Problem 1: Answer the following question concerning vectors.

Part (a) From the given list choose all that are examples of vectors.

1) Force.
2) Speed.
3) Velocity.
4) Mass.
5) Volume.
6) Acceleration.
7) Temperature.

Answer: 1) Force 2) Velocity and 6) Acceleration are vectors. All other quantities do not have direction and they are scalars.
Problem 2: Consider the two vectors $A$ and $B$. You know the magnitudes of these vectors (1 m and 10 m respectively), but you do not know anything about their directions.

Part (a) If a vector $C$ is defined to be the sum of these two vectors (i.e. $C = A + B$) which of the following are true about the magnitude of $C$? Choose all that apply.

MultipleSelect:

1) $C_{\text{min}} = 9$
2) $C_{\text{min}} = (1^2 + 10^2)^{0.5}$
3) $C_{\text{max}} = 11$
4) $C_{\text{min}}$ cannot be determined.
5) $C_{\text{max}} = 10$
6) $C_{\text{max}} = (1^2 + 10^2)^{0.5}$
7) $C_{\text{min}} = 0$
8) $C_{\text{min}} = -9$

1) Correct. $C$ is minimum when

2) Wrong. Cannot be right if 1) is correct.

3) Correct. $C$ is maximum when

4) Wrong. Cannot be right if 1) is correct.

5) Wrong. Cannot be right if 3) is correct.

6) Wrong. Cannot be right if 3) is correct.

7) Wrong. Cannot be right if 1) is correct.

8) Wrong. Cannot be right if 1) is correct.
Problem 3: Vector \( \mathbf{A} \) has a given magnitude of \( |\mathbf{A}| \) and points due East. Vector \( \mathbf{B} \) of unknown magnitude points directly North. The sum \( \mathbf{A} + \mathbf{B} \) makes an angle of \( \theta \) degrees North of East.

Part (a) Write an expression for the magnitude of vector \( \mathbf{B} \) in terms of the magnitude of \( \mathbf{A} \), \( |\mathbf{A}| \), and the angle \( \theta \), using any required trigonometric functions.

Part (b) What is the numerical value for the magnitude of the difference of vectors \( \mathbf{A} \) and \( \mathbf{B} \), \( |\mathbf{A} - \mathbf{B}| \)?

Part (c) What is the numerical value for the magnitude of the sum of vectors \( \mathbf{A} \) and \( \mathbf{B} \), \( |\mathbf{A} + \mathbf{B}| \)?

Numeric : A numeric value is expected and not an expression.

Solution:

(a)

\[
\tan \theta = \frac{|\mathbf{B}|}{|\mathbf{A}|} \Rightarrow |\mathbf{B}| = |\mathbf{A}| \tan \theta
\]

(b) I redraw the figure for clarity:
\[
\cos \phi = \frac{|\vec{A}|}{|\vec{A} - \vec{B}|}
\]

But \(\triangle POR\) is congruent to \(\triangle QRO\) (SAS), so \(\theta = \phi\)

\[
\therefore \cos \theta = \frac{|\vec{A}|}{|\vec{A} - \vec{B}|} \Rightarrow |\vec{A} - \vec{B}| = \frac{|\vec{A}|}{\cos \theta}
\]

(c)

[Diagram showing \(\vec{A} + \vec{B}\) and \(\theta\).]

\[
\cos \theta = \frac{|\vec{A}|}{|\vec{A} + \vec{B}|} \Rightarrow |\vec{A} + \vec{B}| = \frac{|\vec{A}|}{\cos \theta}
\]
**Problem 4**: Find the following for path B in the Figure shown where length of one block is $l = L$ m.

**Part (a)** Find the total distance traveled in km.

**Part (b)** Find the magnitude of the displacement in meters.

**Part (c)** Find the angle, in degrees, of the direction of the displacement measured North of East.

**Solution:**

(a) Path B: 4 blocks east, 3 blocks north, and 3 blocks west, total distance traveled = $4+3+3 = 10$ blocks.

∴ total distance traveled = $10L$ meters = $10L/1000 = L/100$ km

(b) $|\vec{r}| = \sqrt{(3L)^2 + (L)^2} = \sqrt{10L^2} = \sqrt{10} L = 3.162L$

(c) $\tan \theta = \frac{3L}{L} = 3 \Rightarrow \theta = \tan^{-1} 3 = 71.57^\circ$
The direction of the displacement is 71.57° North of East.
Problem 5: The figure depicts the sum of two velocities.

