PHY231 Midterm 2 Solutions

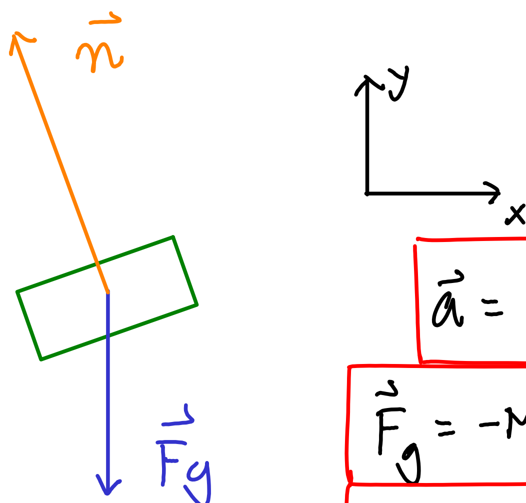
As usual, you may only use a calculator on this test. The final 3 pages have all the formulas you need. No communication between students is allowed during the test. Please address any questions to me.

Name:

ID no:

Problem 1: A curve of radius 200m is banked at an angle $\theta = 15^\circ$ **as shown. The coefficient of static friction between the tires and the road is** μ_s**=0.6. Please solve the problem using the following steps.**

1a. Assume that there is no friction. Draw the FBD, and knowing that the centripetal acceleration points to the center of the curve, find the rated speed of the curve. (10 points)



$$\boxed{\vec{a} = -\frac{v_0^2}{r}\hat{i}}$$

$$\boxed{\vec{F}_g = -Mg\hat{j}}$$

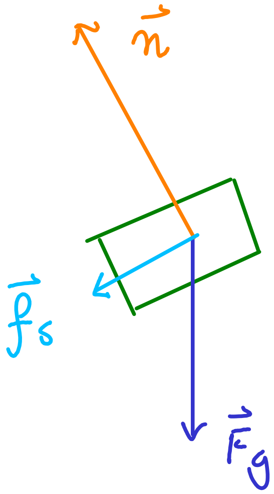
$$\boxed{\vec{n} = -n\sin\theta\,\hat{i} + n\cos\theta\,\hat{j}}$$

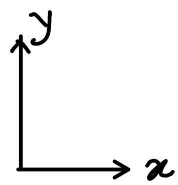
$$\boxed{a_y = 0 \Rightarrow} \quad \boxed{n = \frac{Mg}{\cos\theta}}$$

and $$\boxed{F_{tot,x} = -Mg\tan\theta = -\frac{Mv_0^2}{r}}$$

$$\boxed{v_0 = \sqrt{rg\tan\theta} = 22.92\ m/s}$$

1b. The car travels at 35m/s. Again draw the FBD, and set up the simultaneous equations for n and f_s. Note that f_s will in general not be equal to its maximum value. (10 pts)





$$\boxed{\vec{a} = -\frac{v^2}{r}\hat{i}}$$

$$\boxed{\begin{aligned}\vec{n} &= -n\sin\theta\,\hat{i} + n\cos\theta\,\hat{j}\\ \vec{f}_s &= -f_s\cos\theta\,\hat{i} - f_s\sin\theta\,\hat{j}\end{aligned}}$$

$$a_y = 0 \Rightarrow \quad F_{tot,y} = 0$$

$$\Rightarrow \quad \boxed{n\cos\theta - f_s\sin\theta = Mg}$$

$$a_x = -\frac{v^2}{r} \Rightarrow \boxed{n\sin\theta + f_s\cos\theta = \frac{Mv^2}{r}}$$

1c. Solve the simultaneous equations to find the values of n and f_s of part 1b. (5 pts)

By inspection

$$\boxed{n = Mg\cos\theta + \frac{Mv^2}{r}\sin\theta}$$

$$= M\left[9.8 \times 0.966 + \underbrace{\frac{35^2}{200}}_{6.125\ m/s^2} \times 0.259\right]$$

