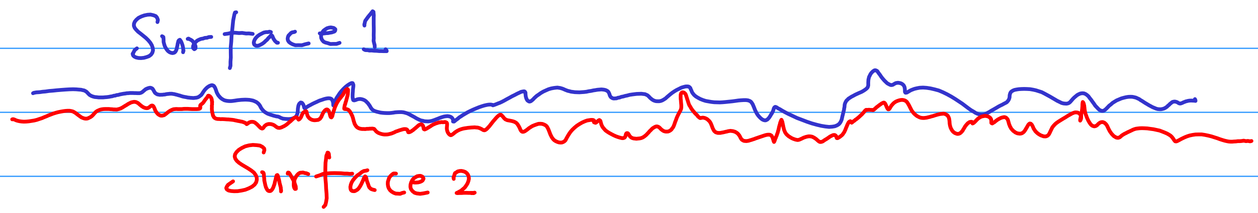
Friction

Friction arises whenever two surfaces are pressed against each other. At the microscopic level one can think of friction as due to small irregularities in the surfaces which "lock" into each other, like the teeth in interlocking gears



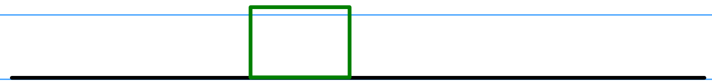
We will be interested in two types of friction.

Static friction applies to the case when the surfaces are at rest w.r.t. each other.

Kinetic friction applies when they move w.r.t. each other

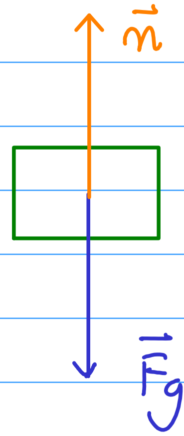
Static friction:

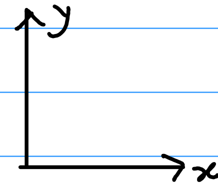
Consider 1st a block sitting on a horizontal surface



The FBD is

$$\boxed{\vec{n} = n\hat{j}} \quad (1)$$





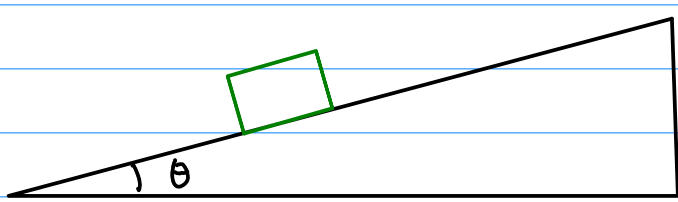
$$\boxed{\vec{F}_g = -mg\hat{j}} \quad (2)$$

$$\boxed{\vec{F}_{tot} = (n - mg)\hat{j}} \quad (3)$$

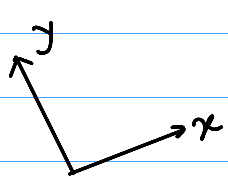
Since $\vec{a} = 0$ we know

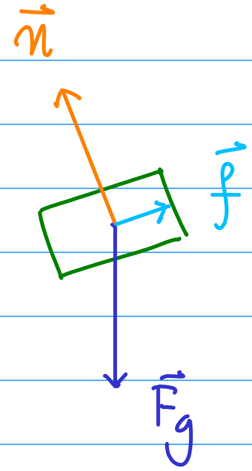
$$\boxed{n = mg} \quad (4)$$

Now let us tilt the plane a bit



Experience tells us that there is a range of angles for which the block remains stationary. Let us draw the FBD





Since $\vec{a} = 0$ we know

$\vec{F}_{tot} = 0$. So we know that there has to be a force of friction $\vec{f}$

$$\boxed{\vec{n} = n\hat{j}} \quad (5) \qquad \boxed{\vec{f} = f\hat{i}} \quad (6)$$

$$\boxed{\vec{F}_g = -mg\sin\theta\,\hat{i} - mg\cos\theta\,\hat{j}} \quad (7)$$

$$\boxed{\vec{F}_{tot} = (f - mg\sin\theta)\hat{i} + (n - mg\cos\theta)\hat{j}} \quad (8)$$

So $\boxed{n = mg\cos\theta}$ (9) $\boxed{f = mg\sin\theta}$ (10)

As the angle changes f changes!!

