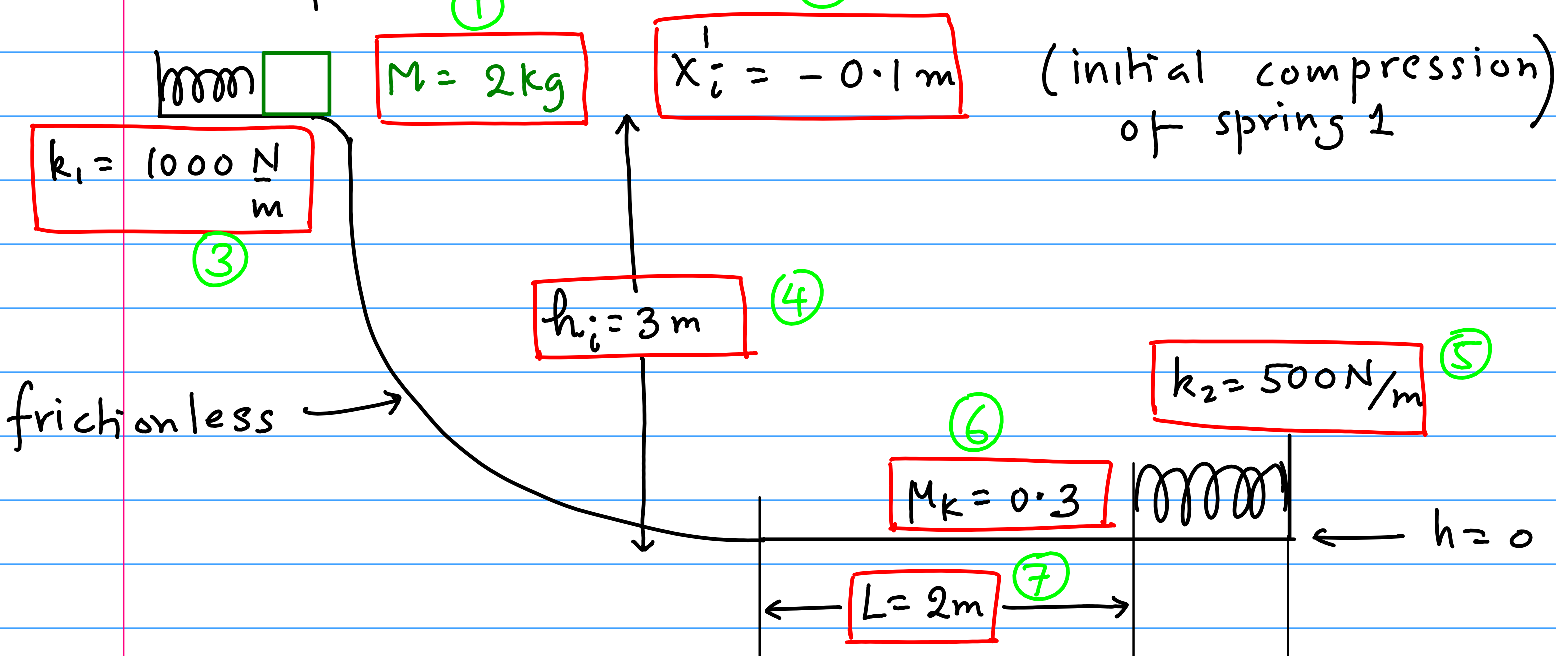
Midterm-type problems in Work, Energy, and momentum

Example 1

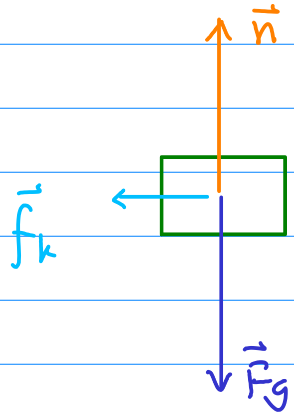


How far does spring 2 compress at its max compression?

Let it be x'_f ⑧

$$W_{nc} = E_{mech,f} - E_{mech,i} \quad ⑨$$

W_{nc} can only come from $\vec{f}_k$ on the horizontal stretch.



