**University of Kentucky**
**Department of Physics and Astronomy**

PHY 520 Introduction to Quantum Mechanics
Spring 2008
Test 2

Name: ____________________________________

Answer all questions. Write down all work in detail.

*Do as many problems as possible. The ideal score of this test is considered as 80. Your actual score will be 80 if your total exceeds this value.*

Time allowed: 50 minutes

1. (25 points)

The eigenfunctions for a potential of the form

$$V(x) = \infty \qquad x < 0;\ x > a$$
$$= 0 \qquad 0 < x < a$$

are of the form

$$u_n(x) = \sqrt{\frac{2}{a}}\,\sin\left(\frac{n\pi x}{a}\right)$$

Suppose a particle in the preceding potential has an initial normalized wave function of the form

$$\psi(x) = A\cos^4\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi x}{a}\right)$$

(a) (10 points)
What is the form of $\psi(x,t)$?

(b) ( 7 points)
Calculate the normalization constant A.

(c) (8 points)

What is the probability that an energy measurement yields $E_3$, where

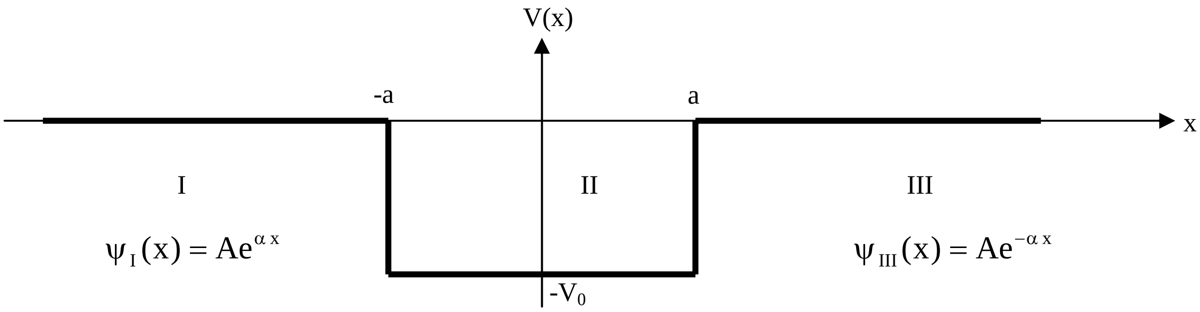
$$E_n = \frac{n^2\pi^2\hbar^2}{2ma^2} \quad ?$$

2. (25 points)

Consider the potential in the form

$$V(x) = -V_0 \qquad -a<x<a$$
$$= 0 \qquad x<-a \text{ or } x>a$$

and energy $E < 0$.




$$\psi_{II}(x) = \quad B\cos qx \quad (\text{Case } 1) \quad \text{or} \quad B\sin qx \quad (\text{Case } 2)$$

(a) (5 points)

With the wave functions given in the above figure, what are q and $\alpha$ in terms of $V_0$, E, and mass m?

(b) (10 points)

For both cases of $\psi_{II}$, derive dispersion relationship between E, $\alpha$, and q.

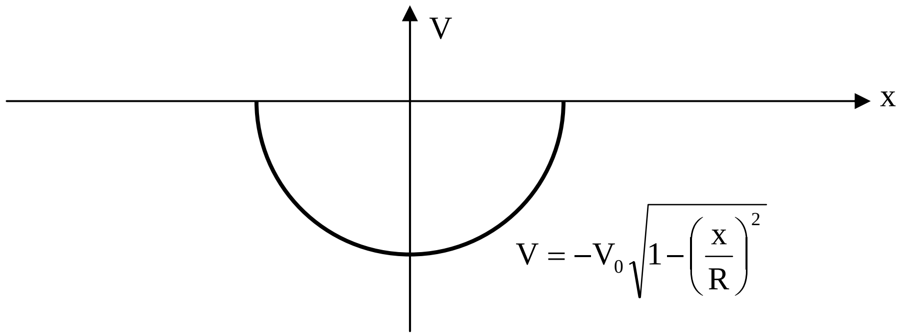
(c) (3 points)

Ground state belongs to which case of $\psi_{II}$ (i.e. $\psi_{II}$ = B cos qx or B sin qx)? Briefly explain your answer. The first excited state belongs to which case of $\psi_{II}$?

(d) (7 points)
Calculate the minimum $V_0$ required to ensure there is at least two bound states in the well.

3. (25 points)
Undergraduate students: do either 3A or 3B, but NOT both.
Graduate students: do 3B only.

