Ch 7: CIRCULAR MOTION

SO FAR WE'VE DONE:

1) LINEAR (STRAIGHT LINE) MOTION (1-D)
2) PROJECTILE MOTION – TRAJECTORIES (2-D)

AND WE CONSIDERED "POINT OBJECTS"

NOW WE COME TO FINITE SIZE OBJECTS

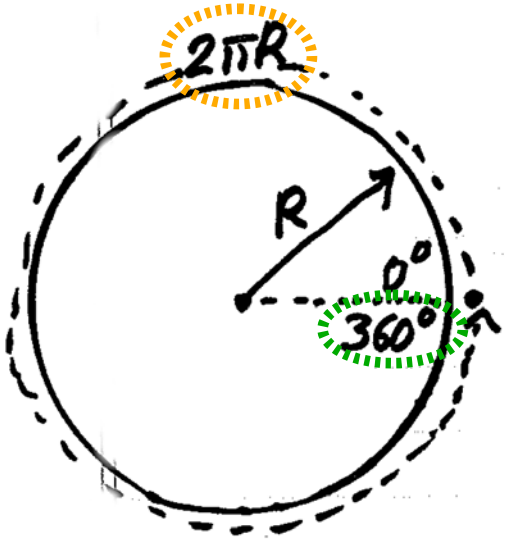
THESE CAN ROTATE ABOUT AN AXIS OR AXLE – CIRCULAR MOTION

E.G. – THE WHEEL!

IT'S A VERY SPECIAL TYPE OF MOTION

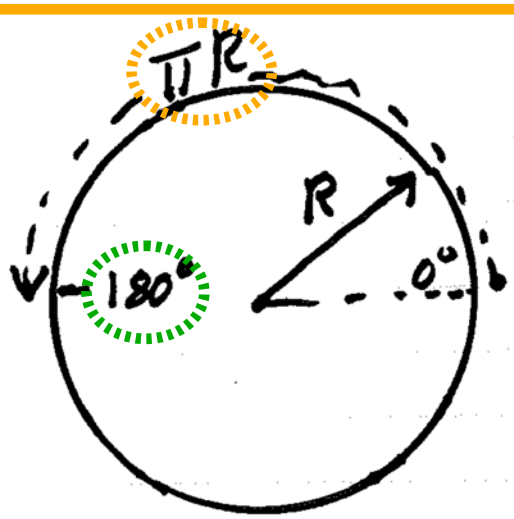
WE NEED TO LOOK AT THE QUANTITIES USED TO DESCRIBE CIRCULAR MOTION

CONSIDER A DISC OF RADIUS R
DISK



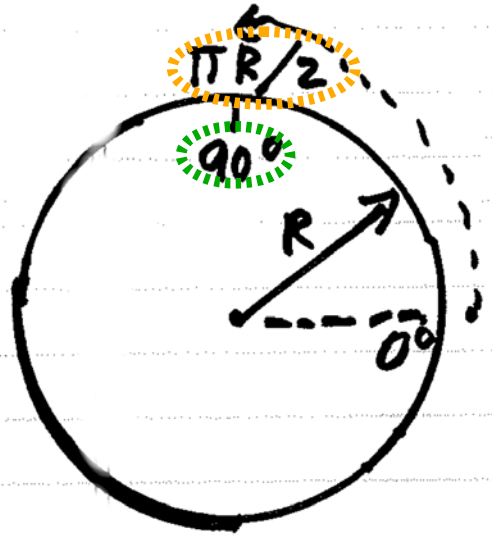
ALL THE WAY ROUND IS 360°

DISTANCE 2πR



HALF WAY " " 180°

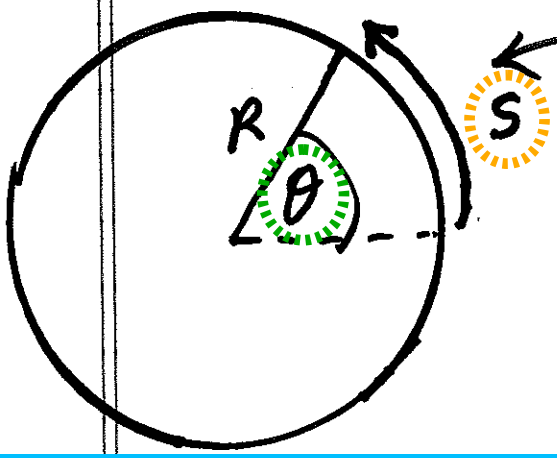
DISTANCE πR



QUARTER " " " 90°

DISTANCE $\frac{\pi R}{2}$

WHAT IS THE LENGTH FOR θ DEGREES?



