MOTION IN A PLANE

PROJECTILE MOTION (Ch 3)

SO FAR 1-D MOTION:

$\longrightarrow x$

v, a, t

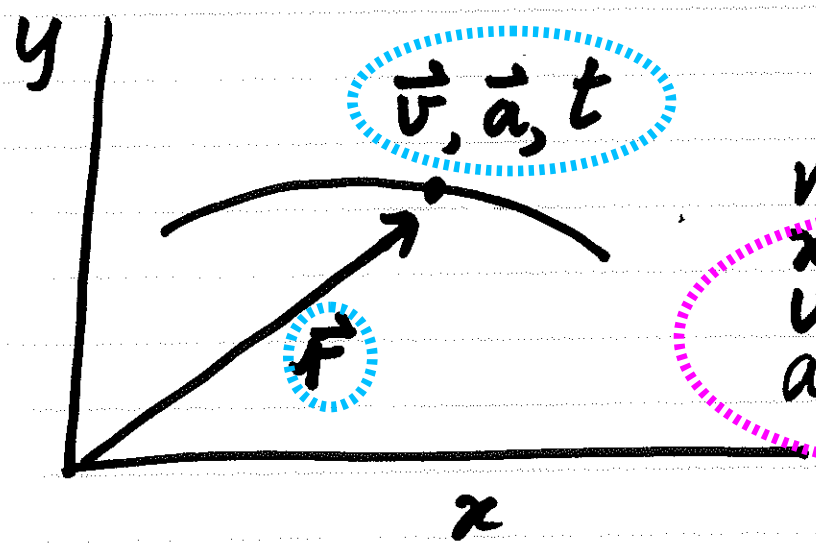
CARS ACCELERATING, ETC.

$y \uparrow$

$v, a = -g, t$

GRAVITY

NOW WE WILL COMBINE THESE: PROJECTILE MOTION



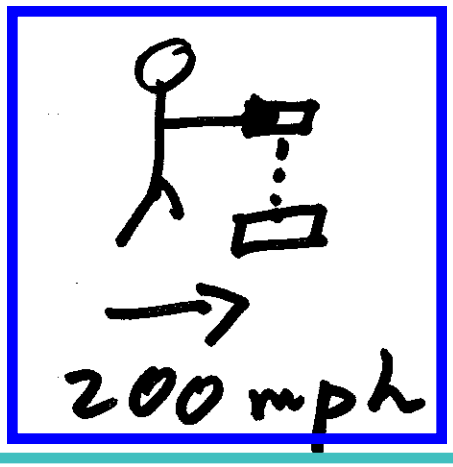
IN FACT WE USE

x, y
v_x, v_y
a_x, a_y, t

CONSIDER THIS:

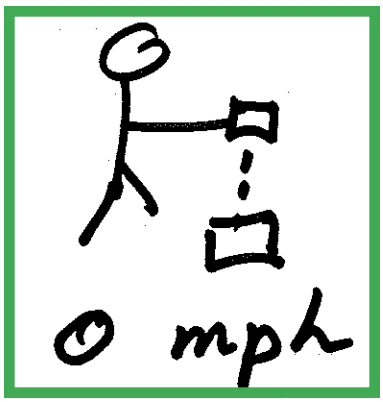
YOU ARE TRAVELING IN A TRAIN AT 200 mph.

1) YOU HOLD OUT A BOOK IN FRONT OF YOU AND LET IT GO.



IT LANDS AT YOUR FEET.

2) THE TRAIN STOPS. YOU DO THE SAME THING.

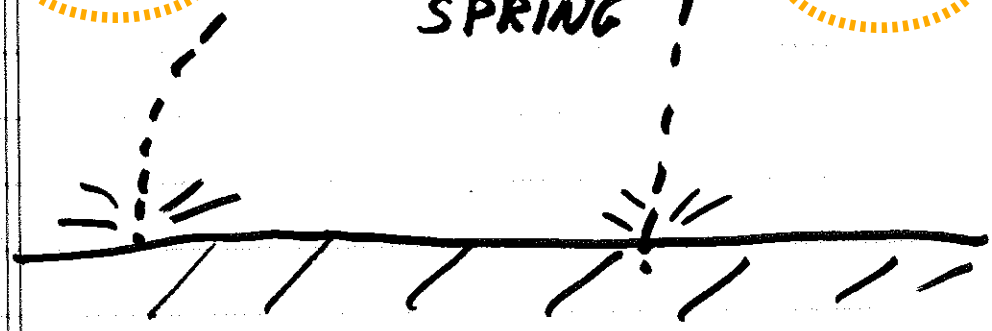


IT LANDS AT YOUR FEET

THE HORIZONTAL MOTION DOES NOT SEEM TO AFFECT THE VERTICAL MOTION!