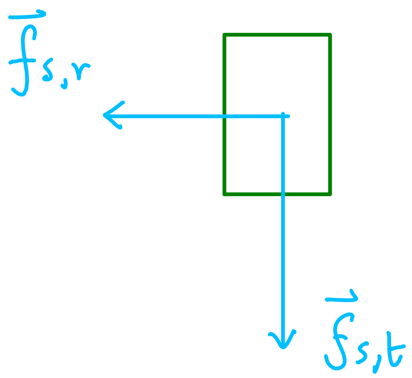
$$\boxed{n = 11.05\ M}$$

$$\boxed{f_s = -Mg\sin\theta + \frac{Mv^2}{r}\cos\theta} = M\left[-9.8 \times 0.259 + 6.125 \times 0.966\right]$$

$$\boxed{f_s = 3.38\ M}$$

1d. The driver (travelling at ~~30m/s~~ 35 m/s) sees a deer in the headlights. How fast can the car brake without skidding? I am looking for a value of the tangential acceleration in m/s^2.
(10 pts)

Draw an FBD looking from the top



We know $\boxed{f_{s,r} = 3.38M}$

and $\boxed{f_{s,t} = Ma_t}$

We want $a_{t,max}$. This happens

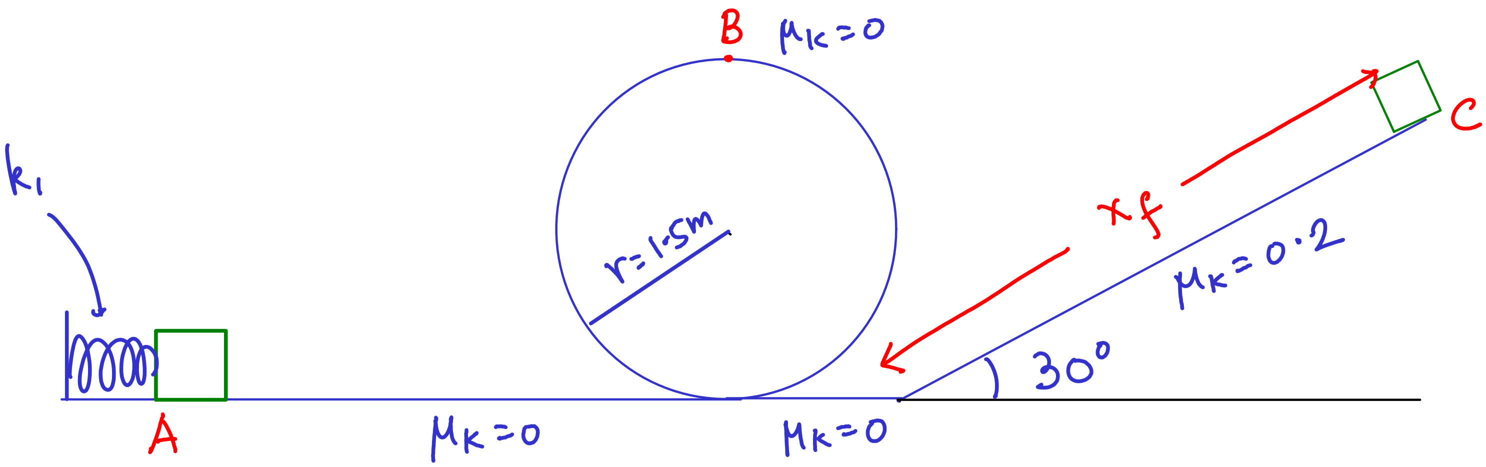
when $\boxed{|\vec{f}_{s,r} + \vec{f}_{s,t}| = f_{s,max} = \mu_s n}$

So $$\boxed{\sqrt{(3.38M)^2 + (Ma_t)^2} = 0.6 \times 11.05M = 6.63M}$$

$$\Rightarrow \boxed{a_t = \sqrt{(6.63)^2 - (3.38)^2} = 5.7 \text{ m/s}^2}$$

This is the maximum tangential acceleration.