Part (a) Find the component of $v_{tot}$ along the x axis in m/s.

Part (b) Find the component of $v_{tot}$ along the y axis in m/s.

Solution:

(a)

$V_{tot\,x} = 6.72 \cos (26.5^\circ + 22.5^\circ)$

$= 6.72 \cos 49^\circ$

$= 4.409 \, \text{m/s}$

(b)

$V_{tot\,y} = 6.72 \sin (26.5^\circ + 22.5^\circ)$

$= 6.72 \sin 49^\circ$

$= 5.072 \, \text{m/s}$
**Problem 5:** The figure depicts the sum of two velocities.  
**Part (a)** Find the component of \( \mathbf{v}_{\text{tot}} \) along an \( x \) axis rotated \( 30^\circ \) counterclockwise relative to those in the Figure.  
**Part (b)** Find the component of \( \mathbf{v}_{\text{tot}} \) along a \( y \) axis rotated \( 30^\circ \) counterclockwise relative to those in the Figure.

Solution:

Let us do some simple geometry first. My first goal is to figure out the angle between \( \mathbf{v}_B \) and the \( x \)-axis.

\[
\theta + 30^\circ = 22.5^\circ + 26.6^\circ \Rightarrow \theta + 30^\circ = 49^\circ \Rightarrow \theta = 19^\circ
\]

(a)

\[
\mathbf{V}_{\text{Tot},x} = 6.72 \cos 19^\circ \\
= 6.354 \text{ m/s}
\]

(b)

\[
\mathbf{V}_{\text{Tot},y} = 6.72 \sin 19^\circ \\
= 2.188 \text{ m/s}
\]
Problem 7: A farmer wants to fence off his four-sided plot of flat land. He measures the first three sides, shown as $A$, $B$, and $C$ in the figure, where $A = A$ km, $B = B$ km, and $C = C$ km and then correctly calculates the length and orientation of the fourth side $D$.

Part (a) What is the length of the vector $D$ in kilometers?
Part (b) What is the orientation of the vector $D$, in degrees W of S?

(a)
\[ \tan \theta = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{|D_x|}{|D_y|} \Rightarrow \theta = \tan^{-1}\left(\frac{|D_x|}{|D_y|}\right) \]

\[ \Rightarrow \theta = \tan^{-1}\left(\frac{-0.9914A + 0.2756B + 0.9455C}{0.1305A - 0.9613B - 0.3256C}\right) \]
Problem 8: A ship sets sail from Rotterdam, The Netherlands, heading due north at \( V_{SW} \) m/s relative to the water. The local ocean current is \( V_w \) m/s in a direction \( \theta \) north of east.

**Part (a)** What is the magnitude of the velocity of the ship relative to the Earth in m/s?

**Part (b)** What is the angle of the velocity of the ship relative to the Earth in degrees N of E?

Solution

\[
\begin{align*}
\text{Velocity of ship relative to water} &= \text{Velocity of ship} - \text{velocity of water} \\
&= \vec{V}_{SW} = \vec{V}_S - \vec{V}_w \\
The problem is asking for \( \vec{V}_S \) \\
&\therefore \vec{V}_S = \vec{V}_{SW} + \vec{V}_w
\end{align*}
\]

(a)