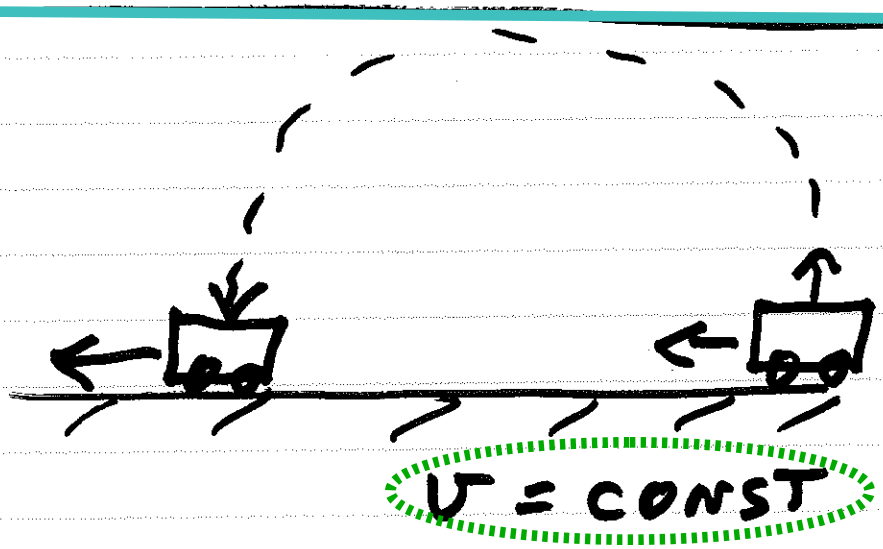
LET'S DO TWO DEMOS:

1) FIRE! v_0 ⇐ SPRING DROP



BOTH LAND AT THE SAME INSTANT

2)



