Non-equilibrium Josephson Oscillations of Trapped Bose-Einstein Condensates

Johann Kroha\textsuperscript{1}

Collaborators:
Mauricio Trujillo Martinez\textsuperscript{1}
Anna Posazhennikova\textsuperscript{2}

\textsuperscript{1} Institute of Physics, University of Bonn, Germany
\textsuperscript{2} Department of Physics, University of Konstanz, Germany


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Motivation

Measurement of the quantum phase

Merging of two Bose condensates in a double-well potential non-adiabatic

→ Josephson oscillations and damping

With repulsive interaction:
Initial state energy $E_1^0 + E_2^0 \gg E_{merge}^0$ ground state energy of merged system

→ Thermalization in final state?
Where do entropy and energy go (isolated system)?
Time scale(s) ?

→ Bogoliubov quasiparticle excitations

Experiments by Oberthaler Thywissen groups
Particle current oscillations between two weakly coupled, macroscopic, coherent quantum systems

**Conditions:**
1) well defined quantum phase (grand canonical ensemble)
2) weak coupling

\[ I = I_c \sin(\Delta \theta) \]

\[ \Delta \theta = \text{const} \]

\[
\frac{\partial \Delta \theta}{\partial t} = -\frac{\Delta \mu}{\hbar} = -\frac{2e \Delta V}{\hbar}
\]

Introduction: Josephson effect in bulk systems
Dilute and cold systems: contact interaction:

Microscopic (quantum) Hamiltonian:

\[ \hat{H} = \frac{\hbar^2}{2m} \nabla^2 + V_{\text{ext}}(r) \]

\[ \hat{H} = \int \left[ \frac{\hbar^2}{2m} \nabla^2 + V_{\text{ext}}(r) \right] \hat{\psi}^+(r,t) \hat{\psi}(r,t) \, dr \]

\[ \alpha_s = \frac{4\pi\hbar^2 a_s}{m} \]

Cold bosonic gas in a double-well potential.
Relative population imbalance: \( n = \frac{N_1 - N_2}{N_{tot}} \)

Phase difference: \( \Delta \theta \)

Interaction parameter (inverse mass): \( \Lambda = \frac{UN_{tot}}{2J} \)

Classical Josephson Hamilton function:

\[
H / J = \frac{\Lambda}{2} n^2 + \Delta E_n - \sqrt{1 - n^2} \cos(\Delta \theta)
\]
Josephson oscillations in a double-well

Josephson frequency:

Smerzi et al., PRL 79, 4690 (1997)

\[ \omega_J = 2|J| \sqrt{1 + \frac{N_{tot}U}{2J}} \]

\[ \Lambda > \Lambda_c \]
self-trapping: rotating pendulum AC Josephson

delocalized: small-amplitude oscillations
Josephson oscillations: Experiment

M. Albiez, … M. Oberthaler et al., PRL 95, 010402 (2005); Thywissen et al. (2010)
Phase space portrait

Josephson oscillations: experiment

M. Albiez, … M. Oberthaler et al., PRL 95, 010402 (2005); Thywissen et al. (2010)
Bose Josephson junction out of equilibrium

Instantaneous switching on the Josephson coupling

\[ J(t) = J\theta(t) \]

Coupling to Bogoliubov quasiparticle excitations
Bose Josephson junction out of equilibrium

Bogoliubov quasiparticles out of single-particle states of the trap

\[ \hat{\Psi}(\mathbf{r}, t) = \phi_1(\mathbf{r}) a_1(t) + \phi_2(\mathbf{r}) a_2(t) + \sum_{n \neq 0} \varphi_n(\mathbf{r}) \hat{b}_n(t) \]

Classical condensate amplitudes:

\[ a_\alpha(t) = \sqrt{N_\alpha} e^{i\theta_\alpha(t)} \]