Static friction is a self-adjusting force!

As one increases the angle, there comes a point when the block begins to slide.

So friction has an upper limit which it cannot exceed. This is known as the maximum static friction, and is proportional to the force of normal reaction

$$\boxed{f_{s,max} = \mu_s n} \quad (11)$$

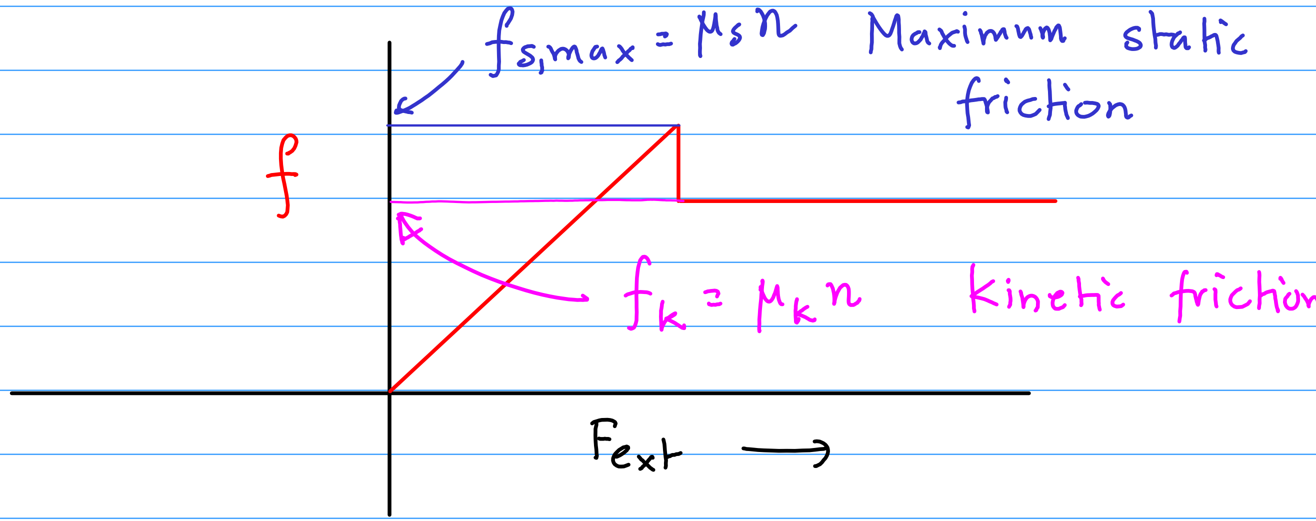
$$\boxed{\mu_s = \text{coefficient of static friction}} \quad (12)$$

μ_s (pronounced mu-sub-s) is a dimensionless number, usually between 0.1 and 1.

Kinetic friction:

Friction usually decreases when the surfaces move w.r.t. to each other, because the irregularities don't have time to lock.

To see what happens let us plot f versus the external horizontal force F_{ext} which is trying to accelerate the object.



For $F_{ext} < f_{s,max} = \mu_s n$ $f = F_{ext}$. So f cancels F_{ext} and the object doesn't move.

