de Hass - van Alphen Effect. (a) Write down an expression in relating the flux quanta Φ_0 to fundamental physical constants $\hbar$, e, and c. (b) Consider an electron moving in close orbits in both k- and real space under a constant external magnet field B. Show that $S = \left(\frac{eB}{\hbar c}\right)^2 A$, where S and A are area enclosed by the orbits in k- and real space respectively. (c) The following figure shows data for De Haas-van Alphen oscillations in silver The magnetic field is along a <111> direction. The vertical axis is magnetic moment and the horizontal axis is $\frac{1}{B}$. Estimate the ratio of the areas of the two extremal orbits responsible for the oscillations as shown in the lower figure.

(a) $\Phi_0 = \dfrac{2\pi\hbar c}{e}$

(b) $\hbar\dot{\vec{k}} = -\dfrac{e}{c}\,\vec{v} \times \vec{B} \quad \Rightarrow \quad \hbar\vec{k} = -\dfrac{e}{c}\,\vec{r} \times \vec{B}$

$$\Rightarrow \quad \vec{k} = -\frac{e}{\hbar c}\,\vec{r} \times \vec{B}$$

$\therefore \ \vec{k} \perp \vec{B}$

If $r_\perp$ is the component of $\vec{r}$ perpendicular to $\vec{B}$, then

$$k = -\frac{e}{\hbar c} r_\perp B$$

$$S = \oint \frac{1}{2} k^2 d\theta = \oint \frac{1}{2}\left(-\frac{e}{\hbar c} r_\perp B\right)^2 d\theta$$

$$= \left(\frac{eB}{\hbar c}\right)^2 \oint \frac{1}{2} r_\perp^2 d\theta$$

$$= \left(\frac{eB}{\hbar c}\right)^2 A$$

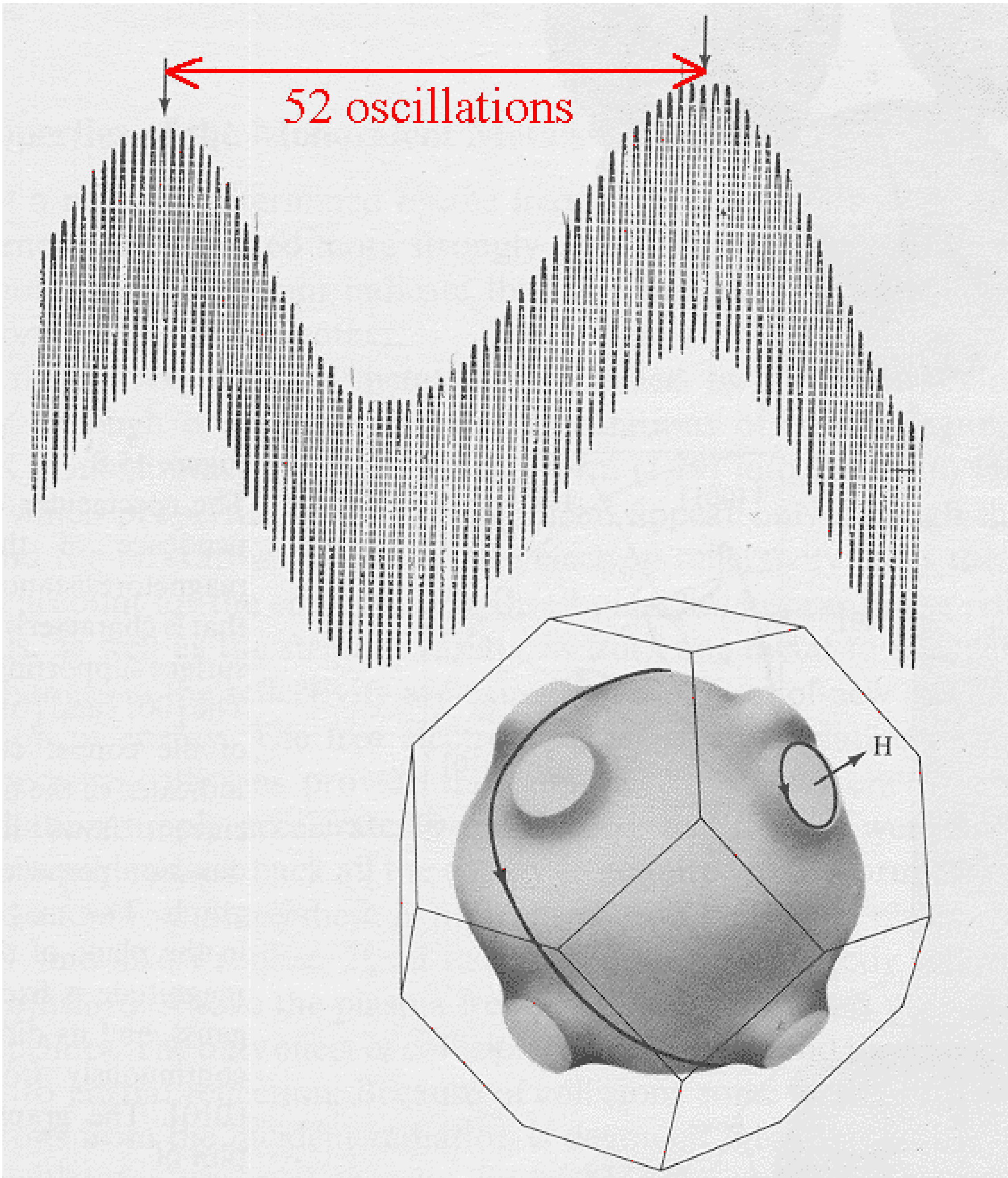
(c) From the above equation, one can induce that

$$S\frac{1}{B} = \left(\frac{e}{\hbar c}\right)^2 \Phi \ \Rightarrow \ S\Delta\left(\frac{1}{B}\right) = \left(\frac{e}{\hbar c}\right)^2 \Delta\Phi = \left(\frac{e}{\hbar c}\right)^2 \Phi_0$$

$$\Rightarrow \ S \propto \left[\Delta\left(\frac{1}{B}\right)\right]^{-1}$$

In the figure, there are two periodicity in $\frac{1}{B}$, the faster oscillation corresponds to a smaller area (because it takes a smaller field to produce a flus quanta for larger area). In one slow oscillation, there are 52 fast oscillations (by counting).

$$\therefore \ \frac{A_{belly}}{A_{neck}} = 52$$


52 oscillations
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