3A. (25 points)
Consider a particle of mass m in a "semi-circular" potential as shown in the following diagram




(a) (10 points)

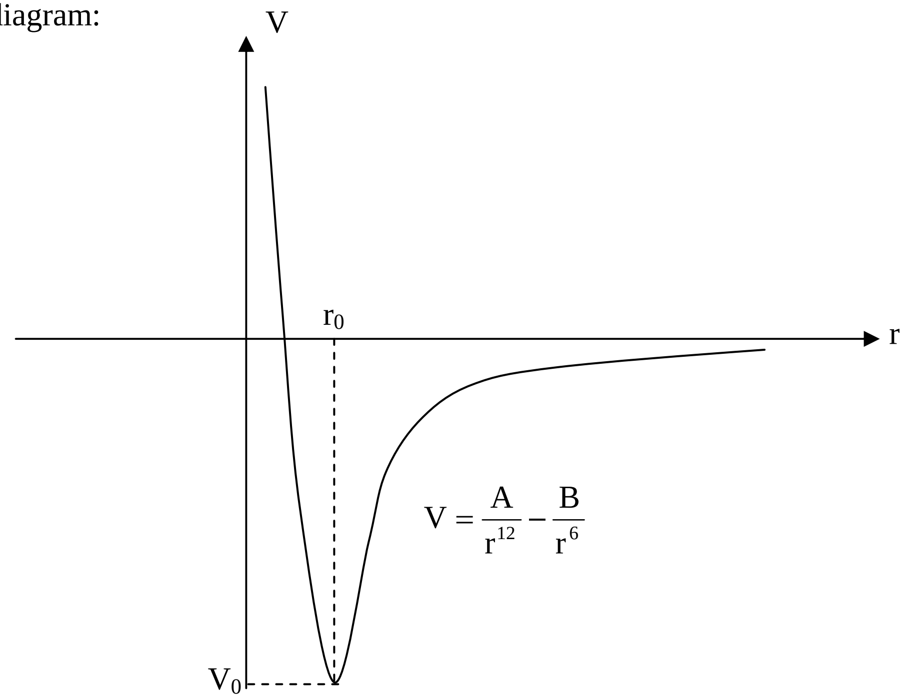
Approximate above potential at its minimum with another potential $V=Ax^2+B$. Write A and B in terms of R and $V_0$.

(b) (10 points)
Estimate the ground state energy.

(c) (5 points)
What are $\langle x \rangle$ and $\langle p \rangle$?

3B. (25 points)

Consider a particle of mass m in a "6-12" potential as shown in the following diagram:




(a) (5 points)
Determine $r_0$ and $V_0$ in terms of A and B.

(b) (10 points)
Approximate above potential at its minimum with another potential $V=k(r-r_0)^2/2+C$. Write k and C in terms of A and B.

(c) (10 points)
Estimate the ground state energy.

4. (25 points)

Answer all five questions. Circle the right answer.

4A. The ladder opperator of a simple harmonic oscillator is defined as

$$a = \left( \sqrt{\frac{m\omega}{2\hbar}} x - i \frac{p}{\sqrt{2m\omega}} \right)$$

Which of the following is not correct?

A. $[a^+, a] = 0$

B. $[H, a^+ a] = 0$

C. $[H, a^+] = \hbar\omega a^+$

D. $H = \left( a^+ a + \frac{1}{2} \right) \hbar\omega$

E. $a \,|\, 0> = 0$ where $|\, 0>$ is the ground state

4B. Which of the following is not correct about the Hermitian operator A:

A. All eigenvalues of A must be real.

B. All eigenvalues of A must have different values. No two of them have the same value.

C. For continuous eigenfunctions, $\int \psi (A\phi\psi_n)^* dx = \int \phi^* A^+ \psi dx$

D. For any operator X (not necessary Hermitian), $A = X^+ X$ is Hermitian.

E. In quantum mechanics, the operator of a physical observable must be Hermitian..

4C. Which of the following expression is not correct:

A. $\sum_n \,|\, n >< n \,|= 1$

B. $\sum_n \, < n \,|\, n >= 1$

C. $< \psi \,|\, \phi >^* = < \phi \,|\, \psi >$

D. $< \phi \,|\, \alpha\psi_1 + \beta\psi_2 >= \alpha < \phi \,|\, \psi_1 > + \beta < \phi \,|\, \psi_2 >$

E. $< \psi \,|\, A \,|\, \phi >^* = < \phi \,|\, A^+ \,|\, \psi >$

4D. Which of the following is a possible eigenstate of the simple harmonic potential. In all these equations, $y = (2\pi m\omega/h)^{1/2}$ and $\omega=(k/m)^{1/2}$. A is the normalization constant.

A. $\Psi(x) = A(32y^6 - 160y^3 + 129y)\exp(y^2/2)$

B. $\Psi(x) = A(16y^4 + 48y^2 + 12)\exp(-y^2/2)$

C. $\Psi(x) = A(4y^2 - 2)\exp(-y/2)$

D. $\Psi(x) = A(2y)\exp(-y^2/2)$

E. $\Psi(x) = A(8y^3 - 12y)\ \exp(y^2/2)$

4E. What is the energy eigenvalue of the simple harmonic eigenstate

$$\psi(x) = A(64y^6-480y^4+720x^2-120)\exp(-y^2/2)$$

where $y = (2\pi m\omega/h)^{1/2}$, $\omega=(k/m)^{1/2}$, and A is the normalization constant?

A. $\frac{1}{2}\hbar\omega$

B. $\frac{3}{2}\hbar\omega$

C. $\frac{7}{2}\hbar\omega$

D. $6\hbar\omega$

E. $\frac{13}{2}\hbar\omega$