However, when $F_{ext} > f_{s,max}$ the object starts moving. The frictional force now drops a bit. It is still proportional to n, but with a different coefficient

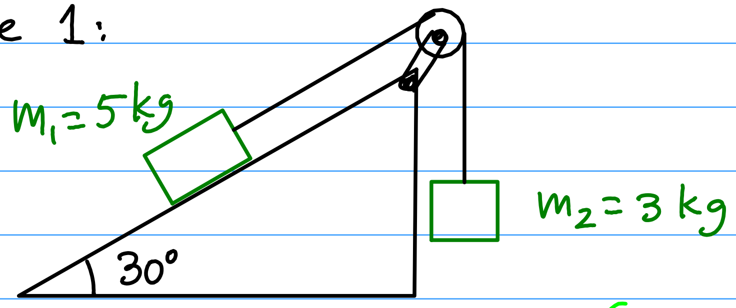
$$\boxed{f_k = \mu_k n} \quad (13)$$

(14)

$$\boxed{\mu_k = \text{coefficient of kinetic friction}}$$

$$\mu_k < \mu_s$$

Example 1:



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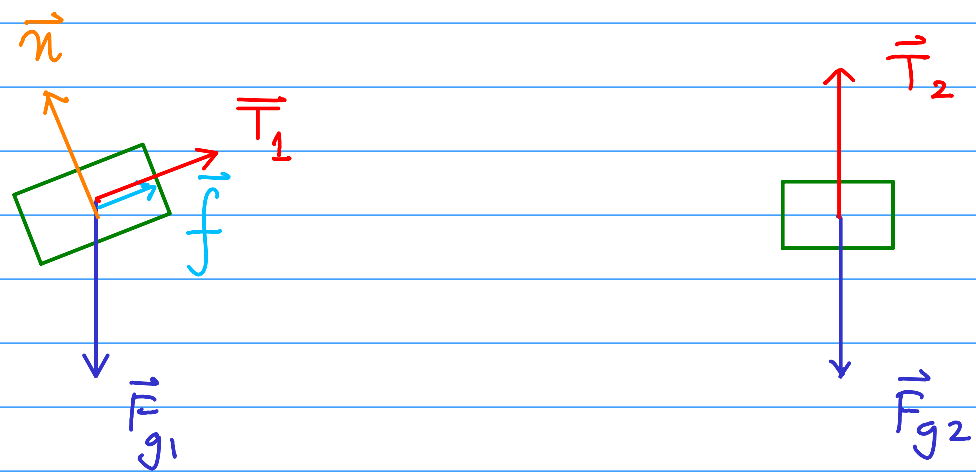
$\boxed{\mu_s = 0.2}$

16

$\boxed{\mu_k = 0.15}$

Suppose the masses are initially at rest. Do they start accelerating? Let's find out.



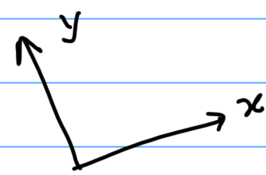
Let us assume that the blocks are not accelerating and see what value of f is required. If this value of f is less than $\mu_s n$, then our assumption was correct, and the blocks remain stationary. If the required value of $f > \mu_s n$ our assumption is wrong and the blocks do accelerate.

We know from the fact that the string is massless and inextensible that

$$\boxed{|\vec{a}_1| = |\vec{a}_2| = a} \tag{17}$$

$$\boxed{|\vec{T}_1| = |\vec{T}_2| = T} \tag{18}$$

For body 1 choose



$$\boxed{\vec{n} = n\,\hat{j}} \tag{19}$$

$$\boxed{\vec{T}_1 = T\,\hat{i}} \tag{20}$$

$$\boxed{\vec{f} = f\,\hat{i}} \tag{21}$$

$$\boxed{\vec{F}_{g1} = -m_1 g \sin\theta\,\hat{i} - m_1 g \cos\theta\,\hat{j}} \tag{22}$$

$$\boxed{\vec{F}_{tot,1} = \hat{i}\,(T + f - m_1 g \sin\theta) + \hat{j}\,(n - m_1 g \cos\theta)} \tag{23}$$

