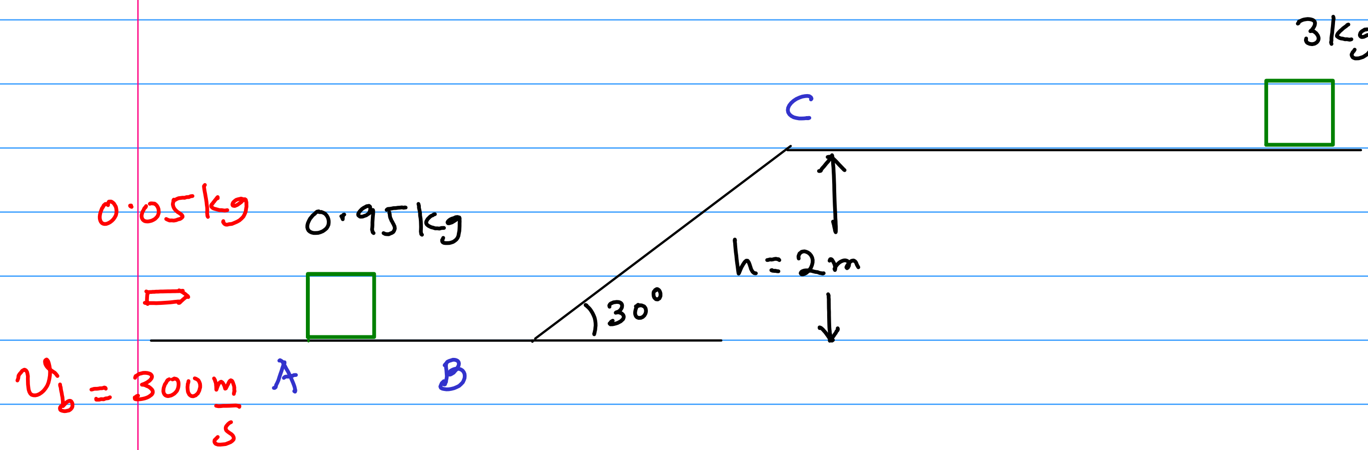
Potential Midterm problem

Collisions



Two blocks of masses 0.95 kg and 3kg are initially at rest. A bullet of mass 0.05kg travelling at 300m/s slams into the 0.95 kg block and embeds itself. The block + bullet then proceeds up the frictionless slope and has a perfectly elastic collision with the 3 kg block. Find their final velocities.

The 1st collision is totally inelastic.

=) Only momentum is conserved. Let the mass of block 1 + bullet = M_1 and its velocity after collision be v_1

$$\boxed{P_i = 0.05\,kg \times 300\,m/s = 15\,kg\,m/s} \quad ①$$

$$\boxed{P_f = M_1 v_1 = 1\,kg \times v_1 = P_i = 15\,kg\,m/s} \quad ②$$

=) $\boxed{v_1 = 15\,m/s}$ ③

Now M1 travels up the slope. There is no friction, so $\boxed{W_{nc}=0}$ ④. The work-energy thm says

$$\boxed{W_{nc} = \Delta E_{mech}} \quad ⑤$$

So, if M1 at the bottom of the slope is state B and M1 at the top is state C

$$\boxed{E_{mech,B} = E_{mech,C}} \quad ⑥$$

$$\boxed{\frac{1}{2} M_1 v_1^2 = \frac{1}{2} M_1 v_{1c}^2 + M_1 g h} \quad ⑦$$

or $v_1^2 = v_{1c}^2 + 2gh$

$$225 = v_{1c}^2 + 2\times 9.8\times 2 = v_{1c}^2 + 39.2$$

$$\Rightarrow \boxed{v_{1c} = \sqrt{185.8} = 13.63\ m/s} \quad ⑧$$

Now consider the perfectly elastic collision

$$\boxed{\begin{array}{ll} M_1 = 1kg & v_{1i} = 13.63\ m/s \\ M_2 = 3kg & v_{2i} = 0 \end{array}} \quad ⑨$$