\[
\begin{align*}
V_{sx} &= V_{swx} + V_{wx} \quad \Rightarrow \quad V_{sx} = 0 + V_w \cos \theta = V_w \cos \theta \quad \text{ (} V_{swx} = 0 \text{)} \\
V_{sy} &= V_{swy} + V_{wy} \quad \Rightarrow \quad V_{sy} = V_{sw} + V_w \sin \theta \quad \text{ (} V_{swy} = V_{sw} \text{)} \\
\therefore |\vec{V}_S| &= \sqrt{V_{sx}^2 + V_{sy}^2} = \sqrt{(V_w \cos \theta)^2 + (V_{sw} + V_w \sin \theta)^2} \\
&= \sqrt{V_w^2 \cos^2 \theta + (V_{sw}^2 + 2V_{sw}V_w \sin \theta + V_w^2 \sin^2 \theta)^2} \\
&= \sqrt{V_w^2 \cos^2 \theta + (V_{sw}^2 + 2V_{sw}V_w \sin \theta + V_w^2 \sin^2 \theta)^2} \\
&= \sqrt{V_w^2 + V_{sw}^2 + 2V_{sw}V_w \sin \theta}
\end{align*}
\]
\[\begin{align*}
V_{sx} &= V_{swx} + V_{wx} \quad \Rightarrow \quad V_{sx} = 0 + V_w \cos \theta = V_w \cos \theta \quad (V_{swx} = 0) \\
V_{sy} &= V_{swy} + V_{wy} \quad \Rightarrow \quad V_{sy} = V_{sw} + V_w \sin \theta \quad (V_{swy} = V_{sw}) \\
\tan \phi &= \frac{|V_{sy}|}{|V_{sx}|} = \frac{V_{sw} + V_w \sin \theta}{V_w \cos \theta} \quad \Rightarrow \quad \phi = \tan^{-1} \left( \frac{V_{sw} + V_w \sin \theta}{V_w \cos \theta} \right)
\end{align*}\]
**Problem 9:** An ice hockey player is moving at $V_{\text{player}}$ m/s when he hits the puck toward the goal. The speed of the puck relative to the player is $V_{\text{puck}}$ m/s. The line between the center of the goal and the player makes a $90.0^\circ$ angle relative to his path as shown in the figure.

**Part (a)** What angle must the puck’s velocity make relative to the player (in his frame of reference) to hit the center of the goal in degrees?

**Solution:**

The given figure and the way it names variables is actually kind of confusing, so let me clarify here:

- $V_{\text{player}}$ is the velocity of player relative to the Earth, no confusion here.
- $V_{\text{puck}}$ is actually the velocity of the puck relative to the player, so be careful here.
- Actually it is clear if you read the problem text carefully.
- $V_{\text{tot}}$ is the velocity of the puck relative to the Earth.

So Velocity of puck relative to player = velocity of puck – velocity of player will give $V_{\text{puck}} = V_{\text{tot}} - V_{\text{player}}$.

\[
\begin{align*}
    \vec{V}_{\text{puck}} &= \vec{V}_{\text{tot}} - \vec{V}_{\text{player}} \\
    \therefore \quad V_{\text{puck}} \cos \theta &= V_{\text{tot}} - 0 \quad (V_{\text{tot}} = V_{\text{tot}} \text{ and } V_{\text{player}} = 0) \\
    \Rightarrow \quad V_{\text{puck}} \cos \theta &= V_{\text{tot}} \quad - - - (1) \\
    V_{\text{puck}} \sin \theta &= V_{\text{tot}} - V_{\text{player}} \quad (V_{\text{tot}} = 0 \text{ and } V_{\text{player}} = V_{\text{player}}) \\
    \Rightarrow \quad V_{\text{puck}} \sin \theta &= V_{\text{player}} \quad - - - (2) \\
\end{align*}
\]

\[
(2)/(1) \Rightarrow \frac{V_{\text{puck}} \sin \theta}{V_{\text{puck}} \cos \theta} = \frac{V_{\text{player}}}{V_{\text{tot}}} \\
\Rightarrow \tan \theta = \frac{V_{\text{player}}}{V_{\text{tot}}} \\
\Rightarrow \theta = \tan^{-1} \left( \frac{V_{\text{player}}}{V_{\text{tot}}} \right)
\]
Problem 10: A famous golfer strikes a golf ball on the ground, giving it an initial velocity \( \mathbf{v} = v_0x \mathbf{i} + v_0y \mathbf{j} \). Assume the ball moves without air resistance and its motion is described using a Cartesian coordinate system with its origin located at the ball’s initial position.

Part (a) Determine the maximum height above ground, \( h_{\text{max}} \) in meters, attained by the golf ball.

Part (b) Express the total horizontal distance, \( x_{\text{max}} \), the ball travels when it returns to ground level in terms of \( v_{0x}, v_{0y}, \) and \( g \).

Part (c) Evaluate the total horizontal distance, \( x_{\text{max}} \) in meters, the ball travels when it returns to ground level.

