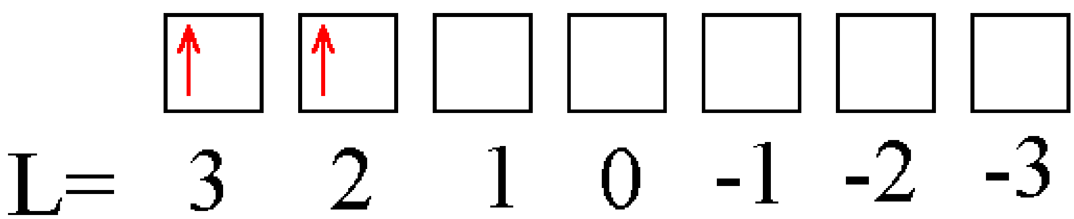
2. ***Magnetism of rare earth ions*** The basic electronic configurations of Pr^{3+}, Ho^{3+}, and Cr^{2+} are $4f^2$, $4f^{10}$, and $3d^4$ respectively. (a) Determine the ground state of these three ions with Hund's rule. (b) Assume the same ion density n=N/V, compare the susceptibility of a Ho^{3+} compound to that of Pr^{3+} at the same temperature. (c) Assume the same ion density n=N/V, compare the susceptibility of a Cr^{2+} compound to that of Pr^{3+} at the same temperature. The g-factor is given by the Lande equation:

$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}.$$

(a)

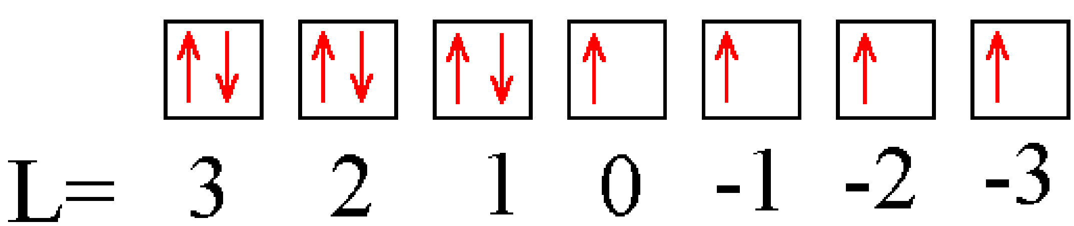
Pr^{3+} : $4f^2$



$\Sigma L = 5$
$\Sigma S = 1$
$J = |L - S| = 4$

Ground state: $\underline{\underline{{}^3H_4}}$

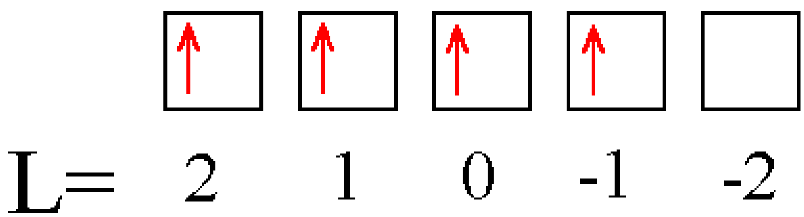
Ho^{3+} : $4f^{10}$



$\Sigma L = 6$
$\Sigma S = 2$
$J = L + S = 8$

Ground state: $\underline{\underline{{}^5I_8}}$

Cr^{2+} : $3d^4$



$\Sigma L = 2$
$\Sigma S = 2$
$J = |L - S| = 0$

Ground state: $\underline{\underline{{}^5D_0}}$

(b) For the same n and T, $\chi \propto p^2$, where $p = g\sqrt{j(j+1)}$.
g is given by the Lande equation:

$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

For Pr^{3+}, $L = 5$, $S = 1$, $J = 4$. $g(Pr^{3+}) = 1 + \frac{20 + 2 - 30}{40} = 0.8$

$p(Pr^{3+}) = 0.8 \times \sqrt{20} = 3.58$

For Ho^{3+}, $L = 6$, $S = 2$, $J = 8$. $g(Ho^{3+}) = 1 + \frac{72 + 6 - 42}{144} = 1.25$

$p(Ho^{3+}) = 1.25 \times \sqrt{72} = 10.6$

$$\therefore \frac{\chi(Ho^{3+})}{\chi(Pr^{3+})} = \left(\frac{p(Ho^{3+})}{p(Pr^{3+})}\right)^2 = \left(\frac{10.6}{3.58}\right)^2 = \underline{\underline{8.8}}$$

(c) We need to realize that Cr is a transition metal, and we cannot use the j obtained from Hund's rule. Instead, it is more accurate to set J = S and L = 0.

$\therefore$ For Cr^{2+}, J = S = 2 and L = 0. $g(Cr^{2+}) = 1 + \dfrac{6+6-0}{12} = 2$

$$p(Ho^{3+}) = 2 \times \sqrt{6} = 4.90$$

- From part (b), for Pr^{3+}, $p(Pr^{3+}) = 3.58$

$$\therefore \frac{\chi(Cr^{2+})}{\chi(Pr^{3+})} = \left(\frac{p(Cr^{2+})}{p(Pr^{3+})}\right)^2 = \left(\frac{4.90}{3.58}\right)^2 = \underline{\underline{1.87}}$$