Semiconductor crystal. For simplicity, let us assume there is only one conduction band, one valance band, and one donor band (i.e. degeneracy = 1 for each case) in this problem. The effective densities of state of the valance (N_v) and conduction (N_c) band are given as $N_v = 2\left(\frac{m_h k_B T}{2\pi\hbar^2}\right)^{\frac{3}{2}}$ and $N_c = 2\left(\frac{m_e k_B T}{2\pi\hbar^2}\right)^{\frac{3}{2}}$ respectively, where m_e and m_h are masses of electron and hole respectively, and T is temperature. Now consider an intrinsic semiconductor of energy gap E_g = 1 eV, $m_h = 0.5\ m_e$, and $N_c = 3 \times 10^{19}$ cm^{-3} at T=300K. (a) Calculate n and p in cm^{-3}, the carrier density of electron and hole respectively. (b) Calculate the Fermi energy E_F. The semiconductor is now doped with a donor impurity at a concentration of 10^{13} cm^{-3}. The donor ionization energy is so small that they are essentially 100% ionized (i.e. $N_D^+ = N_D$). (c) What is the new hole density (p) in cm^{-3}? (d) What is the new Fermi energy?

(a)

$$n = N_c \exp\left(-\frac{E_c - E_F}{k_B T}\right)$$

$$p = N_v \exp\left(-\frac{E_F - E_v}{k_B T}\right)$$

$$n = p \Rightarrow n^2 = N_c N_v \text{sxp}\left(-\frac{E_c - E_v}{k_B T}\right) = N_c N_v \exp\left(-\frac{E_g}{k_B T}\right)$$

$$N_v = \left(\frac{m_h}{m_e}\right)^{\frac{3}{2}} N_c = (0.5)^{\frac{3}{2}} \times 3 \times 10^{19} = 1.06 \times 10^{19} \text{ cm}^{-3}$$

$$\therefore\ n^2 = 3 \times 10^{19} \times 1.06 \times 10^{19} \times \exp\left(-\frac{1}{8.617 \times 10^{-5} \times 300}\right) = 5.044 \times 10^{21}$$

$$\Rightarrow n = p = 7.1 \times 10^{10} \text{ cm}^{-3}$$

(b)

$$n = N_c \exp\left(-\frac{E_c - E_F}{k_B T}\right) \Rightarrow 7.1 \times 10^{10} = 3 \times 10^{19} \exp\left(-\frac{E_c - E_F}{8.617 \times 10^{-5} \times 300}\right)$$

$$\Rightarrow \left(-\frac{E_c - E_F}{8.617 \times 10^{-5} \times 300}\right) = \ln(2.367 \times 10^{-9}) = -19.86$$

$$\Rightarrow E_c - E_F = 8.617 \times 10^{-5} \times 300 \times 19.86 = 0.513 \text{ eV}$$

$\therefore$ The Fermi energy is 0.513 eV below the conduction band edge.

(c)

Because of the donor impurity, n is now
$n = 10^{13}\ cm^{-3}$

$\therefore\ np = n_i p_i \quad \Rightarrow\ 10^{13} p = \ = 5.044 \times 10^{21}$
$\Rightarrow\ p = \underline{\underline{5.044 \times 10^{8}\ cm^{-3}}}$

(d)

$$n = N_c \exp\left(-\frac{E_c - E_F}{k_B T}\right) \quad \Rightarrow \quad 10^{13} = 3\times 10^{19} \exp\left(-\frac{E_c - E_F}{8.617\times 10^{-5} \times 300}\right)$$

$$\Rightarrow \left(-\frac{E_c - E_F}{8.617\times 10^{-5} \times 300}\right) = \ln(3.333\times 10^{-7}) = -14.914$$

$$\Rightarrow\ E_c - E_F = 8.617\times 10^{-5} \times 300 \times 14.914 = 0.386\ eV$$

The Fermi energy is now only 0.386 eV below the conduction band edge.