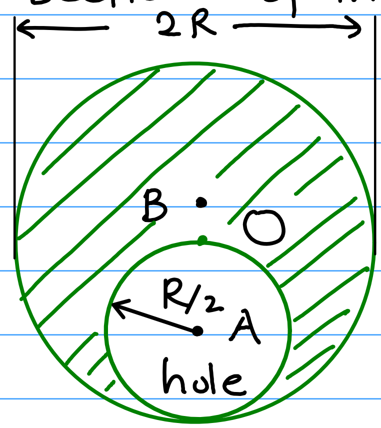
The Cam and shaft problem.

The cross section of the cam is



First let us find its center of mass, located at B. The Mass = M and area of cam

$$\boxed{A_{cam} = \pi R^2 - \pi\left(\frac{R}{2}\right)^2 = \frac{3\pi R^2}{4}} \quad (1)$$

$\Rightarrow$ areal density $\boxed{\rho_A = \frac{4M}{3\pi R^2}}$ (2)

Think of the cam as composed of a solid disk of density $\boxed{\rho_A}$ (3), radius $\boxed{R}$ (4), and mass

$$\boxed{M_+ = \frac{4}{3} M} \quad (5)$$

and another disk of density $\boxed{-\rho_A}$ (6), radius $\boxed{\frac{R}{2}}$ (7) and mass

$$\boxed{M_- = -\frac{M}{3}} \quad (8)$$

located with its center at A

Together, these two disks make up the cam

Choose the origin at O

$$\Rightarrow \quad \boxed{X_{+,CM} = Y_{+,CM} = 0} \quad (9)$$

(CM coordinates of M_+)

$$\boxed{X_{-CM} = 0 \qquad Y_{-CM} = -\frac{R}{2}} \quad (10)$$

(CM of M_-)

So $X_{CM} = 0$ for the cam

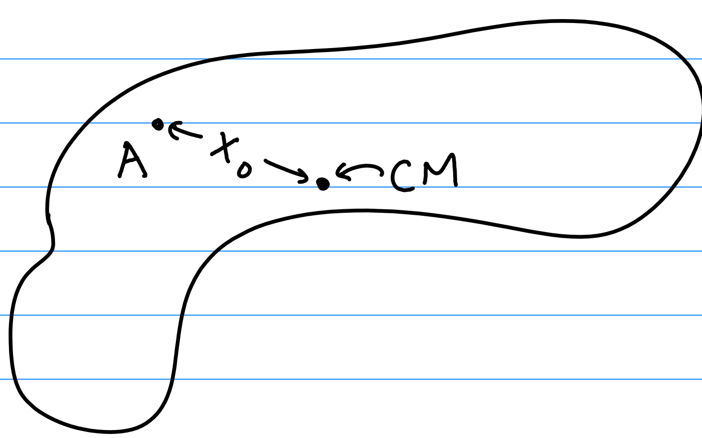
$$\boxed{\begin{aligned} Y_{CM} &= \frac{M_+ Y_{+,CM} + M_- Y_{-CM}}{M_+ + M_-} \\ &= \frac{0 - \left(\frac{M}{3}\right)\left(-\frac{R}{2}\right)}{\frac{4M}{3} - \frac{M}{3}} = \frac{R}{6} \end{aligned}} \quad (11)$$

So B is $\frac{R}{6}$ above O.

Now find the moment of inertia of the cam about O.

We will need the parallel axis theorem

$$\boxed{I_A = I_{CM} + M x_0^2} \quad (12)$$



$$\boxed{I_O = I_{+,O} + I_{-,O}} \quad (13)$$

$$\boxed{I_{+,O} = \frac{1}{2} M_+ R^2 = \frac{2}{3} MR^2} \quad (14)$$

$$\boxed{I_{-,O} = I_{-,cm} + M_-\left(\frac{R}{2}\right)^2} \quad (15)$$

parallel axis thm

$$= \frac{1}{2}\left(-\frac{M}{3}\right)\left(\frac{R}{2}\right)^2 + \left(-\frac{M}{3}\right)\left(\frac{R}{2}\right)^2$$

$$\boxed{I_{-,O} = -\frac{M}{3}\,\frac{R^2}{4}\,\frac{3}{2} = -\frac{MR^2}{8}} \quad (16)$$

(17)

So $$\boxed{I_O = \frac{2}{3}MR^2 - \frac{MR^2}{8} = \frac{MR^2}{24}(16-3) = \frac{13\,MR^2}{24}}$$

Now we need to find the moment of inertia of the cam about its CM, which is B.

Use the parallel axis thm again

$$\boxed{I_O = I_B + M\left(\frac{R}{6}\right)^2} \quad (18)$$

$$\Rightarrow I_B = I_O - M\left(\frac{R}{6}\right)^2 = MR^2\left[\frac{13}{24} - \frac{1}{36}\right]$$

$$\boxed{I_B = \frac{MR^2\,37}{72}} \quad (19)$$

Finally, we need I_A, the moment of inertia of the cam around A. Use the parallel axis thm again

$$\boxed{I_A = I_B + M\left(\frac{R}{2}+\frac{R}{6}\right)^2 = MR^2\,\frac{37}{72} + MR^2\,\frac{4}{9} = MR^2\,\frac{69}{72} = MR^2\,\frac{23}{24}}$$ 20

To this one has to add the moment of inertia of the shaft about its axis of symmetry. $= \boxed{\frac{1}{2}M\left(\frac{R}{2}\right)^2 = \frac{MR^2}{8}}$ 21

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$$\boxed{I_{tot} = MR^2\,\frac{23}{24} + MR^2\,\frac{1}{8} = MR^2\,\frac{26}{24} = MR^2\,\frac{13}{12}}$$

So total $\boxed{KE = \frac{1}{2} I_{tot}\,\omega^2 = \frac{13}{24} MR^2\omega^2}$ 23