Solution:

Equations of motion:

\[
\begin{align*}
    x &= x_i + v_{xi} \, t \quad \Rightarrow \quad x = v_{0x} \, t & \text{--- (1)} \\
    v_x &= v_{xi} & \text{--- (2)} \\
    y &= y_i + v_{yi} \, t + \frac{1}{2} a_y t^2 \quad \Rightarrow \quad y = v_{0y} \, t - \frac{1}{2} g \, t^2 & \text{--- (3)} \\
    v_y &= v_{yi} + a_y \, t \quad \Rightarrow \quad v_y = v_{0y} - gt & \text{--- (4)} \\
\end{align*}
\]

(a)
At highest point \( v_y = 0 \)

\[
(4) \Rightarrow 0 = v_{0y} - gt \quad \Rightarrow \quad t = \frac{v_{0y}}{g}
\]
Substitute this into (3) \( \Rightarrow y_{\text{highest}} = v_{oy} \left( \frac{v_{oy}}{g} \right) - \frac{1}{2} g \left( \frac{v_{oy}}{g} \right)^2 \)

\[
= \frac{v_{oy}^2}{g} - \frac{1}{2} \left( \frac{v_{oy}}{g} \right)^2
\]

\[
= \frac{1}{2} \frac{v_{oy}^2}{g}
\]

(b) and (c):
When the golf ball returns to ground, \( y = 0 \)

\( (3) \Rightarrow 0 = v_{oy} t - \frac{1}{2} g t^2 \Rightarrow 0 = t(v_{oy} - \frac{1}{2} gt) \)

\( \Rightarrow t = 0 \) or \( v_{oy} - \frac{1}{2} gt = 0 \)

\( \Rightarrow t = 0 \) or \( t = \frac{2v_{oy}}{g} \)

\( t = 0 \) corresponds to the launching point, so the ball will hit the ground again when \( t = \frac{2v_{oy}}{g} \)

Substitute this into (1) \( \Rightarrow x_{\text{range}} = v_{ox} \left( \frac{2v_{oy}}{g} \right) \)

\[
= \frac{2v_{ox} v_{oy}}{g}
\]
Problem 11: A quarterback throws a football with an initial velocity $v$ at an angle $\theta$ above horizontal. Assume the ball leaves the quarterback’s hand at ground level and moves without air resistance. All portions of this problem will produce algebraic expressions in terms of $v$, $\theta$, and $g$. Let the origin of the Cartesian coordinate system be the ball’s initial position.

Part (a) Write an expression for the magnitude of the football’s initial vertical velocity $v_{0y}$.

Part (b) Find an expression for the magnitude of the football’s initial horizontal velocity $v_{0x}$.

Part (c) Write an expression for the total time, $t_{total}$, the football is above the ground.

Solution:

Equations of motion:

\[ x = x_i + v_{xi} t \Rightarrow x = v_{0x} t \quad \text{--- (1)} \]

\[ v_x = v_{xi} \quad \text{--- (2)} \]

\[ y = y_i + v_{yi} t + \frac{1}{2} a_y t^2 \Rightarrow y = v_{0y} t - \frac{1}{2} g t^2 \quad \text{--- (3)} \]

\[ v_y = v_{yi} + at \Rightarrow v_y = v_{0y} - gt \quad \text{--- (4)} \]

(a) \[ v_{0y} = v_0 \sin \theta \]

(b) \[ v_{0x} = v_0 \cos \theta \]

(c) When the football returns to ground, $y = 0$

\[ (3) \Rightarrow 0 = v_{0y} t - \frac{1}{2} g t^2 \Rightarrow 0 = t(v_{0y} - \frac{1}{2} g t) \Rightarrow t = 0 \text{ or } 0 \quad v_{0y} - \frac{1}{2} g t = 0 \]

\[ \Rightarrow t = \frac{2v_{0y}}{g} \]

\[ \Rightarrow t = \frac{2v_0 \sin \theta}{g} \]

\[ \therefore \text{total time for the football above ground} = t_{total} = \frac{2v_0 \sin \theta}{g} - 0 = \frac{2v_0 \sin \theta}{g} \]
Problem 12: An arrow is fired upward with an initial velocity $v_0$ at an angle $\theta_0$ above horizontal. Assume the arrow moves without air resistance. Use a Cartesian coordinate system with the origin at the arrow’s initial position to analyze the arrow’s motion. 