KNOWN AS THE ARC LENGTH. S

(IE. HOW FAR HAS AN ANT WALKED)

IF 360° IS $S = 2\pi R$

THEN 1° IS $S = \dfrac{2\pi R}{360}$

SO θ° IS $S = \dfrac{2\pi\theta}{360} R$ (HORRIBLE!)

REWRITE AS $S = \dfrac{\theta}{\left(\frac{360^\circ}{2\pi}\right)} R$ ← DEGREES

WHAT IS THE ANGLE FOR WHICH $\underline{S = R}$?

ANSWER: WHEN $\theta / (360^\circ / 2\pi) = 1$

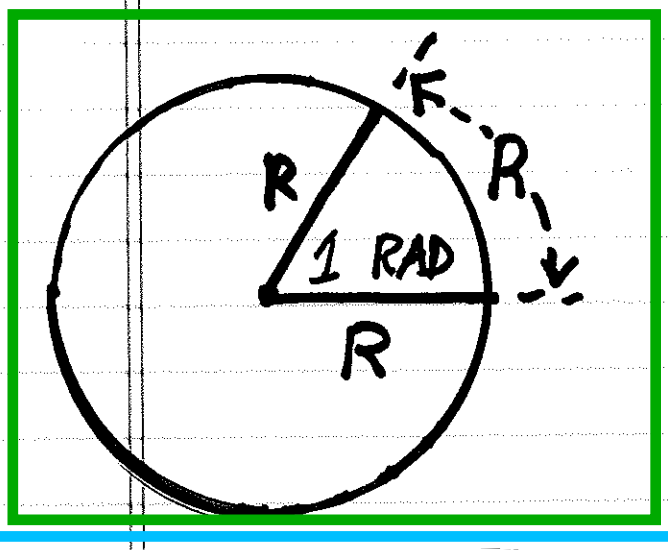
OR $\theta = \dfrac{360^\circ}{2\pi} \approx 57^\circ$

THESE EXPRESSIONS BECOME MUCH MORE ELEGANT IF WE DEFINE (AND USE) A NEW UNIT OF ANGULAR MEASURE:

THE RADIAN

1 RADIAN IS THE ANGLE FOR WHICH

$$s = R$$



⇐ DEFINITION OF THE RADIAN

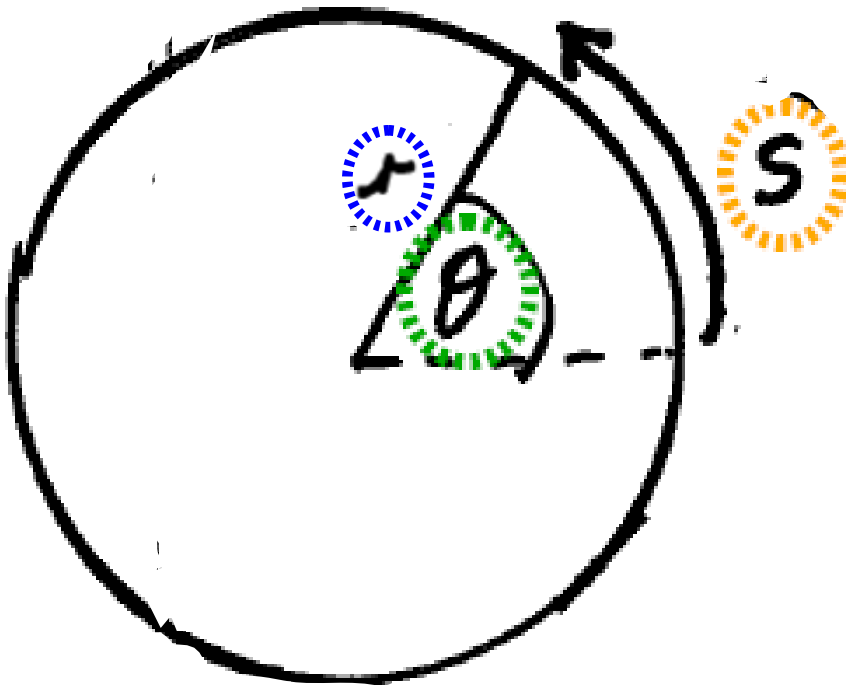
1 RADIAN $\simeq 57^\circ$

IF θ IS IN RADIANS:

$$s = R\theta$$

$$\theta = \frac{s}{R}$$

DIMENSIONLESS — BUT WE STILL SAY RADS



IF θ IS IN RADIANS:

$$\boxed{s = r\theta} \qquad \boxed{\theta = \frac{s}{r}}$$

DIMENSIONLESS — BUT WE STILL SAY RADS

HOW MANY RADIANS MAKE A FULL CIRCLE?

