Newton's Laws

So far we have been describing motion, and now we want to understand its causes.

The 1st law says that if the net force on an object is zero it does not accelerate

$$\boxed{\text{If} \quad \vec{F}_{tot} = 0 \qquad \vec{a} = 0} \quad (1)$$

Note that $\vec{a} = 0$ does not necessarily mean that the object is at rest.

$$\boxed{\vec{a} = 0 \Rightarrow \frac{d\vec{v}}{dt} = 0 \Rightarrow \vec{v} = \text{constant}} \quad (2)$$

This seems to run against our intuition, which suggests that objects that don't feel a net force eventually come to rest. This intuition is wrong, because it ignores friction. So objects that come to rest do have a net force acting on them.

Galileo discovered this law, and here's how he thought of it.

Remember the Principle of Relativity: The laws of Physics should be the same in two frames moving at constant velocity with respect to each other. Now imagine you are on a ship and a smooth ball is resting on a smooth table inside the ship. The ship is moving at constant velocity, so you don't feel any vibration or other cues that you are moving. Your experience on a plane tells you that the ball will remain at its position on the table, in the moving ship.

Now consider an observer who is "stationary" on the ground, watching the ship go by. With X-ray vision, he can see into the ship. To him it appears that the ball is moving at constant velocity.

Both observers agree that $\bar{F}_{tot} = 0$ on the ball. This shows that $\vec{F}_{tot} = 0$ implies constant velocity.

Newton's second law tells us what happens when $\vec{F}_{tot}$ is not zero. It says

$$\boxed{\bar{F}_{tot} = m\bar{a}} \quad (3)$$

It is very important to take the vector sum of all the forces acting on a body to find the total, or net, force.

What kinds of forces do we know about? First there are contact forces, basically pulling and pushing. Then there are forces that act at a distance, and do not need contact. Gravity is just such a force. Next semester you will learn about other such forces, electric and magnetic.

An important class of forces is friction. This comes in several types. Static friction is applicable when two surfaces are in in contact with each other and at rest with respect to each other.
Kinetic friction applies when two surfaces move with respect to each other while in contact.
There are yet other types such an rolling friction and viscous friction.

The force of gravity is simple near the Earth's surface.

$$\boxed{\vec{F}_g = -mg\,\hat{j}} \quad (4)$$

where $\hat{j}$ is the unit vector upwards and

$$\boxed{g = 9.8\ m/s^2} \quad (5) \quad \text{(approximately)}$$

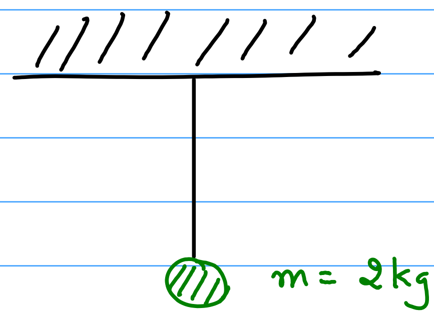
Friction is more complicated, because it is "self-adjusting", so let us leave it for later.

An important contact force is the force of normal reaction $\vec{n}$. This is the force with which two surfaces are pressing each other. It acts normal to the surface of contact.

Now we turn to the most important tool that you will use to solve problems in this course

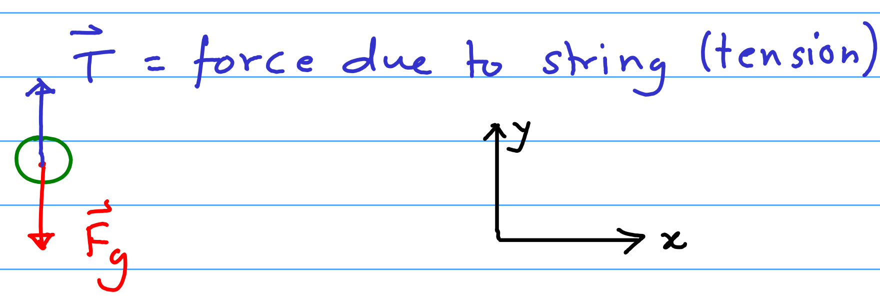
Free body diagrams (FBDs)

Consider **Example 1:** A bob of mass 2kg is hanging vertically from the ceiling by a string.