BALL LANDS BACK IN CAR

THE SIMPLEST EXPLANATION THAT FITS THE FACTS

WE CAN SEPARATE OUT VERTICAL AND HORIZONTAL MOTION

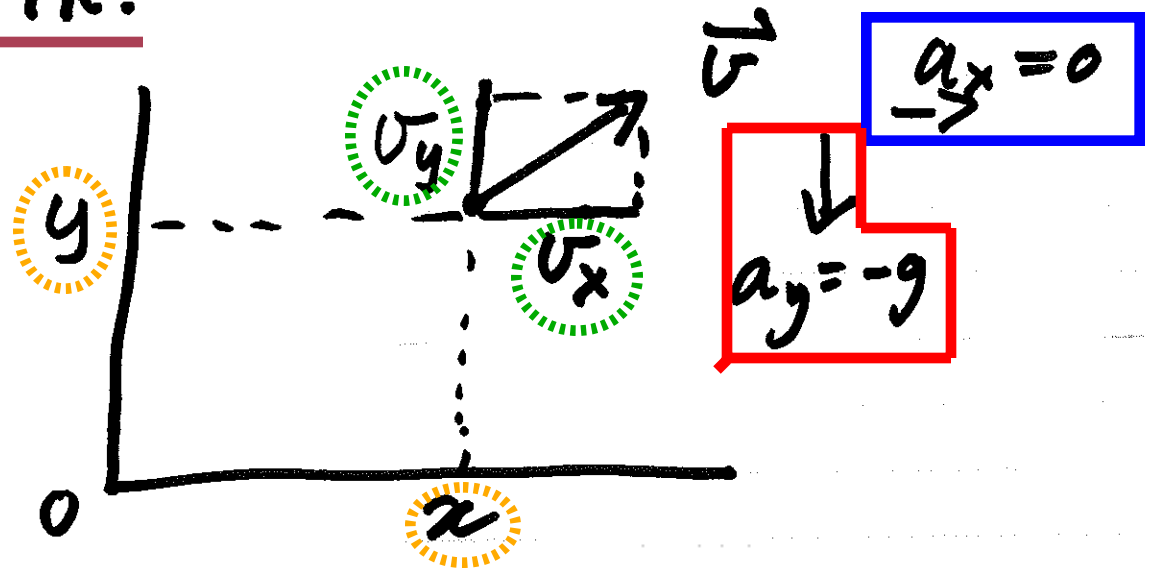
VERTICAL MOTION GIVEN BY GRAVITY

$$a_y = -g$$

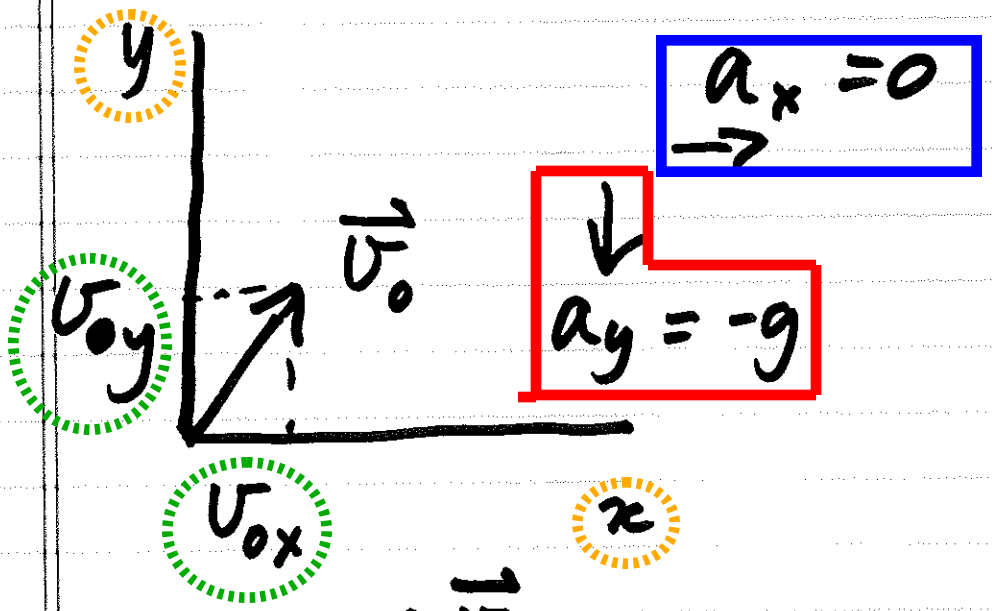
HORIZONTAL MOTION GIVEN BY

$$a_x = 0$$

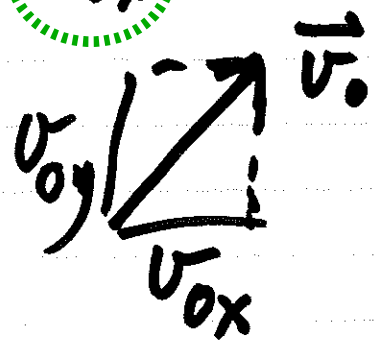
FOR AN OBJECT IN THE AIR:



IF THROWN FROM THE ORIGIN



IE



EQUATIONS OF MOTION

1) HORIZONTAL:

$\boxed{a_x = 0}$

$$v_x = v_{0x} + \cancel{a_x t}$$
$$v_x^2 = v_{0x}^2 + \cancel{2 a_x x}$$
$$x = v_{0x} t + \cancel{\tfrac{1}{2} a_x t^2}$$
$$x = \frac{v_{0x} + v_x}{2} t$$