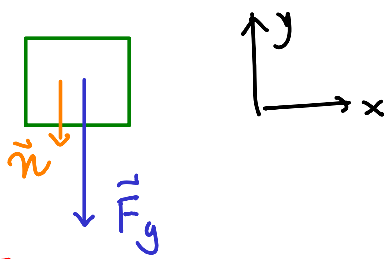
Problem 2. A block of mass M=1.5kg is held against a compressed spring of force constant k1=2000N/m. The compression is unkown. The mass is released and it is noted that it barely goes around the loop-de-loop of radius 1.5m shown. After this it travels up the slope shown and ends up at rest at the point C. Solve the problem using these steps.



2a. Draw a FBD of the block at the very top of the loop, marked B. Find the minimum speed it must have in order to not lose contact with the loop. (10 pts)

FBD at B

We know



$$\boxed{\vec{a} = -\frac{v_B^2}{r}\hat{j}}$$

$$\Rightarrow \boxed{n + Mg = \frac{M v_B^2}{r}}.$$

The minimum v_B will be when $n = 0$ $\Rightarrow$

$$\boxed{v_{B,min} = \sqrt{rg} = 3.834\ m/s}$$

2b. By using the conservation of mechanical energy between A and B find the initial compression of the first spring. (10 pts)

$$\boxed{E_{mech,A} = E_{mech,B}}$$

No W_{nc}!

$K_A = 0$ (at rest)

$U_{gA} = 0$ (at zero height)

$U_{sA} = \frac{1}{2}k(x_i')^2$ x_i' = initial compression of spring

$$\boxed{E_{mech,A} = \frac{1}{2}k(x_i')^2}$$

At B $\boxed{K_B = \frac{1}{2}Mv_B^2 = \frac{1}{2}Mgr}$ $(v_B^2 = rg)$

$\boxed{U_{g,B} = Mg(2r)}$ $(h_B = 2r)$

$U_{s,B} = 0$

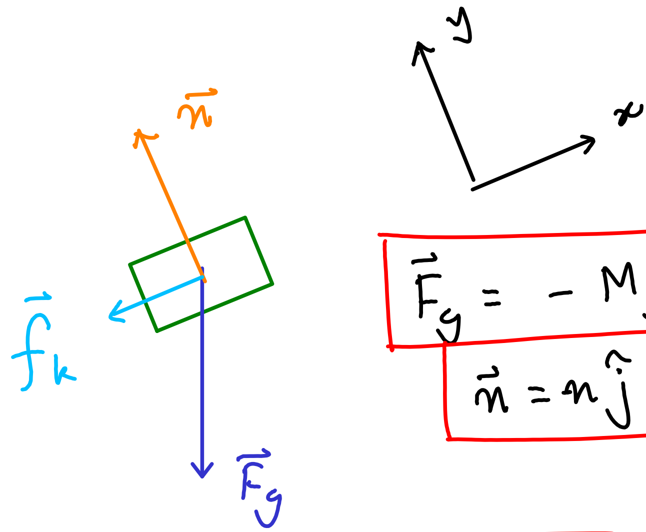
$$\Rightarrow \boxed{E_{mech,B} = Mg\left(2r + \frac{1}{2}r\right) = \frac{5}{2}Mgr = \frac{5}{2} \times 1.5\,kg \times 9.8\,\frac{m}{s^2} \times 1.5\,m = 55.125 \text{ Joules}}$$