In the FBD you will replace the environment by forces. Only the body itself appears explicitly in the FBD.

In this case there are two forces acting on the bob, the force of gravity vertically down and the force due to the string vertically up. Here is the FBD



Choose a convenient coordinate system

$$\boxed{\vec{F}_g = -mg\,\hat{j}} \quad (6) \qquad \boxed{\vec{T} = T\,\hat{j}} \quad (7)$$

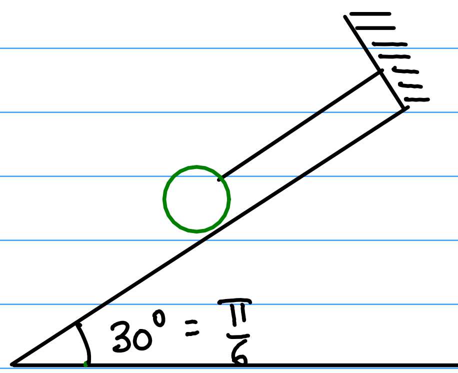
Total force on the bob and apply Newton II

$$\boxed{\vec{F}_{tot} = \hat{j}\,(-mg + T) = m\vec{a}} \quad (8)$$

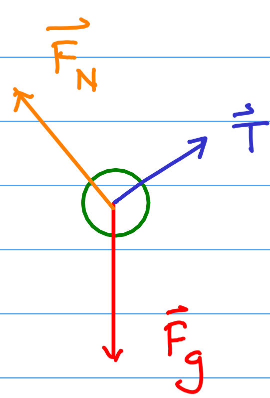
We know that the bob is at rest and remains at rest. So $\vec{a} = 0 \Rightarrow \vec{F}_{tot} = 0$

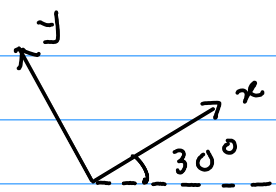
So, in this case $\boxed{T = mg}$ (9)

Let us complicate this a bit in Example 2:
The bob is at rest on a frictionless inclined plane



Now in addition to $\vec{F}_g$ and $\vec{F}_s$ there is a force of normal reaction from the plane acting on the bob.





It is more convenient to choose the coordinates as shown

(10) $\boxed{\vec{T} = T\,\hat{i}}$ $\boxed{\vec{n} = n\,\hat{j}}$ (11)

$$\boxed{\begin{aligned}\vec{F}_g &= -mg\,\sin\left(\frac{\pi}{6}\right)\hat{i} - mg\,\cos\left(\frac{\pi}{6}\right)\hat{j} \\ &= -\frac{mg}{2}\,\hat{i} - \frac{mg\sqrt{3}}{2}\,\hat{j}\end{aligned}}$$ (12)

$$\vec{F}_{tot} = \vec{F}_g + \vec{T} + \vec{n} = -\frac{mg}{2}\hat{i} - \frac{mg\sqrt{3}}{2}\hat{j} + T\,\hat{i} + n\,\hat{j}$$

$$\boxed{\vec{F}_{tot} = \left(-\frac{mg}{2}\right)\hat{i} + \left(n - \frac{mg\sqrt{3}}{2}\right)\hat{j}}$$ (13)

We know that the bob is not accelerating

$$\boxed{\vec{a} = 0 \Rightarrow a_x = 0,\ a_y = 0}$$ (14)

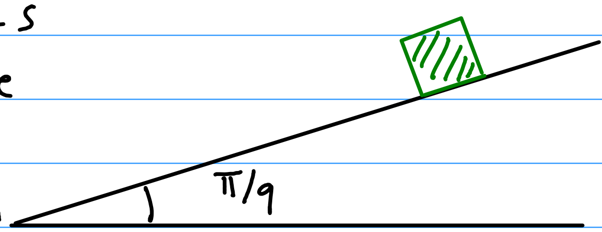
So, $\boxed{\vec{F}_{tot} = 0}$ (15) $\Rightarrow F_{tot,x} = 0,\quad F_{tot,y} = 0$