USE $s = R\theta = \theta R$ WITH $S = C = 2\pi R$

SO $\boxed{\theta = 2\pi \text{ IS A FULL CIRCLE}}$

SO $\boxed{90^\circ = \frac{\pi}{2} \text{ RADIANS}}$ $\left(\frac{C}{4}\right)$

$\boxed{180^\circ = \pi \text{ RADIANS}}$

$\boxed{270^\circ = \frac{3}{2}\pi \text{ RADIANS}}$

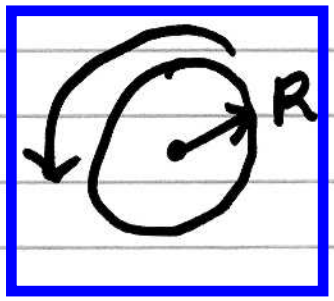
SO FAR STATIONARY OBJECTS

NOW CIRCULAR MOTION

E.G. EARTH ORBITING THE SUN

OR: A ROTATING WHEEL

LET'S CONSIDER A ROTATING DISC:



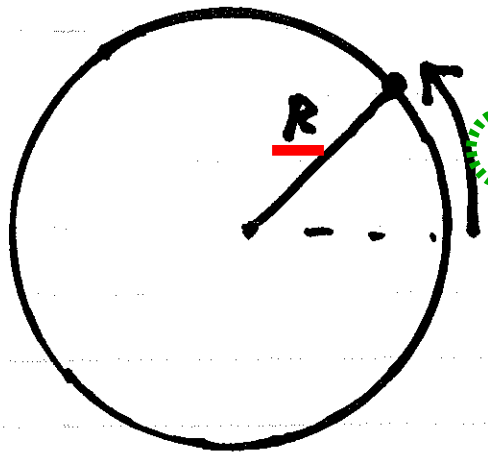
WE COULD DESCRIBE ITS MOTION BY THE NUMBER OF TURNS IT MAKES PER SECOND

BUT THERE IS A MORE USEFUL DESCRIPTION!

ROTATING DISC

WHAT IS THE SPEED ON THE EDGE OF THE DISC?

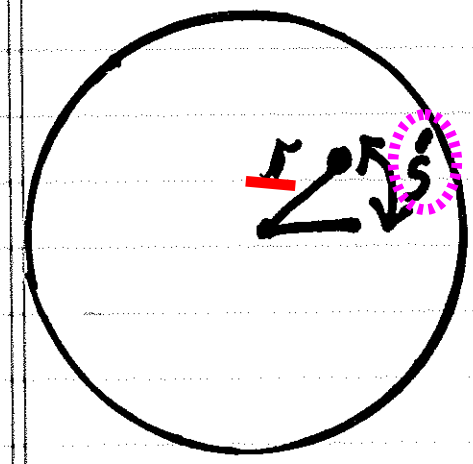
IT TRAVELS s IN t SECONDS.



s IN t

$$\boxed{v = \frac{s}{t}}$$

WHAT IS THE SPEED OF A POINT FURTHER IN AT r?



s' IN THE SAME t.

s' IN t

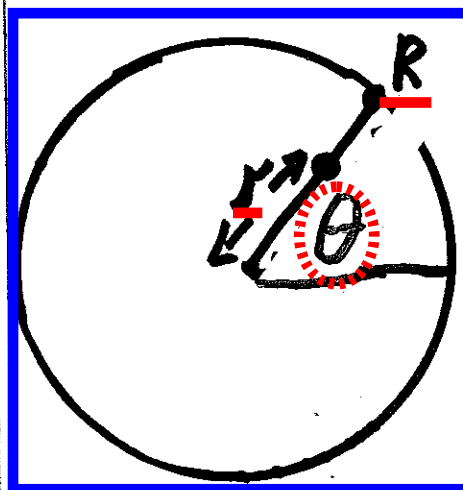
$$\boxed{v' = \frac{s'}{t}}$$

BUT $s' < s$ SO $v' \neq v$.

SO IF WE WANT TO DESCRIBE THE MOTION OF ALL POINTS ON THE DISC, THE SPEED v OF A POINT IS NOT A UNIVERSAL QUANTITY.