$\Rightarrow$

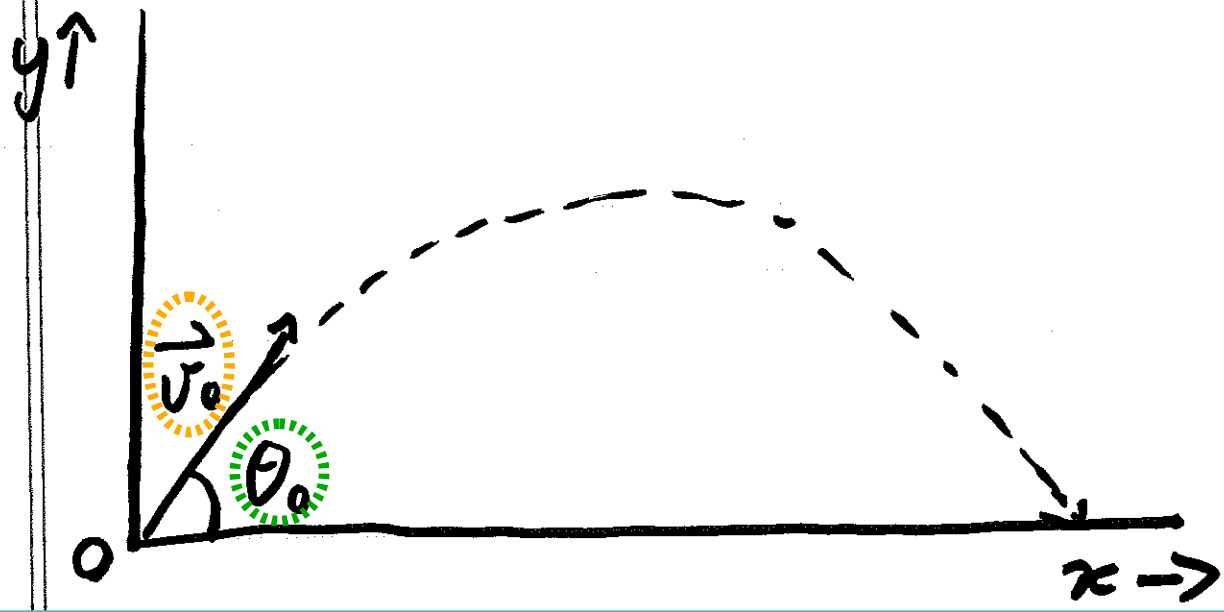
$$\boxed{\begin{aligned} v_x &= v_{0x} \\ x &= v_{0x} t \end{aligned}}$$

2) VERTICAL:

$\boxed{a_y = -g}$

$$v_y = v_{0y} - gt$$
$$v_y^2 = v_{0y}^2 - 2gy$$
$$y = v_{0y} t - \tfrac{1}{2} g t^2$$
$$y = \frac{v_{0y} + v_y}{2} t$$

PROJECTILE MOTION – GROUND TO GROUND



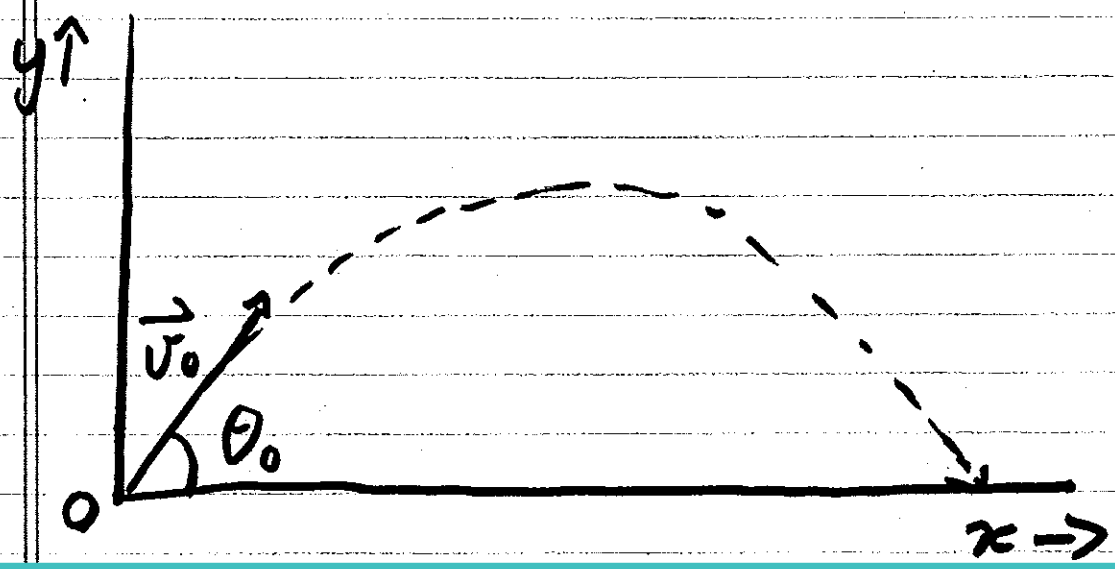
STARTS FROM ORIGIN AT $t=0$
WITH $\vec{v}_0$ AT θ_0