Part (a) Some time later the arrow makes an angle of $\theta$ degrees with respect to the horizontal. Write an expression for the time, $t$, that passes between when the arrow was fired and this later point.

Solution:

Equations of motion:
\[ \begin{align*}
    x &= x_i + v_{xi} t \quad \Rightarrow \quad x = v_0 t \cos \theta_0 \quad \text{---(1)} \\
    v_x &= v_{xi} = v_0 \cos \theta_0 \quad \text{---(2)} \\
    y &= y_i + v_{yi} t + \frac{1}{2} a_y t^2 \quad \Rightarrow \quad y = v_0 t \sin \theta_0 - \frac{1}{2} g t^2 \quad \text{---(3)} \\
    v_y &= v_{yi} + at \quad \Rightarrow \quad v_y = v_0 \sin \theta_0 - gt \quad \text{---(4)}
\end{align*} \]

If the velocity at the point where the arrow makes an angle $\theta$ with respect to ground is $v$. $v_x = v \cos \theta$ and $v_y = v \sin \theta$

\[ \begin{align*}
    (2) \quad &\Rightarrow \quad v \cos \theta = v_0 \cos \theta_0 \quad \Rightarrow \quad v = v_0 \frac{\cos \theta_0}{\cos \theta} \\
    (4) \quad &\Rightarrow \quad v_y = v_0 \sin \theta_0 - gt \quad \Rightarrow \quad v \sin \theta = v_0 \sin \theta_0 - gt \\
    \quad &\Rightarrow \quad v_0 \frac{\cos \theta_0}{\cos \theta} \cdot \sin \theta = v_0 \sin \theta_0 - gt
\end{align*} \]
\[ \Rightarrow gt = v_0 \sin \theta_0 - v_0 \frac{\cos \theta_0}{\cos \theta} \cdot \sin \theta \]

\[ \Rightarrow t = \frac{v_0}{g} \left( \sin \theta_0 - \frac{\cos \theta_0 \cdot \sin \theta}{\cos \theta} \right) \]

\[ \Rightarrow t = \frac{v_0}{g} \left( \frac{\sin \theta_0 \cos \theta - \cos \theta_0 \sin \theta}{\cos \theta} \right) \]

\[ \Rightarrow t = \frac{v_0 \sin (\theta_0 - \theta)}{g \cos \theta} \]

\[ \Rightarrow t = \frac{v_0 \sin (\theta_0 - \theta)}{g \cos \theta} \]
Problem 13: A student throws a stone from the top of a cliff at a distance $d$ m above a horizontal plane with initial velocity $v_0$ m/s at an angle $\theta$ below horizontal. The stone moves without air resistance; use a Cartesian coordinate system with the origin at the stone’s initial position.

Part (a) With what speed, $v_f$ in m/s, does the stone strike the ground?

Part (b) If the stone had been thrown from the clifftop in the same manner but upward instead of downward, would its impact velocity be different?

Solution:

Equations of motion:

1. $x = x_i + v_{xi} t \implies x = v_0 t \cos \theta$  \hspace{1cm} --- (1)
2. $v_x = v_{xi} = v_0 \cos \theta$  \hspace{1cm} --- (2)
3. $y = y_i + v_{yi} t + \frac{1}{2} a_y t^2 \implies y = -v_0 \sin \theta - \frac{1}{2} g t^2$  \hspace{1cm} --- (3)
4. $v_y = v_{yi} + at \implies v_y = -v_0 \sin \theta - gt$  \hspace{1cm} --- (4)

When the stone hits the bottom of the cliff, $y = -d$

(3) $\Rightarrow -d = -v_0 \sin \theta - \frac{1}{2} g t^2$

$\Rightarrow -2d = -(2v_0 \sin \theta)t - g t^2$

$\Rightarrow g t^2 + (2v_0 \sin \theta)t - 2d = 0$

$\Rightarrow t = \frac{-2v_0 \sin \theta \pm \sqrt{(2v_0 \sin \theta)^2 - 4g(-2d)}}{2g}$

$\Rightarrow t = \frac{-v_0 \sin \theta \pm \sqrt{v_0^2 \sin^2 \theta + 2gd}}{g}$
\[
\sqrt{v_0^2 \sin^2 \theta + 2gd} > v_0 \sin \theta
\]

so \( t \) has two roots, one positive and one negative. We will take the positive root.