So $\boxed{T = \frac{mg}{2}}$ (16) $\boxed{n = \frac{mg\sqrt{3}}{2}}$ (17)

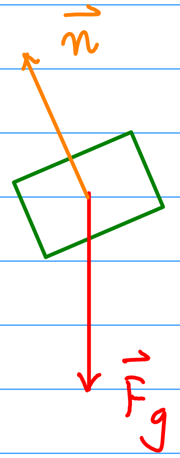
So far we have talked about cases where $\vec{a} = 0$. Let us move on to a case where $\vec{a} \neq 0$ in

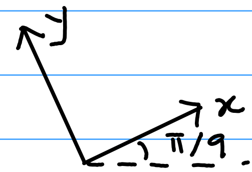
Example 3: The block slides frictionlessly.

The only forces are $\vec{F}_g$ and the force of normal reaction $\vec{n}$



Here is the FBD. Choose the coordinates below





$$\boxed{\vec{n} = n\,\hat{j}} \tag{18}$$

$$\boxed{\vec{F}_g = -mg\sin\left(\frac{\pi}{9}\right)\hat{i} - mg\cos\left(\frac{\pi}{9}\right)\hat{j}} \tag{19}$$

$$\boxed{\vec{F}_{tot} = \vec{F}_g + \vec{n} = -mg\sin\left(\frac{\pi}{9}\right)\hat{i} + \hat{j}\left(n - mg\cos\left(\frac{\pi}{9}\right)\right)} \tag{20}$$

What do we know about $\vec{a}$? We know that since the block accelerates along the plane

$$\boxed{a_y = 0} \tag{21}$$

$$\Rightarrow \quad \boxed{F_{tot,y} = 0 \quad \Rightarrow \quad n = mg\cos\left(\frac{\pi}{9}\right)} \tag{22}$$

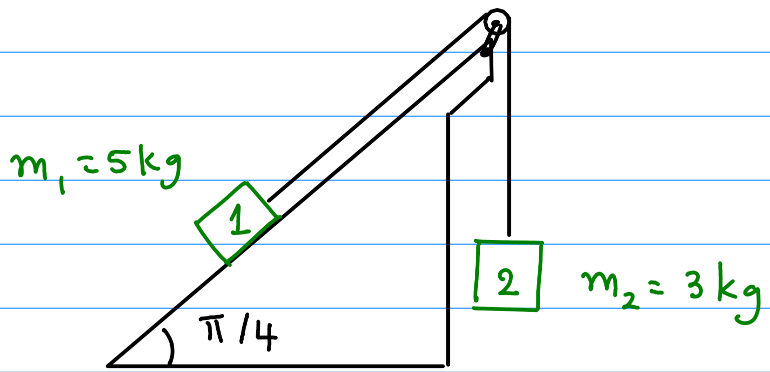
$$\text{So} \quad \boxed{\vec{a} = a_x\,\hat{i}} \tag{23}$$

$$\text{and} \quad \vec{F}_{tot} = -mg\sin\left(\frac{\pi}{9}\right)\hat{i} = m a_x\,\hat{i}$$

$$\boxed{a_x = -g\sin\left(\frac{\pi}{9}\right) = -9.8\times\sin\left(\frac{\pi}{9}\right) = -3.35\ m/s^2} \tag{24}$$

Minus because downslope is in the $-x$ direction.

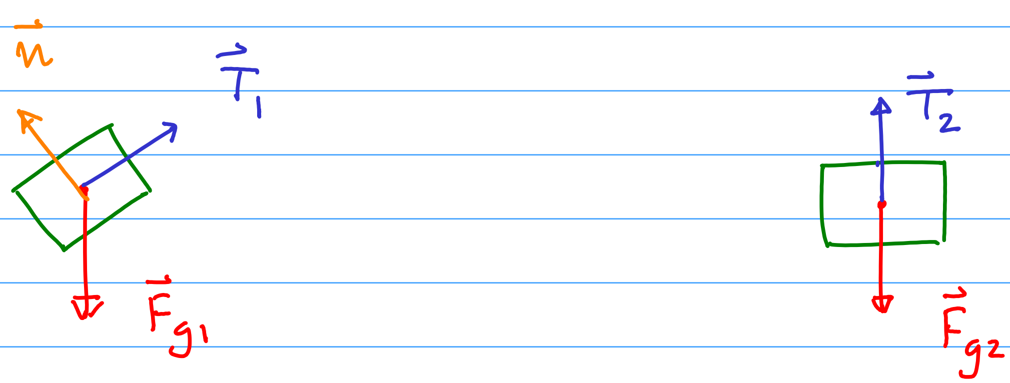
Sometimes there are two or more bodies involved, which leads us to Example 4:



The string is massless and cannot be stretched and the pulley is massless and frictionless. The plane is also frictionless. Since the string is inextensible the two masses must always remain the same distance apart, and must therefore have the same magnitude of velocity and acceleration.

$$\boxed{|\vec{a}_1| = |\vec{a}_2|} \quad (25)$$

The 1st step is always to draw the FBDs

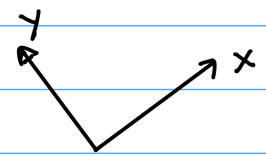


Since the string is massless, no force is "used up" in accelerating it. So

$$\boxed{|\vec{T}_1| = |\vec{T}_2| = T} \tag{26}$$

Let us choose convenient coordinates for the two bodies separately

Body 1:



$$\boxed{\vec{n} = n\,\hat{j}} \quad \boxed{\vec{T}_1 = T\,\hat{i}} \tag{27}$$

$$\boxed{\vec{F}_{g1} = -\frac{m_1 g}{\sqrt{2}}\,\hat{i} - \frac{m_1 g}{\sqrt{2}}\,\hat{j}} \tag{28}$$

$$\boxed{\begin{aligned}\vec{F}_{tot,1} = {} & \hat{i}\left(T - \frac{m_1 g}{\sqrt{2}}\right) \\ & + \hat{j}\left(n - \frac{m_1 g}{\sqrt{2}}\right)\end{aligned}} \tag{29}$$

(30) $\boxed{a_{1y} = 0}$ (does not acc. $\perp$ to plane)

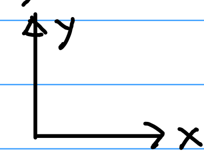
$$\boxed{\vec{a}_1 = +a\,\hat{i}} \tag{31}$$

Same a

$$\Rightarrow \boxed{n = \frac{m_1 g}{\sqrt{2}}} \tag{32}$$

$$\boxed{T - \frac{m_1 g}{\sqrt{2}} = m_1 a} \tag{33}$$

Body 2



$$\boxed{\vec{T}_2 = T\,\hat{j}} \tag{34}$$

$$\boxed{\vec{F}_{g2} = -m_2 g\,\hat{j}} \tag{35}$$

$$\boxed{\begin{aligned}\vec{F}_{tot,2} &= \hat{j}\,(T - m_2 g) \\ &= m_2 \vec{a}_2\end{aligned}} \tag{36}$$

Assume mass 2 accelerates down.

$$\boxed{\vec{a}_2 = -a\,\hat{j}} \tag{37}$$

$$\boxed{T - m_2 g = -m_2 a} \tag{38}$$

So we have the simultaneous eq^{n}s

$$\boxed{\begin{aligned} T - \frac{m_1 g}{\sqrt{2}} &= m_1 a \\ -T + m_2 g &= m_2 a \end{aligned}} \quad (39)$$

add the two eq^{n}s

$$\boxed{m_2 g - \frac{m_1 g}{\sqrt{2}} = (m_1 + m_2) a} \quad (40)$$

Plug in numbers

$$9.8 \left(3 - \frac{5}{\sqrt{2}}\right) = (3+5) a$$

Solve

$$\boxed{a = -0.656 \ m/s^2} \quad (41)$$

The minus sign tells us that we guessed wrong about the direction of the acceleration. The answer is still correct. The mass m_1 accelerates down the slope at $0.656 \ m/s^2$ while the mass m_2 accelerates vertically up at $0.656 \ m/s^2$.