$a_y = 0 \Rightarrow$ $n = Mg$ ⑩

$f_k = \mu_k M g$ ⑪

$$\boxed{W_{nc} = -f_k(2 + x'_f)} \quad (12)$$

because the block slides 2 meters to the end of spring 2 and then a further x'_f as it compresses it.

$$\boxed{E_{mech,f} = \frac{1}{2} k_2 {x'_f}^2} \quad (13)$$

$$\boxed{E_{mech,i} = Mgh_i + \frac{1}{2}k_1(x'_i)^2} \quad (14)$$

$$\Rightarrow \boxed{-\mu_k Mg(2+x'_f) = \frac{1}{2}k_2(x'_f)^2 - Mgh_i - \frac{1}{2}k_1(x'_i)^2} \quad (15)$$

Plug in numbers $M = 2\,kg$ $g = 9.8\,m/s^2$ $h_i = 3\,m$

$k_1 = 1000\,\frac{N}{m}$ $x'_i = -0.1\,m$

$$-0.3 \times 2 \times 9.8\,(2 + x'_f) = 250(x'_f)^2 - 2 \times 9.8 \times 3 - \frac{1}{2} \times 1000 \times 0.01$$

$$\Rightarrow 250(x'_f)^2 + 5.88\,(2 + x'_f) - 63.8 = 0$$

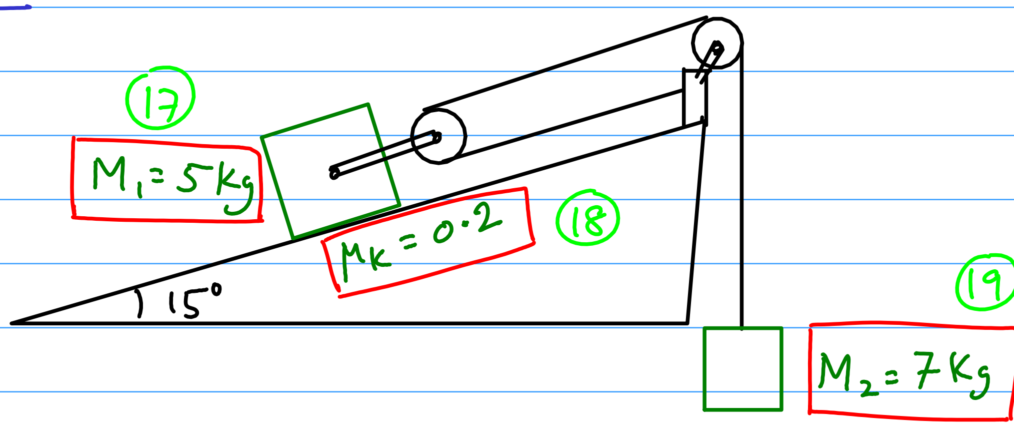
$$\text{or} \quad 250(x'_f)^2 + 5.88\,x'_f - 52.04 = 0$$

$$x'_f = \frac{-5.88 \pm \sqrt{(5.88)^2 + 1000 \times 52.04}}{500}$$

Need $x'_f > 0$ choose + solution

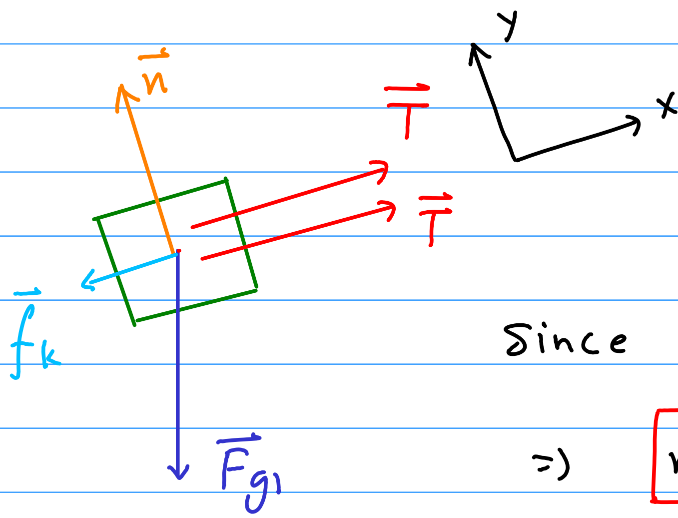
$$\boxed{x'_f = 0.445\,m} \quad (16)$$

Example 2



The masses start at rest. What are their speeds when M_2 has dropped by 1 m?