\[
\Rightarrow t = \frac{-v_0 \sin \theta + \sqrt{v_0^2 \sin^2 \theta + 2gd}}{g}
\]

(4) \( \Rightarrow v_{fy} = -v_0 \sin \theta - \frac{-v_0 \sin \theta + \sqrt{v_0^2 \sin^2 \theta + 2gd}}{g} \)

\[
\Rightarrow v_{fy} = -v_0 \sin \theta - (-v_0 \sin \theta + \sqrt{v_0^2 \sin^2 \theta + 2gd})
\]

\[
\Rightarrow v_{fy} = -v_0 \sin \theta + v_0 \sin \theta - \sqrt{v_0^2 \sin^2 \theta + 2gd}
\]

\[
\Rightarrow v_{fy} = -\sqrt{v_0^2 \sin^2 \theta + 2gd}
\]

(2) \( \Rightarrow v_{fx} = v_0 \cos \theta \)

\[
\Rightarrow |v_f| = \sqrt{v_{fx}^2 + v_{fy}^2} = \sqrt{(v_0 \cos \theta)^2 + (-\sqrt{v_0^2 \sin^2 \theta + 2gd})^2}
\]

\[
= \sqrt{v_0^2 \cos^2 \theta + v_0^2 \sin^2 \theta + 2gd}
\]

\[
= \sqrt{v_0^2 + 2gd} \quad (\cos^2 \theta + \sin^2 \theta = 1)
\]
(b) If the stone is thrown in similar manner but upward in the of downward, this will only affect the sign of some terms in the equations of motion. Equation (3) and (4) has to be rewritten as:

\[ y = y_1 + v_{yi} t + \frac{1}{2} a_y t^2 \Rightarrow y = + v_0 t \sin \theta - \frac{1}{2} g t^2 \quad - - - (3) \]

\[ v_y = v_{yi} + at \Rightarrow v_y = + v_0 \sin \theta - gt \quad - - - (4) \]

New (3) \Rightarrow \quad -d = +v_0 t \sin \theta - \frac{1}{2} g t^2

\[ \Rightarrow g t^2 - (2v_0 \sin \theta)t - 2d = 0 \]

\[ \Rightarrow t = \frac{2v_0 \sin \theta \pm \sqrt{(2v_0 \sin \theta)^2 - 4g(-2d)}}{2g} \]

\[ \Rightarrow t = \frac{v_0 \sin \theta \pm \sqrt{v_0^2 \sin^2 \theta + 2gd}}{g} \]

Taking positive \( t \), \( t = \frac{v_0 \sin \theta + \sqrt{v_0^2 \sin^2 \theta + 2gd}}{g} \)

New (4) \Rightarrow \quad v_{yf} = + v_0 \sin \theta - gt \Rightarrow v_{yf} = + v_0 \sin \theta - g\left(\frac{v_0 \sin \theta + \sqrt{v_0^2 \sin^2 \theta + 2gd}}{g}\right)

\[ \Rightarrow v_{yf} = -\sqrt{v_0^2 \sin^2 \theta + 2gd} \]

So we have the same \( v_{yf} \) as if the stone is thrown downward. We can conclude that the impact velocity should be the same.
Problem 14: A ball is kicked with an initial velocity of $V_0x$ m/s in the horizontal direction and $V_0y$ m/s in the vertical direction.

Part (a) At what speed does the ball hit the ground in m/s?
Part (b) For how long does the ball remain in the air in seconds?
Part (c) What maximum height is attained by the ball in meters?

Solution

Equations of motion:
1. $x = x_i + v_{xi} t \Rightarrow x = v_{0x} t$  --- (1)
2. $v_x = v_{xi} \Rightarrow v_x = v_{x0}$  --- (2)
3. $y = y_i + v_{yi} t + \frac{1}{2} a_y t^2 \Rightarrow y = v_{0y} t - \frac{1}{2} g t^2$  --- (3)
4. $v_y = v_{yi} + at \Rightarrow v_y = v_{oy} - gt$  --- (4)

Answer part (b) first:
(b) When the football returns to ground, $y = 0$

(3) $\Rightarrow 0 = v_{0y} t - \frac{1}{2} g t^2 \Rightarrow 0 = t( v_{0y} - \frac{1}{2} g t) \Rightarrow t = 0$ or $v_{0y} - \frac{1}{2} g t = 0$