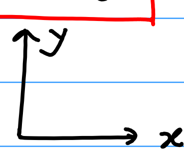
Since $a_y = 0$

$$\boxed{n = m_1 g \cos\theta} \tag{24}$$

Since $a_x = 0$ has been assumed

$$\boxed{T + f - m_1 g \sin\theta = 0} \tag{25}$$

For body 2, choose



$$\boxed{\vec{F}_{tot,2} = (T - m_2 g)\,\hat{j} = m_2 \vec{a} = 0} \tag{26}$$

$$\Rightarrow \quad \boxed{T = m_2 g} \tag{27}$$

Therefore if $a_x = 0$ the required f is

$$f = m_1 g \sin\theta - T = m_1 g \sin\theta - m_2 g$$

$$= 9.8 \frac{m}{s^2} \left(5\,kg \times \frac{1}{2} - 3\,kg \right) = \boxed{-4.9\ N} \quad (28)$$

The minus sign tells us that the actual frictional force is opposite to what we have assumed, but the answer is algebraically correct.

Now

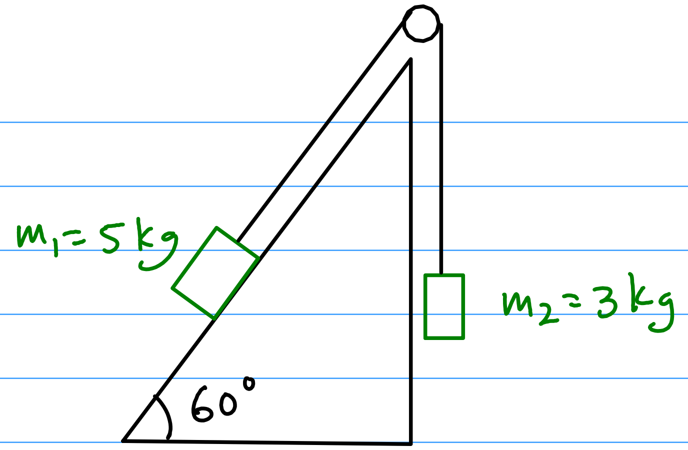
$$\boxed{\begin{aligned} n = m_1 g \cos\theta &= 5\,kg \times 9.8 \frac{m}{s^2} \cdot \frac{\sqrt{3}}{2} \\ &= 42.43\ N \end{aligned}} \quad (29)$$

The maximum static frictional force is

$$\boxed{f_{s,max} = \mu_s n = 8.49\ N} \quad (30)$$

Since $f_{s,max} > |f|$ our assumption that the masses remain stationary is correct.

Example 2:



Now let us increase the angle of the plane to 60°. Again start by assuming $a = 0$. The required friction

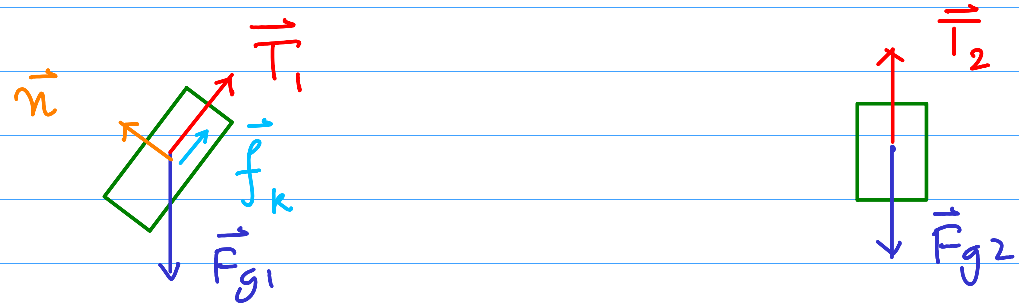
$$\boxed{f = m_1 g \sin\theta - m_2 g = 9.8\,\tfrac{m}{s^2}\left(5\,kg\,\tfrac{\sqrt{3}}{2} - 3\,kg\right) = 13.03\,N}$$ (31)

$$\boxed{n = m_1 g \cos\theta = 24.5\,N}$$ (32)

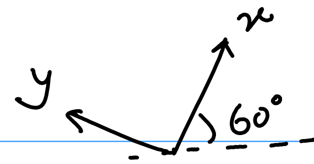
$$\boxed{f_{s,max} = \mu_s n = 4.9\,N}$$ (33)

Since $f > f_{s,max}$ our assumption that the blocks remain stationary is WRONG.