Now we can find $T = m_1 a + \frac{m_1 g}{\sqrt{2}}$

$$= 5(-0.656) + \frac{5 \times 9.8}{\sqrt{2}}$$

$$\boxed{T = 31.37 \ N} \quad (42)$$

Newton's III Law

This is an extremely important law, and is difficult to understand and internalize at first. Define $\vec{F}_{12}$ to be the force due to object 1 on object 2 and $\vec{F}_{21}$ the force due to object 2 on object 1.

Then, Newton's III Law says

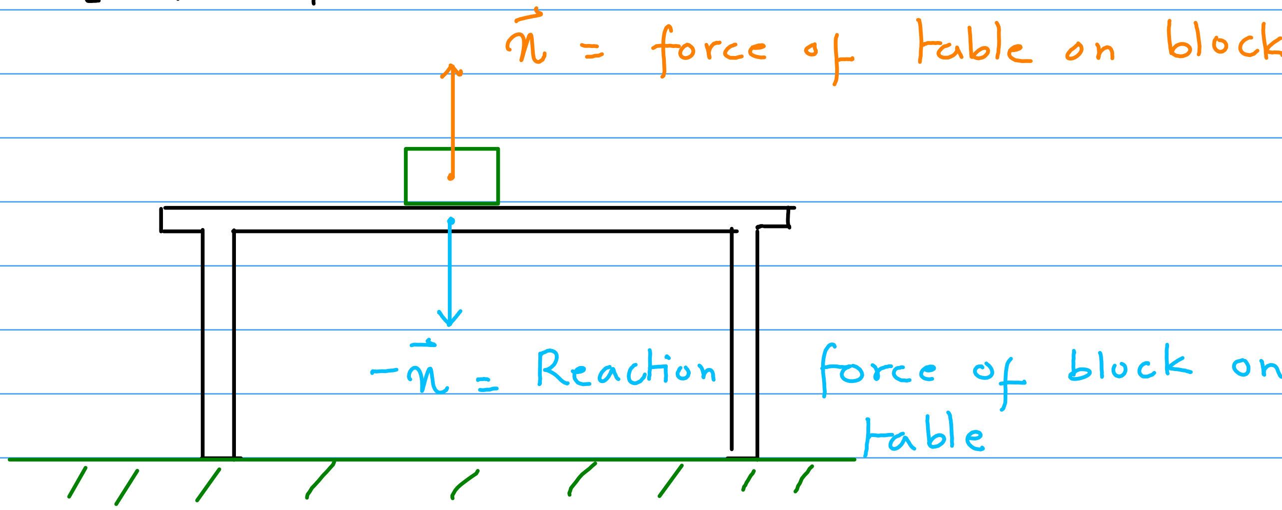
$$\boxed{\vec{F}_{21} = -\vec{F}_{12}} \quad (43)$$

Imagine a baseball comes towards me and I hit it with a bat. According to Newton III the force exerted by the bat on the ball is equal and opposite to the force exerted by the ball on the bat.

How can this be? The ball shoots off very fast in a direction opposite to its initial velocity, whereas the bat continues moving in the same direction.

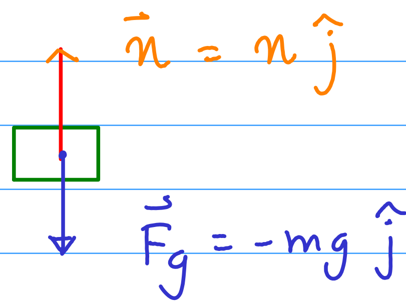
The key to understanding this is that the masses of the ball and bat are very different. The same magnitude of force will cause a much greater acceleration in the ball than it will on the bat.

Another example. Suppose a block of mass m rests at equilibrium on a horizontal table

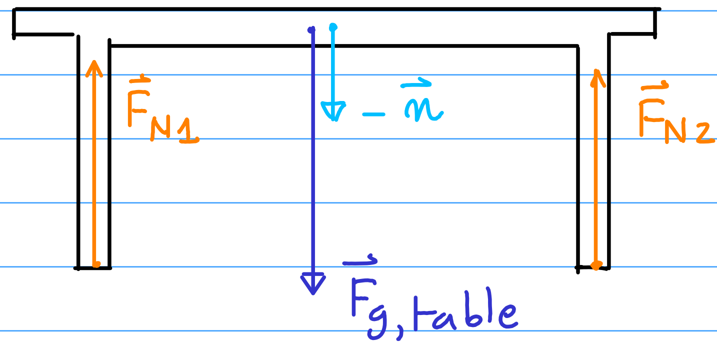


Of course, these are not the only forces acting on either the block or the table.