$\Rightarrow t = 0$ or $t = \frac{2v_{0y}}{g}$

$t = 0$ corresponds to the launching point, so the ball will hit the ground again when $t = \frac{2v_{0y}}{g}$

The ball will stay above ground for a period of $\frac{2v_{0y}}{g} - 0 = \frac{2v_{0y}}{g}$

Now part (a):
(a) Substitute $t = \frac{2v_{0y}}{g}$ into (3) $\Rightarrow v_y = v_{0y} - g\left(\frac{2v_{0y}}{g}\right)$

$\Rightarrow v_y = v_{0y} - 2v_{0y}$

$\Rightarrow v_y = -v_{0y}$ (make sense?)
(2) \( \Rightarrow v_x = v_{x0} \)

\[ \because \text{the ball will hit the ground with a speed} | \vec{v} | = \sqrt{v_x^2 + v_y^2} = \sqrt{v_{x0}^2 + (-v_{oy})^2} = \sqrt{v_{x0}^2 + v_{oy}^2} \]

(c)

At highest point \( v_y = 0 \)

(4) \( \Rightarrow 0 = v_{oy} - gt \Rightarrow t = \frac{v_{oy}}{g} \)

Substitute this into (3) \( \Rightarrow y_{\text{highest}} = v_{oy}\left(\frac{v_{oy}}{g}\right) - \frac{1}{2}g\left(\frac{v_{oy}}{g}\right)^2 \)

\[ = \frac{v_{oy}^2}{g} - \frac{1}{2} \frac{v_{oy}^2}{g} = \frac{1}{2} \frac{v_{oy}^2}{g} \]
**Problem 15:** A football player punts the ball at a \( \theta_0 \) angle. Without an effect from the wind, the ball would travel \( R \) m horizontally.

**Part (a)** What is the initial speed of the ball in m/s?

**Part (b)** When the ball is near its maximum height it experiences a brief gust of wind that reduces its horizontal velocity by \( \Delta v \) m/s. What distance does the ball travel horizontally?

**Solution**

Equations of motion:

\[
x = x_i + v_{xi} t \quad \Rightarrow \quad x = v_0 t \cos \theta_0 \quad \text{--- (1)}
\]

\[
v_x = v_{xi} = v_0 \cos \theta_0 \quad \text{--- (2)}
\]

\[
y = y_i + v_{yi} t + \frac{1}{2} a_y t^2 \quad \Rightarrow \quad y = v_0 t \sin \theta_0 - \frac{1}{2} g t^2 \quad \text{--- (3)}
\]

\[
v_y = v_{yi} + at \quad \Rightarrow \quad v_y = v_0 \sin \theta_0 - gt \quad \text{--- (4)}
\]

(a) When the ball hits the ground again, \( y = 0 \)

\[
(3) \quad \Rightarrow \quad 0 = v_0 t \sin \theta_0 - \frac{1}{2} g t^2 \quad \Rightarrow \quad 0 = t \left( v_0 \sin \theta_0 - \frac{1}{2} g t \right) \quad \Rightarrow \quad t = 0 \text{ or } v_0 \sin \theta_0 - \frac{1}{2} g t = 0
\]

\[
\Rightarrow \quad t = 0 \quad \text{or} \quad t = \frac{2v_0 \sin \theta_0}{g}
\]

\( t = 0 \) corresponds to the launching point, so the ball will hit the ground again when \( t = \frac{2v_0 \sin \theta_0}{g} \)

(1) \quad \Rightarrow \quad R = v_0 \left( \frac{2v_0 \sin \theta_0}{g} \right) \cos \theta_0
\]

\[
\Rightarrow \quad R = \frac{2v_0^2 \sin \theta_0 \cos \theta_0}{g}
\]

\[
\Rightarrow \quad R = \frac{v_0^2 \sin 2 \theta_0}{g} \quad \text{ (sin}2\theta_0 = 2 \sin \theta_0 \cos \theta_0) \]

\[
\Rightarrow \quad v_0^2 = \frac{gR}{\sin 2 \theta_0} \quad \text{(} \theta_0 \neq 0^\circ , 90^\circ \text{)}
\]

\[
\Rightarrow \quad v_0 = \sqrt{\frac{gR}{\sin 2 \theta_0}}
\]
(b) At highest point \( v_y = 0 \)

\[
(4) \Rightarrow 0 = v_{0y} - gt \Rightarrow t = \frac{v_0 \sin \theta_0}{g}
\]