WE WANT $\vec{r}$ AND $\vec{v}$
AT ANY LATER TIME t

I.E. WE WANT x, y
v_x, v_y

AT TIME t

PROJECTILE MOTION – GROUND TO GROUND



INITIAL CONDITIONS:

$$\boxed{\begin{aligned} v_{0x} &= v_0 \cos\theta_0 \\ v_{0y} &= v_0 \sin\theta_0 \end{aligned}}$$

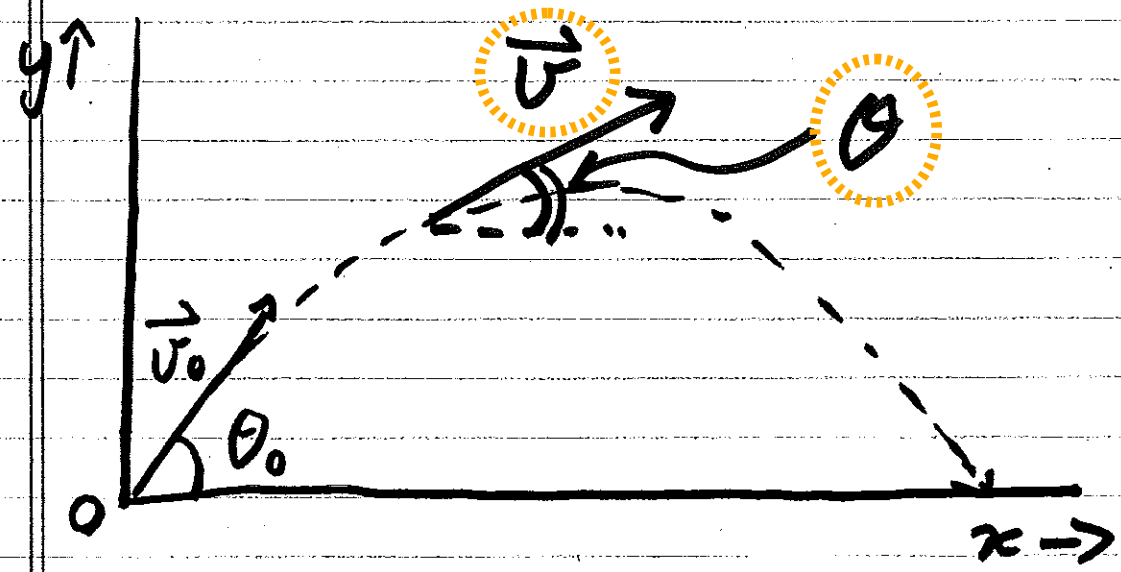
$t = 0$

TO GET $\vec{v}$ AT t WE USE EQUATIONS OF MOTION:

$$\boxed{v_x = v_{0x} \quad (= \text{CONST})}$$

$$\boxed{\begin{aligned} v_y &= v_{0y} + a_y t \\ &= v_0 \sin\theta_0 - g t \end{aligned}}$$