The FBD for the block looks like this



while the FBD for the table is



$\vec{F}_{N1}$, $\vec{F}_{N2}$ are the normal forces acting on the legs due to the ground.

Here is an arguement to see why Newton III has to be true.

Let us imagine that Newton III is not true for a pair of objects 1 and 2. In other words, let us assume that

$$\vec{F}_{12} \neq -\vec{F}_{21}$$

Let us take these two objects far away from our galaxy, so that there are no gravitational or other forces acting on them. Let us attach them to each other. The total force on them is

$$\vec{F}_{12} + \vec{F}_{21} \neq 0 \quad \text{(by assumption)}$$

So, for the combined object, there is a self-generated net force, and it will start accelerating! So we could extract infinite amounts of energy if Newton III were not true.

Let us now do some simple examples where one needs both Newton II and Newton III

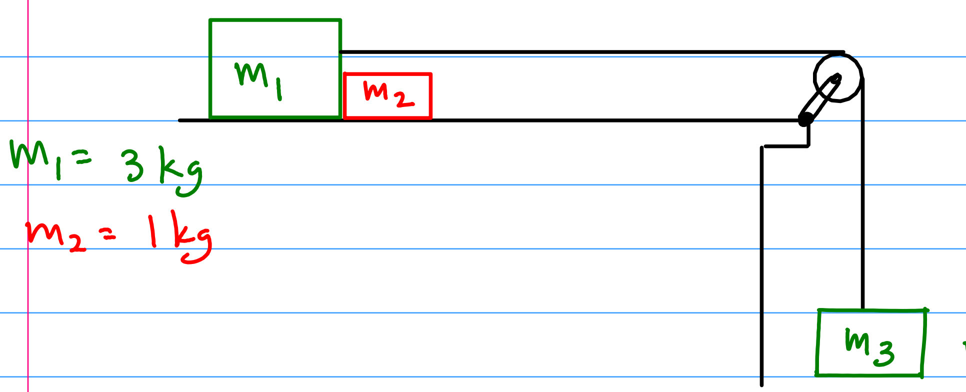
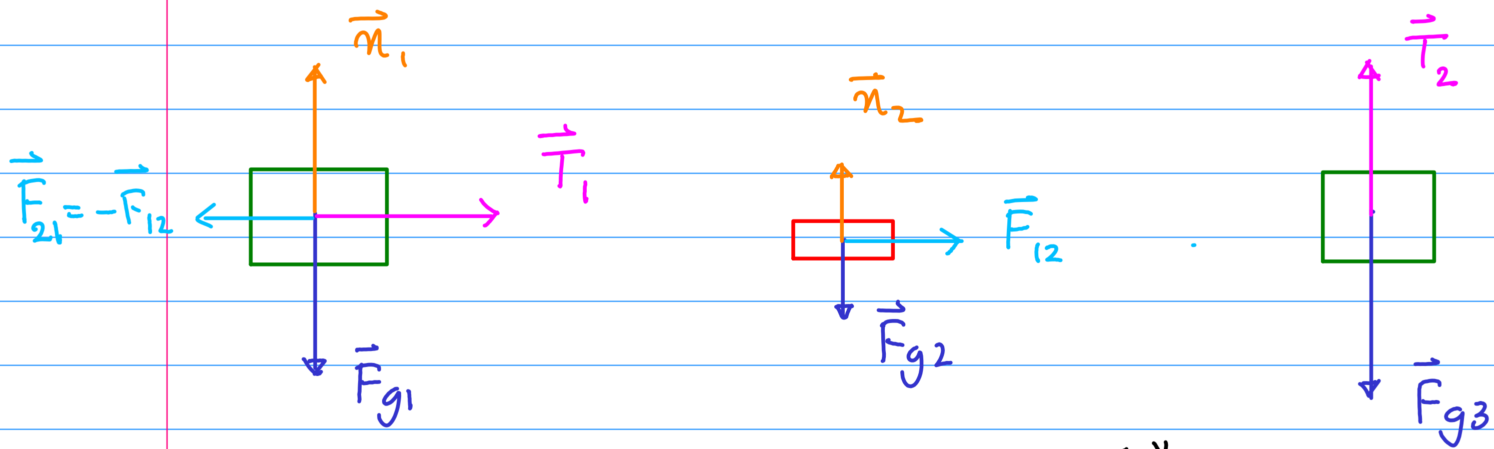
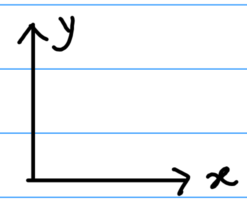