Now we know that they move, we need to use kinetic friction to find the acceleration. Draw the FBDs



For body 1. choose



$$\vec{n} = n\hat{j} \quad (34) \qquad \vec{T}_1 = T\,\hat{i} \quad (35) \qquad \vec{f}_k = \mu_k n\,\hat{i} \quad (36)$$

$$\vec{F}_{g_1} = -m_1 g \sin\theta\,\hat{i} - m_1 g \cos\theta\,\hat{j} \quad (37)$$

$$\vec{F}_{tot,1} = (T + \mu_k n - m_1 g \sin\theta)\,\hat{i} + (n - m_1 g \cos\theta)\,\hat{j} = m_1 \vec{a}_1 \quad (38)$$

$\vec{a}_1$ is along the plane $\quad \vec{a}_1 = -a\,\hat{i} \quad (39)$

the minus indicates that it is accelerating down the plane

$\Rightarrow$ Since $\quad a_{1y} = 0 \Rightarrow F_{tot,1,y} = 0 \quad (40)$

$$\Rightarrow \quad n = m_1 g \cos\theta = 5\,kg \times 9.8\,\frac{m}{s^2} \times \frac{1}{2} = 24.5\,N \quad (41)$$

$$\Rightarrow \quad f_k = \mu_k n = 0.15 \times 24.5\,N = 3.68\,N \quad (42)$$

So from the x-component of Newton's Law

$$T + 3.68 - 5 \times 9.8 \times \frac{\sqrt{3}}{2} = -5a \quad (43)$$

or $\quad T - 38.76 = -5a \quad (44)$

Now body 2. $\qquad T - m_2 g = m_2 a$

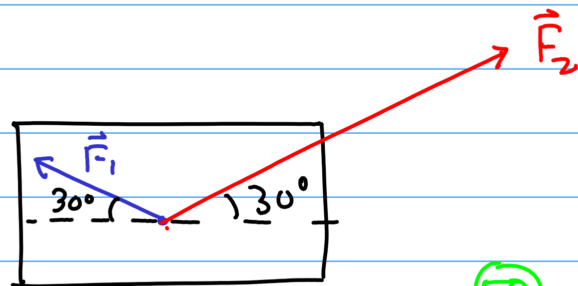
$$T - 29.4 = 3a \quad (45)$$

$\Rightarrow$ $$\boxed{\begin{aligned} -T + 38.76 &= +5a \\ T - 29.4 &= 3a \end{aligned}}$$ 46

Add $\boxed{8a = 9.36}$ 47

$\boxed{a = 1.17 \ m/s^2}$ 48

Example 3: The suitcase again, but with friction.