Substitute this into (3) \( \Rightarrow y_{\text{highest}} = v_{0y} \left( \frac{v_0 \sin \theta_0}{g} \right) - \frac{1}{2} g \left( \frac{v_0 \sin \theta_0}{g} \right)^2
\]

\[
= \frac{v_0^2 \sin \theta_0}{g} - \frac{1}{2} \left( \frac{v_0 \sin \theta_0}{g} \right)^2
\]

\[
= \frac{1}{2} \frac{v_0^2 \sin \theta_0}{g}
\]

\[
= \frac{1}{2} \frac{\sin \theta_0}{g} \cdot \frac{gR}{\sin 2 \theta_0} \quad (\theta_0 \neq 0^\circ, 90^\circ, v_0^2 = \frac{gR}{\sin 2 \theta_0} \text{ from part (a)})
\]

\[
= \frac{1}{2} \frac{\sin \theta_0}{g} \cdot \frac{gR}{2 \sin \theta_0 \cos \theta_0} = \frac{R}{4 \cos \theta_0}
\]

Redefine coordinates and reset time after reaching the highest point:

\[
\begin{align*}
\text{Equations of motion:} & \\
x &= x_0 + v_{x_0} t \quad \Rightarrow \quad x = (v_0 \cos \theta_0 - \Delta v) t & \quad \quad \text{--- (5)} \\
v_x &= v_{x_0} = v_0 \cos \theta_0 - \Delta v & \quad \quad \text{--- (6)} \\
y &= y_0 + v_{y_0} t + \frac{1}{2} a_y t^2 \quad \Rightarrow \quad y = \frac{R}{4 \cos \theta_0} + 0t - \frac{1}{2} g t^2 \quad \Rightarrow \quad y = \frac{R}{4 \cos \theta_0} - \frac{1}{2} g t^2 & \quad \quad \text{--- (7)} \\
v_y &= v_{y_0} + at \quad \Rightarrow \quad v_y = -gt & \quad \quad \text{--- (8)}
\end{align*}
\]

(a) When the ball hits the ground again, \( y = 0 \)

\[
(3) \Rightarrow 0 = \frac{R}{4 \cos \theta_0} - \frac{1}{2} g t^2
\]
\[ \Rightarrow \frac{1}{2} gt^2 = \frac{R}{4 \cos \theta_0} \]
\[ \Rightarrow t^2 = \frac{R}{2 g \cos \theta_0} \]
\[ \Rightarrow t = \sqrt{\frac{R}{2 g \cos \theta_0}} \]

(5) \[ \Rightarrow x = (v_o \cos \theta_0 - \Delta v) t \]
\[ \Rightarrow x = (v_o \cos \theta_0 - \Delta v) \sqrt{\frac{R}{2 g \cos \theta_0}} \]

But from part (a) \[ v_o = \sqrt{\frac{g R}{\sin 2 \theta_0}} \]
\[ \therefore R' = (v_o \cos \theta_0 - \Delta v) \sqrt{\frac{R}{2 g \cos \theta_0}} = \left( \sqrt{\frac{g R}{\sin 2 \theta_0}} \cos \theta_0 - \Delta v \right) \sqrt{\frac{R}{2 g \cos \theta_0}} \]
\[ = \left( \sqrt{\frac{g R}{2 \sin \theta_0 \cos \theta_0}} \cos^2 \theta_0 - \Delta v \right) \sqrt{\frac{R}{2 g \cos \theta_0}} \]
\[ = \left( \sqrt{\frac{g R}{2 \tan \theta_0}} - \Delta v \right) \sqrt{\frac{R}{2 g \cos \theta_0}} \]

\[ \therefore \text{Horizontal distance travelled by the football} = \frac{R}{2} + R' \]
\[ = \frac{R}{2} + \left( \sqrt{\frac{g R}{2 \tan \theta_0}} - \Delta v \right) \sqrt{\frac{R}{2 g \cos \theta_0}} \]