So $\frac{1}{2}k(x_i')^2 = 55.125\,J$

$(x_i')^2 = \frac{55.125}{1000}$

$$\boxed{x_i' = 0.235\ m}$$

2c. Draw a FBD of the block traversing the slope, and determine the numerical value of the force of friction. (5 pts)



$$\boxed{\vec{F}_g = -Mg\sin 30^\circ\,\hat{i} - Mg\cos 30^\circ\,\hat{j}}$$

$$\boxed{\vec{n} = n\,\hat{j}} \qquad \boxed{\vec{f}_k = -f_k\,\hat{i}}$$

$$\boxed{a_y = 0} \Rightarrow \boxed{n = Mg\cos 30^\circ} \Rightarrow \boxed{\begin{aligned} f_k &= \mu_k n \\ &= \mu_k Mg\cos 30^\circ \end{aligned}}$$

$$\boxed{f_k = 0.2 \times 1.5\,\text{kg} \times 9.8\,\frac{\text{m}}{\text{s}^2} \times 0.866 = 2.54\ \text{N}}$$

2d. Use the Work-Energy theorem to find the distance it travels up the slope before coming to rest. (10 pts)

$$\boxed{W_{nc} = E_{mech,C} - E_{mech,B}}$$

$$\boxed{E_{mech,B} = 55.125 \text{ J}}$$

$$E_{mech,C} = \underbrace{\cancel{K_C}}_{0} + U_{gC} + \underbrace{\cancel{U_{s,C}}}_{0}$$

The height at C is $\boxed{h_C = x_f \sin 30^\circ}$

$$\boxed{E_{mech,C} = Mgh_C = Mgx_f \sin 30^\circ}$$

$$\boxed{W_{nc} = -f_k x_f = -2.54\, x_f}$$

So $\boxed{-2.54 x_f = Mg x_f \sin 30^\circ - 55.125}$

or $55.125 = x_f (2.54 + 7.35) = 9.89 x_f$

$$\boxed{x_f = 5.57 \text{ m}}$$

Problem 3. Two blocks of masses M1=2.95kg and M2=2kg are at rest initially on a frictionless surface. A bullet b of mass 50gm slams into the block M1 at a velocity of 300m/s. It embeds itself in the block. The combination then continues to M2 and has a perfectly elastic collision with M2. Follow these steps.

3a. The first collision is between the bullet and M1. Use conservation of momentum to find the final velocity of the combination M1+b. (10 pts)

$$\boxed{P_{i,tot} = m_b v_b = 0.05\,kg \times 300\,\frac{m}{s} = 15\,kg\,m/s}$$

$$P_f = (2.95\,kg + 0.05\,kg)\,v_{1i} = 15\,kg\,m/s$$

$$\Rightarrow \boxed{v_{1i} = 5\,m/s}$$

3b. The second collision is the perfectly elastic one between M1+bullet and M2. Find the velocity of the COM frame. (10 pts)

$M_1 = 3\,kg$ $\quad v_{1i} = 5\,m/s$ $\quad M_2 = 2\,kg$ $\quad v_{2i} = 0$

$$\boxed{P_{i,tot} = 15\,kg\,m/s = (M_1 + M_2)\,v_0 = 5\,kg\,v_0}$$

where v_0 is the velocity of the COM frame

$$\Rightarrow \boxed{v_0 = 3\,m/s}$$

3c. Using your knowledge of how a perfectly elastic collision proceeds in the COM frame, find the final velocities of M1+b and M2 in the COM frame, and thus the lab frame. (10 pts)

$\boxed{v_{1i}' = v_{1i} - v_0 = 2\,m/s}$ $\boxed{v_{2i}' = v_{2i} - v_0 = -3\,m/s}$

$\Rightarrow$ $\boxed{v_{1f}' = -v_{1i}' = -2\,m/s}$ $\boxed{v_{2f}' = -v_{2i}' = +3\,m/s}$

Velocities reverse in COM frame

$\boxed{v_{1f} = v_{1f}' + v_0 = -2\,m/s + 3\,m/s = 1\,m/s}$

$\boxed{v_{2f} = v_{2f}' + v_0 = 6\,m/s}$

Back to lab frame

Check if $P_{f,tot} = P_{i,tot}$

$P_{i,tot} = 15\,kg\,m/s$

$\boxed{P_{f,tot} = 3\,kg \times 1\,\frac{m}{s} + 2\,kg \times 6\,\frac{m}{s} = 15\,kg\,m/s}$

Works!