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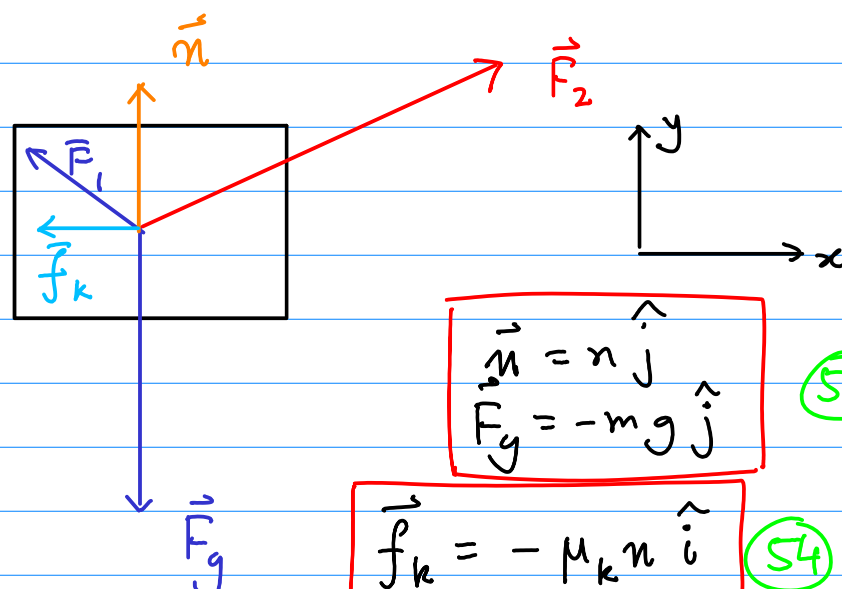
$\boxed{\begin{aligned} F_1 &= 50 N \\ m &= 50 kg \end{aligned}}$ but now $\boxed{\mu_k = 0.2}$ 50 . Find F_2

such that $\boxed{a_x = 2 m/s^2}$ 51 $\boxed{a_y = 0}$ 52

Draw the FBD



$\boxed{\begin{aligned} \vec{n} &= n \hat{j} \\ \vec{F}_g &= -mg \hat{j} \end{aligned}}$ 53

$\boxed{\vec{f}_k = -\mu_k n \hat{i}}$ 54

$$\boxed{\begin{aligned}\vec{F}_1 &= -F_1 \cos 30^\circ \,\hat{i} + F_1 \sin 30^\circ \,\hat{j} \\ \vec{F}_2 &= F_2 \cos 30^\circ \,\hat{i} + F_2 \sin 30^\circ \,\hat{j}\end{aligned}}$$

(55)

(56)

$$\boxed{\begin{aligned}\vec{F}_{tot} = {} & \hat{i}\left(F_2 \cos 30^\circ - F_1 \cos 30^\circ - \mu_k n\right) \\ & + \hat{j}\left(F_1 \sin 30^\circ + F_2 \sin 30^\circ + n - mg\right) = m\vec{a}\end{aligned}}$$

We know that $\boxed{a_y = 0 \Rightarrow F_{tot,y} = 0}$ (57)

$$\boxed{F_1 \sin 30^\circ + F_2 \sin 30^\circ + n - mg = 0}$$ (58)

Plug in the numbers we know

$$\boxed{25 + \frac{F_2}{2} + n - 490 = 0}$$ (59)

or $\boxed{\frac{1}{2} F_2 + n = 465}$ (60)

x-component $a_x = 2\,m/s^2$

(61)

$$\Rightarrow \boxed{\begin{aligned}F_2 \cos 30^\circ - F_1 \cos 30^\circ - \mu_k n &= 50\,kg \times 2\,\frac{m}{s^2} \\ &= 100\ N\end{aligned}}$$

$$\Rightarrow \quad F_2 \frac{\sqrt{3}}{2} - 0.2\,n - 50 \times \frac{\sqrt{3}}{2} = 100$$

or $0.866\,F_2 - 0.2n - 43.3 = 100$

$$\boxed{0.866\,F_2 - 0.2n = 143.3}$$ (62)

So we need to solve a set of simultaneous eqns for F_2 and n

$$\boxed{\begin{aligned} 0.5\,F_2 + n &= 465 \\ 0.866\,F_2 - 0.2n &= 143.3 \end{aligned}} \quad (63)$$

Multiply 2nd eqn by 5

$$\boxed{4.33\,F_2 - n = 716.5} \quad (64)$$

Add to 1st $\quad \boxed{4.83\,F_2 = 1181.5} \quad (65)$

$$\Rightarrow \boxed{F_2 = 244.6 \text{ N}} \quad (66)$$

and $\quad \boxed{n = 342.7 \text{ N}} \quad (67)$