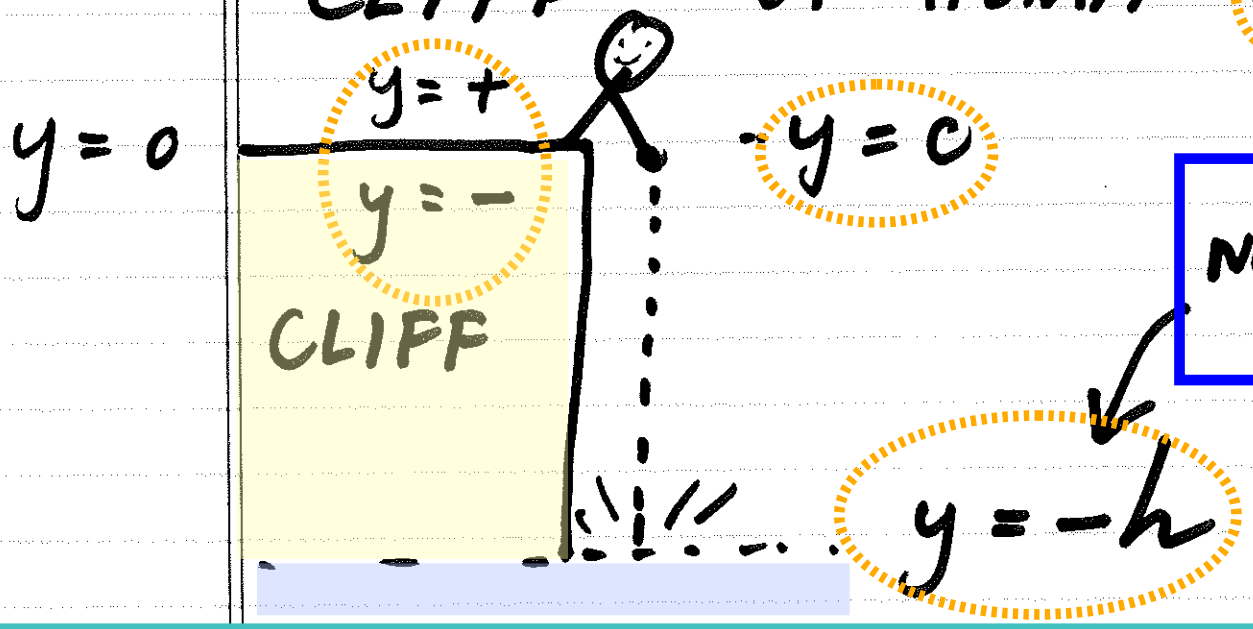
LK3 2012

GRAVITY

LET'S WORK OUT SOME EXAMPLES

EXAMPLE 1

1) DROP SOMETHING OVER A CLIFF OF HEIGHT h



NOTE ⊖ SIGN!

HOW LONG DOES IT TAKE TO REACH THE SEA?

$$\boxed{y = v_0 t - \frac{1}{2} g t^2}$$

DROP $= v_0 = 0$

$y = -\frac{1}{2} g t^2$ AND $y = -h$

$-h = -\frac{1}{2} g t^2 \Rightarrow \boxed{t = \sqrt{\frac{2h}{g}}}$

②

HOW FAST IS IT GOING WHEN IT LANDS?

$$\boxed{v^2 = v_0^2 - 2gy}$$

$y = -h$

$v_0 = 0$

$$v^2 = -2g(-h) = +2gh$$

$$v = \pm\sqrt{2gh}$$

CAREFUL! $\pm$ FOR $\sqrt{}$

THE PHYSICS SAYS v IS DOWNWARDS $\downarrow$

SO v IS $\ominus$

$$\boxed{v = -\sqrt{2gh}}$$

EXAMPLE 2

2) THROW SOMETHING VERTICALLY UPWARDS WITH v_0

a) HOW HIGH DOES IT GO?

WE KNOW v_0 $v = 0$

$a = -g$

SO: $\boxed{v^2 = v_0^2 - 2gh}$

$0 = v_0^2 - 2gh$

$v_0^2 = 2gh$

$$\boxed{h = \frac{v_0^2}{2g}}$$

b) WHAT IS v WHEN IT COMES BACK DOWN TO THE GROUND?

LET'S DO IT TWO WAYS:

FIRST WAY

IT STARTS FROM $y = h$ WITH INITIAL VELOCITY $v_0 = 0$

SAME AS BEING DROPPED OFF A CLIFF OF HEIGHT h!!

SO $v = -\sqrt{2gh}$.