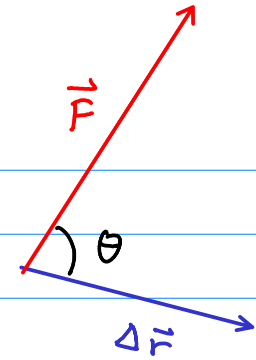
How about $K_i = K_f$?

$\boxed{K_i = \frac{1}{2} \times 3\,kg \times (5\,m/s)^2 = 37.5\,J}$

$\boxed{K_f = \frac{1}{2}(3\,kg)(1\,m/s)^2 + \frac{1}{2}(2\,kg)(6\,m/s)^2 = 1.5\,J + 36\,J = 37.5\,J}$

Works!

Some Useful Formulas



Work $=$ $W = \vec{F}\cdot\Delta\vec{r}$

Geometrically $\quad \vec{F}\cdot\Delta\vec{r} = |\vec{F}|\,|\Delta\vec{r}|\cos\theta$

Algebraically if
$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$
$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z$$

The Work-Energy theorem can be used in two ways.

$$W_{tot} = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F}_{tot}\cdot d\vec{r} = \Delta K = \frac{1}{2}Mv_f^2 - \frac{1}{2}Mv_i^2$$

If some of the forces are conservative, that is

$$\int_{\vec{r}_i}^{\vec{r}_f} \vec{F}\cdot d\vec{r} = -\left(U(\vec{r}_f) - U(\vec{r}_i)\right)$$ independent of path

then one can re-write the Work-Energy theorem

$$W_{nc} = \Delta E_{mech} = E_{mech,f} - E_{mech,i}$$
$$= (K_f + U_f) - (K_i + U_i)$$

W_{nc} is the work done by non-conservative forces, such as friction, or forces exerted by living agents.

We use two kinds of potential energy

$U_g(\bar{r}) = Mgh$ h = height above some chosen zero.

$U_s = \frac{1}{2}k(x')^2$ k = Force constant of spring
x' = Compression/Extention of spring from equilibrium.

If no non-conservative do work, E_{mech} is conserved.

Linear momentum $\vec{p} = M\vec{v}$

In an isolated system of particles, where all the forces are due to other particles, an no external forces act, the total momentum is conserved.

$$\vec{p}_{tot} = M_1\vec{v}_1 + M_2\vec{v}_2 + \cdots + M_N\vec{v}_N$$

$$\frac{d\vec{p}_{tot}}{dt} = 0$$

If external forces do act then

$$\frac{d\vec{p}_{tot}}{dt} = \sum_{\alpha=1}^{N} F_{ext,\alpha}$$

Impulse-Momentum Thm

$$\vec{I}_{tot} = \int_{t_i}^{t_f} \vec{F}_{tot}\, dt = \Delta\vec{p} = \vec{p}_f - \vec{p}_i$$

In any collision, momentum is conserved. In a perfectly elastic collision KE is also conserved.

The COM (Center of Momentum) frame is one in which the total momentum is zero. It moves with velocity v_0 with respect to the lab frame.

v_{1i}, v_{2i} = Initial velocities in Lab frame

v'_{1i}, v'_{2i} = Initial velocities in COM frame

$$v'_{1i} = v_{1i} - v_0 \qquad v'_{2i} = v_{2i} - v_0$$

$$p'_{i\,tot} = M_1 v'_{1i} + M_2 v'_{2i} = p_{1i} + p_{2i} - (M_1 + M_2) v_0$$

So the COM frame is defined by $p'_{i\,tot} = 0$

$$v_0 = \frac{p_{i,tot}}{M_1 + M_2}$$

In the COM frame, after a perfectly elastic collision, the velocities reverse

$$v'_{1f} = -v'_{1i} \qquad v'_{2f} = -v'_{2i}$$