Bose field operators for excitations:

\[ \hat{b}_n(t) \]

\[ H = H_{\text{BEC}} + H_{\text{qp}} + H_{\text{mix}} \]
\[ H = H_{BEC} + H_{qp} + H_{mix} \]

\[ H_{BEC} = E_0 \sum_{\alpha=1}^{2} a_\alpha^* a_\alpha + \frac{U}{2} \sum_{\alpha=1}^{2} (a_\alpha^* a_\alpha^* a_\alpha a_\alpha) - J (a_1^* a_2 + a_2^* a_1) \]

\[ H_{qp} = \sum_{n \neq 0} E_n \hat{b}_n^\dagger \hat{b}_n + \frac{U'}{2} \sum_{n,m} \hat{b}_n^\dagger \hat{b}_n^\dagger \hat{b}_m \hat{b}_m \]

\[ H_{mix} = J' \sum_n \left[ (a_1^* a_2 + a_2^* a_1) \hat{b}_n^\dagger \hat{b}_n + \frac{1}{2} (a_1^* a_2^* \hat{b}_n \hat{b}_n + h.c.) \right] \]

\[ + K \sum_n \left[ \left( \sum_{\alpha=1}^{2} a_\alpha^* a_\alpha \right) \hat{b}_n^\dagger \hat{b}_n + \frac{1}{4} (a_1^* a_1^* \hat{b}_n \hat{b}_n + h.c.) \right] \]
Equations of motion: Keldysh

**Quasiparticles (Hartree-Fock selfenergies)**

\[
i \frac{\partial}{\partial T} G_{nn}^<(T) = \Omega_n(T) F_{nn}^<(T) - \overline{\Omega}_n(T) F_{nn}^<(T),
\]

\[
\left( i \frac{\partial}{\partial T} - 2E_n - 2\Sigma_n(T) \right) F_{nn}^<(T) = \Omega_n(T) \overline{G}_{nn}^<(T) + \Omega_n(T) G_{nn}^<(T)
\]

**Condensates**

\[
i \frac{\partial}{\partial T} a_1(T) = \left[ U |a_1(T)|^2 + KN_b(T) \right] a_1(T) - Ja_2(T)
\]

\[+ J' N_b(T) a_2(T) + \left[ \frac{K}{2} a_1^*(T) + i \frac{J'}{2} a_2^*(T) \right] \sum_{n} F_{nn}^<(T)\]

Josephson oscillation is a periodic perturbation for qp subsystem:

\[ \omega_J < \Delta: \] quasiparticles effectively not excited in low order PT.

\[ \omega_J > \Delta: \] perturbative excitation of quasiparticles: golden rule damping.

Parameter regimes: quasiparticle level spacing

\[ \tau_c \text{ large (non-perturbative)} \]

\[ \tau_c \text{ finite (golden rule)} \]

- small traps, large \( \Delta \)
- large traps, small \( \Delta \)

\[ \Delta \]

e.g. experiments of M. Oberthaler et al.

\[ \Delta \]

e.g. experiments of J. Thywissen et al.

(-private communication)

undamped oscillations (relaxation ??)

oscillations quickly damped
Results: initially delocalized

\( \omega_J < \Delta: \)

population imbalance

\( \Delta < \omega_J < \Delta' \):

population imbalance

\( \omega_J > \Delta' \):

phase difference

quasiparticle occupation number

(5 levels)

Results: initially self-trapped

population imbalance

quasiparticle occupation number

population imbalance

phase difference

(5 levels)
Results: phase diagram and scaling

$J' - K$ scaling
Outlook: time scales for equilibration

\[ E_1^0 + E_2^0 - E_{\text{merge}}^0 = T > \Delta : \]

Characteristic time scales:
- Destruction of Josephson oscillations: \( \tau_c \)
- Quasiparticle damping: \( \tau_{qp} \)
- Thermalization of quasiparticles + BEC: \( \tau_{\text{therm}} \)
Conclusions and outlook

- Dynamics of coupled BECs including Bogoliubov excitations: non-equilibrium Josephson oscillations

- Characteristic time scales (non-exponential):
  Destruction of Josephson oscillations: $\tau_c$
  Quasiparticle damping: $\tau_{qp}$
  Thermalization of quasiparticles + BEC: $\tau_{therm}$ (in progress)

- Relation to inflation in the early universe?

- Strong damping of Josephson oscillations only in large traps: $\omega_f > \Delta$: 