N.B.: FOR STRAIGHT LINE MOTION OF A FINITE SIZED BODY, THE SPEED v IS THE SAME FOR ALL POINTS OF THE BODY

BUT THE TWO POINTS AT R AND r DO HAVE SOMETHING IN COMMON



THE ANGLE THAT BOTH r AND R SWEEP OUT IN A GIVEN TIME t IS THE SAME

AND:

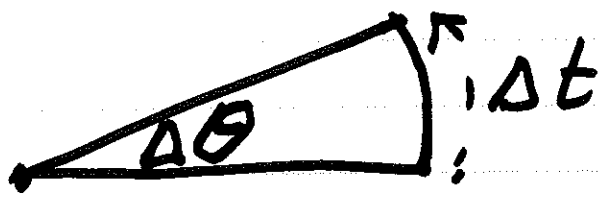
$\frac{\theta}{t}$ IS THE SAME FOR ALL POINTS ON THE DISC

WE DEFINE THE ANGULAR VELOCITY ω BY:

$$\boxed{\omega = \frac{\Delta\theta}{\Delta t}}$$

$\Delta\theta$ IS SWEPT OUT IN TIME Δt

$$\Delta\theta = \theta_f - \theta_i$$



ω ↑ GREEK LOWER CASE OMEGA

TRUE IF ROTATING AT CONSTANT ω (ANGULAR VELOCITY).

IF $\omega \neq$ CONST IN TIME

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$$

AVERAGE $\boxed{\bar{\omega} = \frac{\Delta\theta}{\Delta t}}$

UNITS OF ω: RADIANS/SEC.

WHAT IS THE RELATIONSHIP BETWEEN THE ARC LENGTH s SWEPT OUT IN t, AND ω?

GENERAL POINT

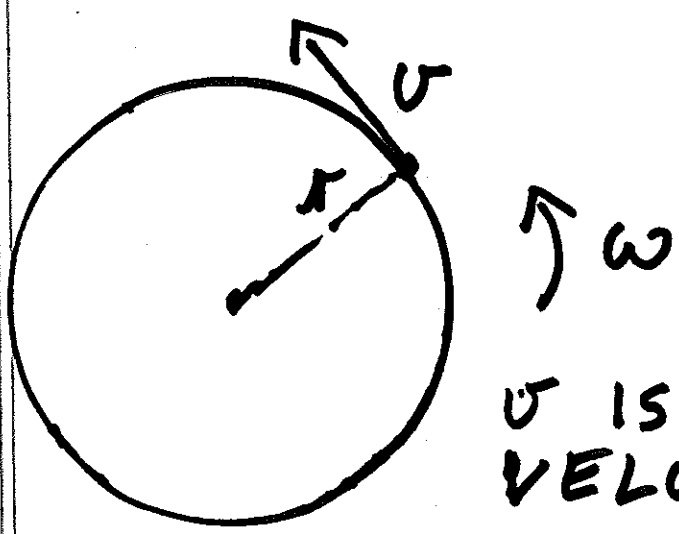
WE HAVE: $\boxed{s = r\theta}$

SO $\boxed{\Delta s = r\Delta\theta}$

ALSO THE SPEED (m/s) OF A POINT AT r IS:

$$v = \frac{\Delta s}{\Delta t} = r\frac{\Delta\theta}{\Delta t} = r\omega$$

IE $\boxed{v = \omega r}$ OR $\boxed{\omega = \frac{v}{r}}$



v IS THE TANGENTIAL VELOCITY

(IE THE POINT AT r STAYS A FIXED DISTANCE r FROM THE CENTER)

$v = \omega r$ STILL TRUE IF DISC IS SPEEDING UP OR SLOWING DOWN

$\omega = \frac{v}{r}$

CONST

ONE MORE THING ABOUT ω:

HOW LONG TO MAKE ONE COMPLETE TURN, OR REVOLUTION?

ω IS RADIANS/SEC ($\frac{1}{\omega}$ IS SEC/RAD)

AND: ONE REVOLUTION IS 2π RADIANS

SO $2\pi \times \frac{1}{\omega}$ IS TIME FOR 1 REV

$$\boxed{T = \frac{2\pi}{\omega}} = \boxed{\text{PERIOD}}$$

ANGULAR ACCELERATION

IF ANGULAR VELOCITY ω IS NOT CONST. IN TIME WE CAN DEFINE AN:

ANGULAR ACCELERATION α

IF α IS CONST (REMEMBER a = CONST)

$$\alpha = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$

IE $\boxed{\alpha = \frac{\Delta\omega}{\Delta t}}$ UNITS RAD/S^2

IF $\alpha \neq$ CONST IN TIME:

AVERAGE $\bar{\alpha} = \frac{\Delta\omega}{\Delta t}$

WHAT IS THE RELATIONSHIP BETWEEN α, r AND:

a_t THE TANGENTIAL ACCELERATION OF A POINT AT r?

