1. ***Surface plasmons.*** Consider a semi-infinite plasma on the positive side of the plane z = 0. The negative side (z<0) is vacuum. Given solution of Laplace's equation $\nabla^2\phi=0$ in the plasma is $\phi_i(x,z) = A \cos kx\, e^{-kz}$ and that of vacuum is $\phi_0(x,z) = A \cos kx\, e^{kz}$. (a) Write down all boundary conditions of electric fields at the boundary z=0. (b) Show that the frequency ω_s of a surface plasma oscillation is $\omega_s = \omega_p / \sqrt{2}$, where ω_p is the plasma frequency. (c) The electron concentration in a copper sample is 8×10^{22} cm^{-3}, mean free path is ~400Å, and the Fermi velocity is 1.6×10^{8} cm s^{-1}. Mass of an electron is 9.11×10^{-28} g. Estimate the plasma frequency at the *surface* of this sample.

Solution:

(a) $(D_\perp)_i = (D_\perp)_0 \quad\Rightarrow\quad (\varepsilon E_\perp)_i = (E_\perp)_0$

$(E_{//})_i = (E_{//})_0$

(b) $(E_\perp)_i = -Ak\cos kx\, e^{-kz}$

$(E_\perp)_0 = Ak\cos kx\, e^{kz}$

$(\varepsilon E_\perp)_i = (E_\perp)_0 \quad\Rightarrow\quad -\varepsilon(\omega_s)\, Ak\cos kx\, e^{-kz} = Ak\cos kx\, e^{kz}$

$$\Rightarrow\ \varepsilon(\omega_s) = -1$$

In general, $\varepsilon(\omega) = 1 - \dfrac{\omega_p^2}{\omega^2} \quad\Rightarrow\quad -1 = 1 - \dfrac{\omega_p^2}{\omega_s^2}$

$$\Rightarrow\ \omega_s^2 = \frac{\omega_p^2}{2} \quad\Rightarrow\quad \underline{\underline{\omega_s = \frac{\omega_p}{\sqrt{2}}}}$$

(c) $$\omega_p^2 = \frac{4\pi n e^2}{m} = \frac{4\pi\times8\times10^{22}\times(4.8\times10^{-10})^2}{9.11\times10^{-28}}$$

$$= 2.54\times10^{32}$$

$$\Rightarrow\quad \omega_p = 1.6\times10^{16}\ /\text{s}$$

$$\therefore\ \omega_s = \underline{\underline{1.1\times10^{16}\ /\text{s}}}$$