Table frictionless. Pulley massless, frictionless. String massless, and inextensible

Now we have to draw three FBDs. The key is that because mass m_1 is pushing mass m_2 to the right, there must be a reaction force on m_1 due to m_2, pushing m_1 to the left



Choose coordinates



Because the string is inextensible

$$\boxed{|\vec{a}_1| = |\vec{a}_3|} \quad (44)$$

Because m_1, m_2 move together

$$\boxed{\vec{a}_1 = \vec{a}_2} \quad (45)$$

From the directions it is clear that

$$\boxed{\vec{a}_1 = \vec{a}_2 = a\,\hat{i}} \quad (46)$$

and

$$\boxed{\vec{a}_3 = -a\,\hat{j}} \quad (47)$$

Since the string is massless, no force is "used up" in accelerating it, so

$$\boxed{|\vec{T}_1| = |\vec{T}_3| = T} \quad (48)$$

$$\boxed{\vec{T}_1 = T\hat{i}} \quad (49) \qquad \boxed{\vec{T}_3 = T\hat{j}} \quad (50)$$

Total force on m_1 is (51)

$$\boxed{\hat{i}\,(T - F_{12}) + \hat{j}\,(n_1 - m_1 g) = \vec{F}_{tot,1} = m_1\vec{a}_1 = m_1 a\,\hat{i}}$$

since $\boxed{a_{y1} = 0}$ (52) $\quad \boxed{n_1 = m_1 g}$ (53)

$$\Rightarrow \boxed{T - F_{12} = m_1 a} \quad (54)$$

Total force on m_2 is (55)

$$\boxed{\hat{i}\,F_{12} + \hat{j}\,(n_2 - m_2 g) = m_2\vec{a}_2 = m_2 a\,\hat{i}}$$

Since $\boxed{a_{y2} = 0}$ (56) $\quad \boxed{n_2 = m_2 g}$ (57)

$$\boxed{F_{12} = m_2 a} \quad (58)$$

Total force on m_3 is

$$\boxed{\hat{j}\,(T - m_3 g) = m_3\vec{a}_3 = -m_3 a\,\hat{j}} \quad (59)$$

$$\Rightarrow \boxed{T - m_3 g = -m_3 a} \quad (60) \text{ or}$$

$$\boxed{m_3 g - T = m_3 a} \quad (61)$$

Rewrite the three simultaneous eq^{n}s

$$\boxed{\begin{aligned} T - F_{12} &= m_1 a \\ F_{12} &= m_2 a \\ m_3 g - T &= m_3 a \end{aligned}} \quad (62)$$

Add all three to get

$$\boxed{m_3 g = (m_1 + m_2 + m_3) a} \quad (63)$$

or $$\boxed{a = \frac{m_3}{m_1 + m_2 + m_3} g} \quad (64)$$

Plug in numbers $$\boxed{a = \frac{9.8 \times 4}{4 + 3 + 1} = 4.9 \frac{m}{s^2}} \quad (65)$$

Now we can go back and find T and F_{12}

$$\boxed{F_{12} = m_2 a = 4.9 \text{ N}} \quad (66)$$

$$\boxed{T = m_3 g - m_3 a = 4 \text{ kg} \times 4.9 \frac{m}{s^2} = 19.6 \text{ N}} \quad (67)$$