COM frame is defined by

$$\boxed{p_{1i} + p_{2i} = (M_1 + M_2) v_0} \quad ⑩$$

or $13.63\ kg\ m/s = 4 v_0$

$$\boxed{v_0 = 3.41\ m/s} \quad ⑪$$

So, in the COM frame

$$\boxed{\begin{aligned} v'_{1i} &= v_{1i} - v_0 = 10.22 \text{ m/s} \\ v'_{2i} &= v_{2i} - v_0 = -3.41 \text{ m/s} \end{aligned}} \quad (12)$$

The velocities reverse themselves after collision in the COM frame

$$\boxed{v'_{1f} = -10.22 \text{ m/s} \qquad v'_{2f} = 3.41 \text{ m/s}} \quad (13)$$

$\Rightarrow$ Go back to lab frame

$$\boxed{\begin{aligned} v_{1f} &= v'_{1f} + v_0 = -6.81 \text{ m/s} \\ v_{2f} &= v'_{2f} + v_0 = 6.82 \text{ m/s} \end{aligned}} \quad (14)$$

Check if momentum and energy are conserved in the lab frame

$$\boxed{P_{i,tot} = 13.63 \text{ kg m/s}} \quad (15)$$

$$\boxed{P_{f,tot} = -6.81 + 3 \times 6.82 = 13.65 \text{ kg m/s}} \quad (16)$$

small numerical error

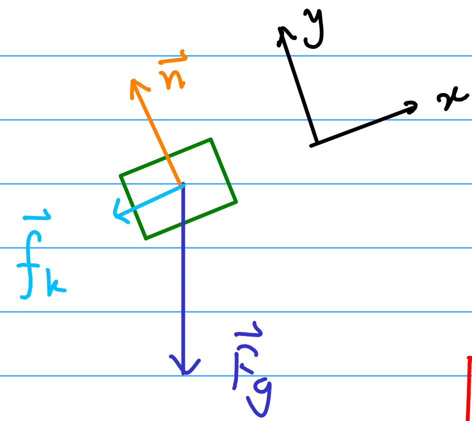
$$\boxed{K_i = \frac{1}{2}(13.63)^2 = 92.89 \text{ J}} \quad (17)$$

$$\boxed{K_f = \frac{1}{2}(6.81)^2 + \frac{1}{2}\, 3\,(6.82)^2 = 92.95 \text{ J}} \quad (18)$$

equal within numerical error.

Example 2: Suppose the slope does have friction. Assume $\boxed{\mu_k = 0.2}$ (19). We will use the work-energy theorem

$$W_{nc} = \Delta E_{mech}.$$



(20)

$$\boxed{\begin{array}{l} a_y = 0 \\ \Rightarrow F_{y,tot} = 0 \end{array}}$$

$$\Rightarrow \quad n - M_1 g \cos 30^\circ = 0$$

$$\boxed{\begin{array}{l} n = M_1 g \cos 30^\circ \\ f_k = \mu_k n \end{array}}$$ (21) (22)

So $$\boxed{f_k = (0.2)(1\,kg)(9.8\,m/s^2)(0.866) = 1.7\,N}$$

The distance over which f_k does work is

$$\boxed{d = \frac{h}{\sin 30^\circ} = 2h = 4m}$$ (23)

$$\boxed{W_{nc} = -f_k d = -1.7\,N\,(4m) = -6.8\,J}$$ (24) (25)

So $$\boxed{-6.8\,J = \frac{1}{2}(1)\,v_{1c}^2 + 1\,(9.8)(2) - \frac{1}{2}(1)(15)^2}$$

$$v_{1c}^2 = 225 - 39.2 - 13.4 = 172.4 \left(\frac{m}{s}\right)^2$$

$$\boxed{v_{1c} = 13.13\,m/s}$$ (26)

Now proceed as usual

$$\boxed{v_0 = \frac{13.13}{4} = 3.28 \text{ m/s}}$$ (27)

$$v_{1i}' = 13.13 - 3.28 = 9.85 \text{ m/s}$$

$$v_{2i}' = -3.28 \text{ m/s}$$ (28)

$$v_{1f}' = -v_{1i}' = -9.85 \text{ m/s}$$

$$v_{2f}' = -v_{2i}' = +3.28 \text{ m/s}$$ (29)

(30)

$$\Rightarrow \boxed{v_{1f} = -6.57 \text{ m/s} \qquad v_{2f} = 6.56 \text{ m/s}}$$