Draw a FBD for M_1



Since $a_y = 0$, $\Rightarrow F_{tot,y} = 0$

$$\Rightarrow \boxed{n = M_1 g \cos\theta} \quad 20$$

$$\Rightarrow \boxed{\begin{aligned} f_k = \mu_k M_1 g \cos\theta &= 0.2 \times 5\,kg \times 9.8\,\frac{m}{s^2} \times 0.966 \\ &= 9.466\,N \end{aligned}} \quad 21$$

Note that when M_2 drops 1m, M_1 moves up the slope 0.5m. Also, its height increases by $\boxed{(0.5m) \sin 15° = 0.1294\ m}$ 22

$$\boxed{W_{nc} = -f_k \times 0.5\,m = -4.733\ J}$$ (23)

$$\boxed{W_{nc} = E_{mech,f} - E_{mech,i}}$$ (24)

let the speed of M_1 be v_{1f} and that of M_2 be $\boxed{v_{2f} = 2v_{1f}}$ (25) (it moves twice the distance M_1 moves)

$$\Delta E_{mech} = \Delta K + \Delta U_g$$

$$= \frac{1}{2} M_1 v_{1f}^2 + \frac{1}{2} M_2 v_{2f}^2 + M_2 g(-1\,m)$$

$$+ M_1 g(0.1294\,m)$$

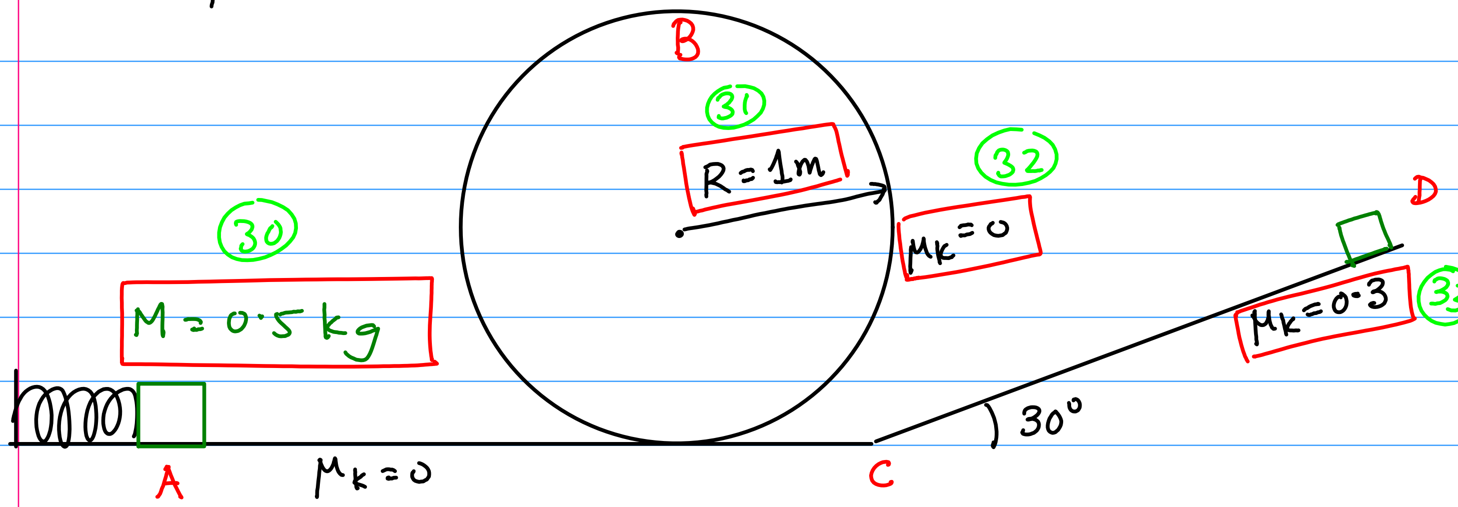
$$\boxed{\Delta E_{mech} = \frac{1}{2} v_{1f}^2 (M_1 + 4M_2) + g(-M_2 + 0.1294\, M_1)}$$ (26)