BUT $h = \dfrac{v_0^2}{2g}$

SO

$$v = -\sqrt{2g\left(\frac{v_0^2}{2g}\right)} = -\sqrt{v_0^2}$$

$$\boxed{v = -v_0}$$

SAME SPEED AS IT STARTED WITH!!

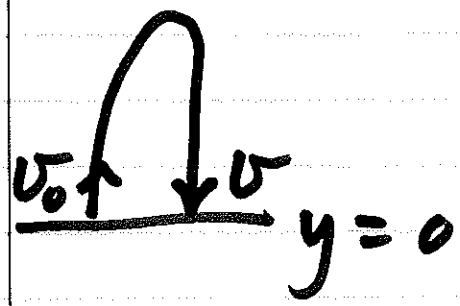
SO THE FIRST WAY DID IT IN TWO SEPARATE STEPS: UP THEN DOWN

UP GAVE h.

DOWN USED h.

SECOND WAY

SINGLE STEP: THE MAGIC OF OUR EQUATIONS.



IT STARTS AT $y=0$.

IT ENDS AT $y=0$

SO $v^2 = v_0^2 - 2gy$ ← $y=0$!!

$v^2 = v_0^2$.

$v = \pm\sqrt{v_0^2}$ $\boxed{v = -v_0}$ "PHYSICS"

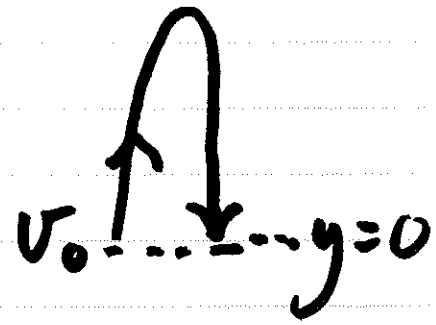
SIMPLE!

HOW LONG IS IT IN THE AIR?

SINGLE STEP:

$$y = v_0 t - \frac{1}{2} g t^2$$

$y = 0$!!



$$\left(v_0 - \frac{1}{2} g t\right) t = 0$$

SO $t = 0$ [IE IT'S AT $y=0$ AT $t=0$ — TRUE!]

OR $v_0 - \frac{1}{2} g t = 0$

$$v_0 = \frac{1}{2} g t$$

$$\boxed{t = \frac{2 v_0}{g}}$$

WHICH IS WHEN IT LANDS.

TIP

USE UNITS, TO CHECK YOUR ANSWER!

WE HAD $h = \frac{v_0^2}{2g}$

UNITS: $\boxed{v_0 \equiv \frac{m}{s}}$

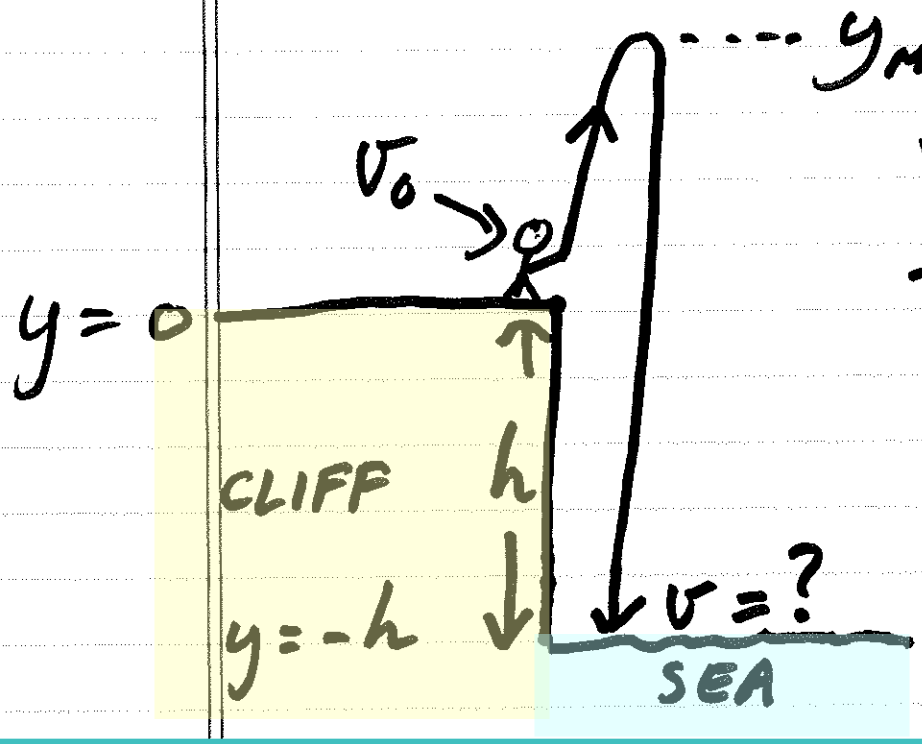
$\boxed{g \equiv \frac{m}{s^2}}$

$$h = \frac{v_0^2}{g} \equiv \frac{\left(\frac{m}{s}\right)^2}{\frac{m}{s^2}} \equiv \frac{m^2}{\cancel{s^2}} \cdot \frac{\cancel{s^2}}{\cancel{m}}$$

$$h \equiv m \checkmark$$

EXAMPLE 3

3) LET'S COMBINE EXAMPLES 1 + 2



WHAT IS v WHEN IT REACHES THE BOTTOM OF THE CLIFF?

