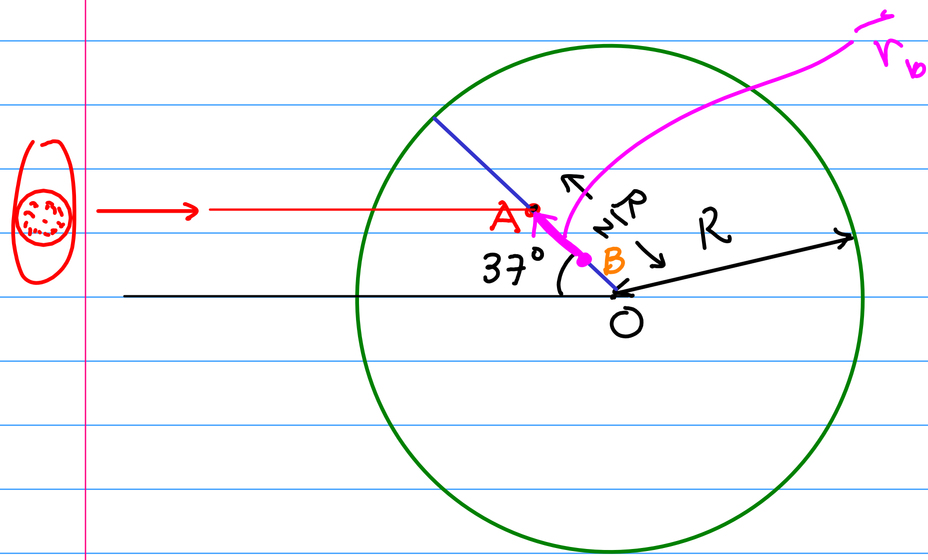
Another example of an angular collision

A large solid disk of mass $\boxed{M_d = 50\text{kg}}$ ① and radius $\boxed{R = 2\text{m}}$ ② sits on a smooth surface. ③ A boy of mass $\boxed{M_b = 30\text{kg}}$ runs up at $\boxed{5\text{ m/s}}$ ④ and lands at the point A shown. Find the subsequent motion of the disk.



In problems of this type the 1st task is to find the position of the CM of the system after the collision. Since no external forces act on the system, the CM moves in a straight line, and the body rotates rigidly around its CM. So we also need the moment of inertia around the final CM.

To find the CM, marked as B, treat the boy as a mass point, and note that the

CM of the solid disk is at O. So

$$\boxed{r_{CM} = \frac{M_d(0) + M_b(R/2)}{M_b + M_d} = \frac{30\text{ kg}\cdot 1\text{m}}{80\text{kg}} = 0.375\text{ m}}$$ 5

How about the moment of inertia of the boy-disk system around B ?

$$\boxed{I_{tot,B} = I_{b,B} + I_{d,B}}$$ 6

Treating the boy as a mass point 7

$$\boxed{I_{b,B} = M_b\left(\frac{R}{2} - r_{CM}\right)^2 = 30\text{ kg}\,(1-0.375)^2\text{m}^2 = 11.72\text{ kgm}^2}$$

For $I_{d,B}$ we use the parallel axis theorem

$$\boxed{I_{d,B} = \underbrace{I_{d,O}}_{} + M_d r_{CM}^2}$$ 8

I around the CM of the disk

$$= \frac{1}{2}M_d R^2 + M_d r_{CM}^2 = 50\text{kg}\left(2\text{m}^2 + (0.375\text{m})^2\right)$$

$$\boxed{I_{d,B} = 107.03\text{ kg m}^2}$$ 9

$$\Rightarrow \boxed{I_{tot,B} = 118.75\text{ kgm}^2}$$ 10

Now we will use the conservation of momentum and angular momentum.

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Initial momentum = $\boxed{M_b v_{bi} \hat{i} = 150 \text{ kg } \frac{m}{s} \hat{i}}$

Final momentum = $\boxed{(M_b + M_d)\vec{v}_f}$ 12

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$\boxed{\vec{P}_i = \vec{P}_f}$ 13 $\Rightarrow$ $\boxed{\vec{v}_f = \frac{150}{80} \text{ m/s } \hat{i} = 1.875 \text{ m/s } \hat{i}}$

For the initial angular momentum, again, it is important to compute it around the final CM, B. Treating the boy as a mass point

$$\boxed{\vec{L}_i = M_b \vec{r}_b \times \vec{v}_b}$$ 15

Clearly L_i is clockwise. Alternately we can say it is in the $-z$ direction. 16

$$\boxed{L_i = -30 \text{kg} \times \left(\frac{R}{2} - r_{cM}\right) \times 5 \text{m/s} \times \sin(180° - 37°)}$$

$$\sin(180° - 37°) = \sin 37° = \frac{3}{5} = 0.6$$

$$\boxed{\begin{aligned} L_i &= -30 \text{ kg} \times 0.625 \text{ m} \times 5 \frac{m}{s} \times 0.6 = \\ &= -56.25 \text{ kg m}^2/s \end{aligned}}$$ 17

By conservation of L this should be $\vec{L}_f$

$$\boxed{L_f = I_{tot,B}\, \omega_f = 118.75 \text{ kg m}^2 \times \omega_f}$$ 18

$\boxed{\omega_f = -0.474 \text{ rad/s}}$ 19 Clockwise rotation