WELL: $\boxed{\omega = \frac{v}{r}}$ SO $\boxed{\Delta\omega = \frac{\Delta v}{r}}$

SO: $\alpha = \frac{\Delta\omega}{\Delta t} = \frac{1}{r}\frac{\Delta v}{\Delta t} = \frac{a_t}{r}$

SO: $\boxed{\alpha = \frac{a_t}{r}}$ $\boxed{a_t = r\alpha}$

TO SUMMARIZE:

LINEAR MOTION	CIRCULAR MOTION	?
t	t	$t = t$!
x (OR s)	θ	$s = r\theta$
v	ω	$v = r\omega$
a (OR a_t)	α	$a_t = r\alpha$

JUST MULTIPLY BY r!

UNITS

t (s)	t (s)
x (m)	θ (RAD)
v (m/s)	ω (RAD/s)
a (m/s^2)	α (RAD/s^2)

REMEMBER THE 1-D EQUATIONS FOR STRAIGHT LINE (LINEAR) MOTION?

$$v = v_0 + at \qquad x = v_0 t + \frac{1}{2} a t^2$$

$$v^2 = v_0^2 + 2ax \qquad x = \frac{v + v_0}{2} t$$

WE CAN ALSO USE THESE FOR CIRCULAR MOTION WITH:

$x \rightarrow s$ $\quad v$ = TANGENTIAL VELOCITY

$a \rightarrow a_t$

I.E. $v = v_0 + a_t t \qquad s = v_0 t + \frac{1}{2} a_t t^2$

$$v^2 = v_0^2 + 2 a_t s \qquad s = \frac{v + v_0}{2} t$$

NOT VERY USEFUL IN THIS FORM, SO WE CONVERT THEM

TO CONVERT THEM TO CONTAIN ANGULAR QUANTITIES WE USE

$$\boxed{\begin{aligned} s &= r\theta \\ v &= r\omega \\ a_t &= r\alpha \end{aligned}} \qquad (t = t!)$$

SO $\boxed{v = v_0 + a_t t}$

BECOMES $\cancel{r}\omega = \cancel{r}\omega_0 + \cancel{r}\alpha t$

OR: $\boxed{\omega = \omega_0 + \alpha t}$

NEXT: $\boxed{s = v_0 t + \frac{1}{2} a_t t^2}$

$\Rightarrow \quad \cancel{r}\theta = \cancel{r}\omega_0 t + \frac{1}{2}\cancel{r}\alpha t^2$

OR: $\boxed{\theta = \omega_0 t + \frac{1}{2}\alpha t^2}$

THEN $$\boxed{s = \frac{v + v_0}{2} t}$$

$$\Rightarrow \quad \cancel{r}\theta = \frac{\cancel{r}\omega + \cancel{r}\omega_0}{2} t$$

OR $$\boxed{\theta = \frac{\omega + \omega_0}{2} t}$$

SO FAR WE'VE HAD EVERY TERM CONTAINS "r" WHICH HAS CANCELLED OUT.

WHAT ABOUT $$\boxed{v^2 = v_0^2 + 2a_t s}$$

WHICH HAS SQUARED TERMS?

LETS SEE. IT BECOMES

$$\cancel{r^2}\omega^2 = \cancel{r^2}\omega_0^2 + 2\underbrace{(\cancel{r}\alpha)(\cancel{r}\theta)}_{=\cancel{r^2}}$$

OR $$\boxed{\omega^2 = \omega_0^2 + 2\alpha\theta}$$

SO FOR CONSTANT α WE HAVE:

$$\boxed{\begin{aligned} \omega &= \omega_0 + \alpha t \\ \theta &= \omega_0 t + \tfrac{1}{2}\alpha t^2 \\ \theta &= \frac{\omega + \omega_0}{2} t \\ \omega^2 &= \omega_0^2 + 2\alpha\theta \end{aligned}}$$

AND WE DON'T HAVE TO MEMORIZE THEM IF WE NOTICE THAT THEY HAVE THE SAME FORM AS THE LINEAR EQUATIONS WITH:

$$\boxed{\begin{aligned} x &\rightarrow \theta \\ v &\rightarrow \omega \\ a &\rightarrow \alpha \end{aligned}}$$