THE MOTION SEEMS COMPLICATED:

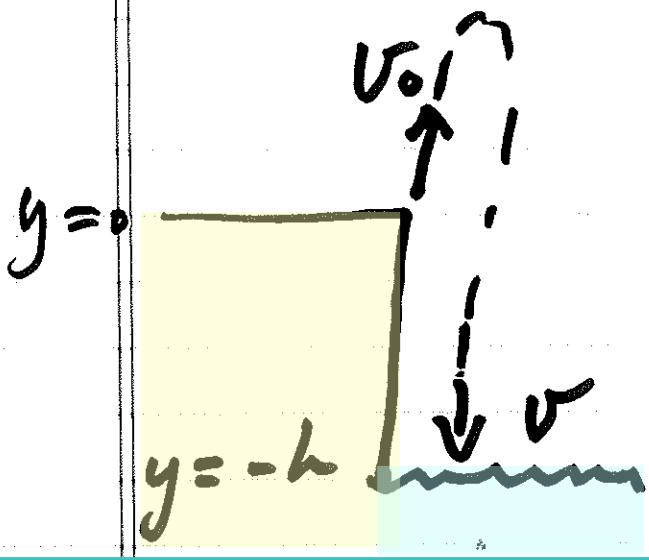
a) GOES UP: $y = 0 \Rightarrow y = y_{MAX}$

b) STOPS

c) COMES DOWN $y = y_{MAX} \Rightarrow y = -h$

IN FACT IT ISN'T!

ONE EQUATION DOES IT ALL IN A SINGLE STEP



WE KNOW

v_0 y $a(=-g)$

NEED v

ONE OF THE FOUR EQUATIONS DOES THE TRICK!

$$v^2 = v_0^2 - 2gy$$
$$= v_0^2 + 2(-g)(-h)$$
$$= v_0^2 + 2gh$$

$$v = -\sqrt{v_0^2 + 2gh}$$

THIS WORKS BECAUSE $a=-g$ ALL THE TIME IT IS AIRBORNE.

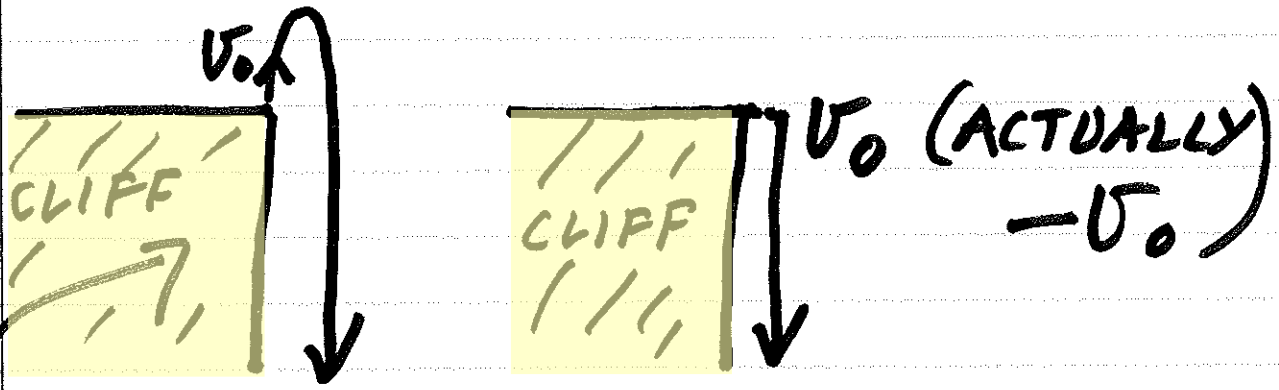
NOTE:

THROWING AN OBJECT UP WITH v_0,

AND:

THROWING AN OBJECT DOWN WITH v_0,

GIVE THE SAME FINAL v!



BECAUSE

$$v^2 = v_0^2 - 2gy$$

$$v^2 = (-v_0)^2 - 2gy$$

ARE THE SAME