So $W_{nc} = \Delta E_{mech}$

$$\Rightarrow \boxed{-4.733\ J = 16.5\, v_{1f}^2 - 62.2594\ J}$$ (27)

$$\Rightarrow \boxed{v_{1f} = 1.867\ m/s}$$ (28) $$\boxed{v_{2f} = 3.734\ m/s}$$ (29)

Example 3

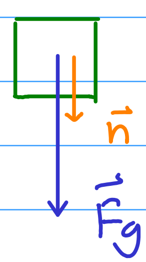


The mass is held against a spring compressed by $\boxed{x_i = -0.1m}$. It is known that it has just enough speed to stay on the circular track at the very top. (34)

i) What is the force constant of the spring?

ii) How far up the slope (with friction) does it go before coming to rest?

At the top B the FBD is



The acceleration is towards the center

$$\boxed{\vec{a} = -\frac{v_B^2}{R}\hat{j}}$$ (35)

$$\Rightarrow \boxed{Mg + n = \frac{Mv_B^2}{R}} \quad (36)$$

as v_B decreases n decreases. The minimum value for n is 0

$$\Rightarrow \boxed{v_{B,min} = \sqrt{Rg}} \quad (37)$$

There is no friction between A and B

So $E_{mech,A} = E_{mech,B}$

or $K_A + U_{g,A} + U_{s,A} = K_B + U_{g,B} + U_{s,B}$ (38)

Since the mass starts from rest $K_A = 0$

Choose the height of A as $h_A = 0 \Rightarrow U_{gA} = 0$

$$U_{sA} = \frac{1}{2}k(x_i')^2$$

At B $U_{s,B} = 0$ $U_{g,B} = Mg\,(2R)$

$$K_B = \frac{1}{2}Mv_B^2 = \frac{1}{2}M\,(gR) = \frac{1}{2}MgR$$

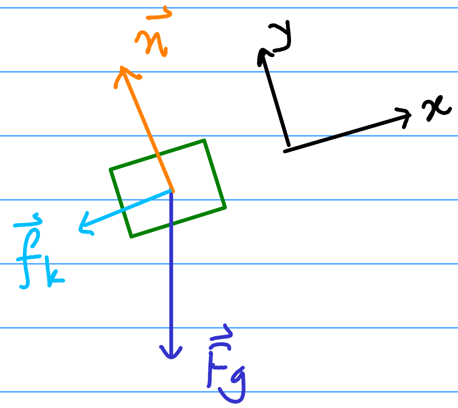
So $$\boxed{\frac{1}{2}k\,(x_i')^2 = \frac{1}{2}MgR + 2MgR = \frac{5}{2}MgR} \quad (39)$$

(40)

$$\boxed{k = \frac{5\,Mg\,R}{(x_i')^2} = \frac{5 \times 0.5\,kg \times 9.8\,m/s^2 \times 1\,m}{0.01\,m^2}}$$

$$\boxed{k = 2450\ N/m}$$ (41)

Now consider the slope. The FBD is



$a_y = 0 \Rightarrow$

$F_{y,tot} = 0$

$\Rightarrow \quad n - Mg\cos(30°) = 0$

$$\boxed{n = Mg\cos(30°)}$$ (42)

$$\Rightarrow \quad \boxed{f_k = \mu_k Mg\cos(30°)}$$ (43)

The Work-Energy Thm says

$$W_{nc} = \Delta E_{mech}$$

Let it travel $\boxed{x_f}$ (44) on the slope. Its final height is

$$\boxed{h_f = x_f \sin(30°)}$$ (45)

$$\boxed{W_{nc} = -f_k x_f = -\mu_k Mg x_f \cos 30°}$$ (46)

$$\boxed{\begin{aligned} \Delta E_{mech} &= E_{mech,D} - E_{mech,B} \\ &= Mgh_f - \left(\tfrac{5}{2} MgR\right) \end{aligned}}$$ (47)

(48)

So $$\boxed{-\mu_k Mg x_f \cos(30°) = Mg x_f \sin 30° - \tfrac{5}{2} MgR}$$

or $x_f \left[\mu_k \cos(30^\circ) + \sin 30^\circ \right] = \frac{5}{2} R$

$$\boxed{x_f = \frac{\frac{5}{2} \times 1\,m}{0.3 \times 0.866 + 0.5} = 3.29\ m}$$

(49)