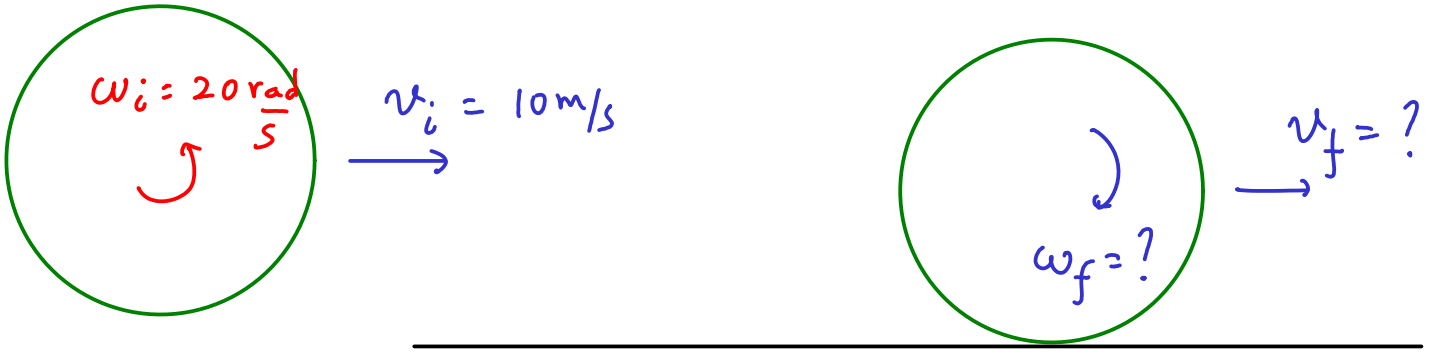
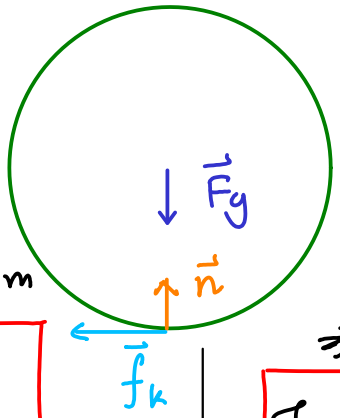
Quiz 11

A solid sphere just above a horizontal surface has an initial CM velocity of 10m/s, and an initial counterclockwise angular velocity of 20rads/sec. The mass is M=10kg and the radius is R=0.1m. The sphere then touches the surface and eventually starts rolling without slipping. Use the Impulse-Momentum theorem in its angular and linear forms to find its final velocity.



$$p_{ix} = M v_i \qquad p_{fx} = M v_f$$

$$L_i = I\omega_i \qquad L_f = -I\frac{v_f}{R}$$ because $$\omega_f = -\frac{v_f}{R}$$



$a_y = 0 \Rightarrow n = Mg$

$$\vec{F}_{tot} = -f_k \hat{i}$$

Impulse - Momentum Thm

$$\mathcal{J}_x = -\int_{t_i}^{t_f} f_k\, dt = \Delta p_x = M(v_f - v_i)$$

$$\Rightarrow \quad \mathcal{J}_{ang,tot} = -\int_{t_i}^{t_f} f_k R\, dt = R\mathcal{J}_x$$

Only f_k produces torque

$$\mathcal{J}_{ang,tot} = \Delta L = I(\omega_f - \omega_i)$$

So use $$\mathcal{J}_{ang,tot} = R\mathcal{J}_{x,tot} = RM(v_f - v_i)$$

$$\Rightarrow \quad MR(v_f - v_i) = \frac{2}{5}MR^2\left(-\frac{v_f}{R} - \omega_i\right) \Rightarrow v_f - v_i = -\frac{2}{5}v_f - \frac{2}{5}\omega_i R$$

$$v_f = \frac{5}{7}\left(v_i - \frac{2}{5}\omega_i R\right) = \frac{5}{7}\left(10 - 0.4 \times 20 \times 0.1\right) = 6.57\ m/s$$