PROJECTILE MOTION – GROUND TO GROUND



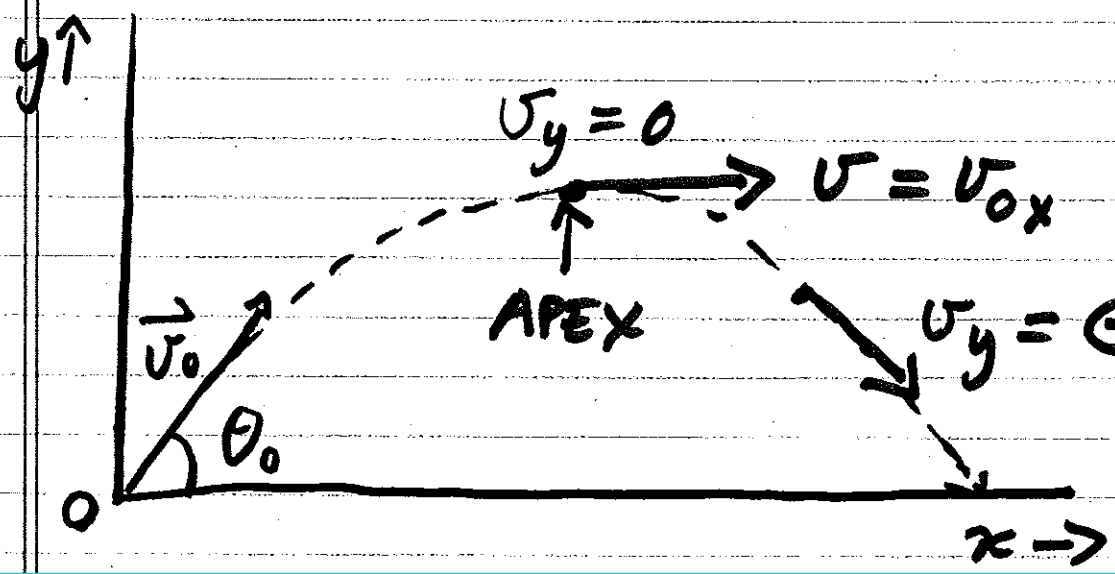
$$\boxed{\begin{aligned} v_x &= v_{0x} = v_0 \cos\theta_0 \\ v_y &= v_0 \sin\theta_0 - gt \end{aligned}}$$

THEN

$$v = \sqrt{v_x^2 + v_y^2} = \text{UGH!}$$

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) \text{ OR } \arctan\left(\frac{v_y}{v_x}\right)$$

PROJECTILE MOTION – GROUND TO GROUND



AT HIGHEST POINT (APEX):

$$\boxed{v_y = 0}$$

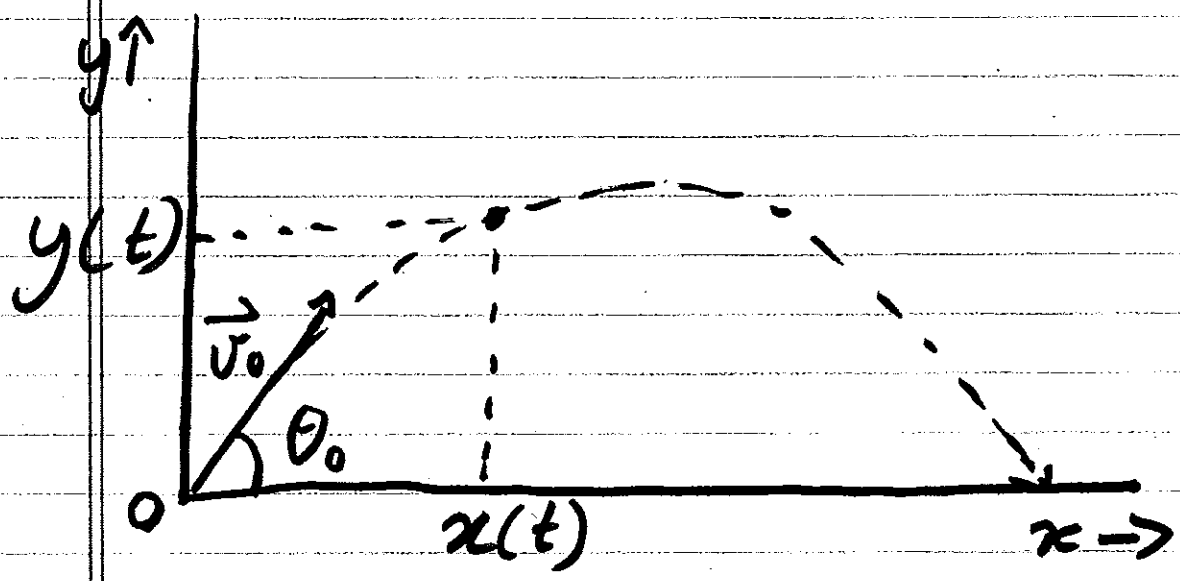
AFTER APEX v_y IS NEGATIVE

NOTICE THAT

$$\boxed{v_x = v_{0x} \text{ FOR ALL } t!}$$

(UNTIL IT HITS THE GROUND!)

PROJECTILE MOTION – GROUND TO GROUND



LETS FIND POSITION AT t:

HORIZONTAL: ($x=0$ AT $t=0$)

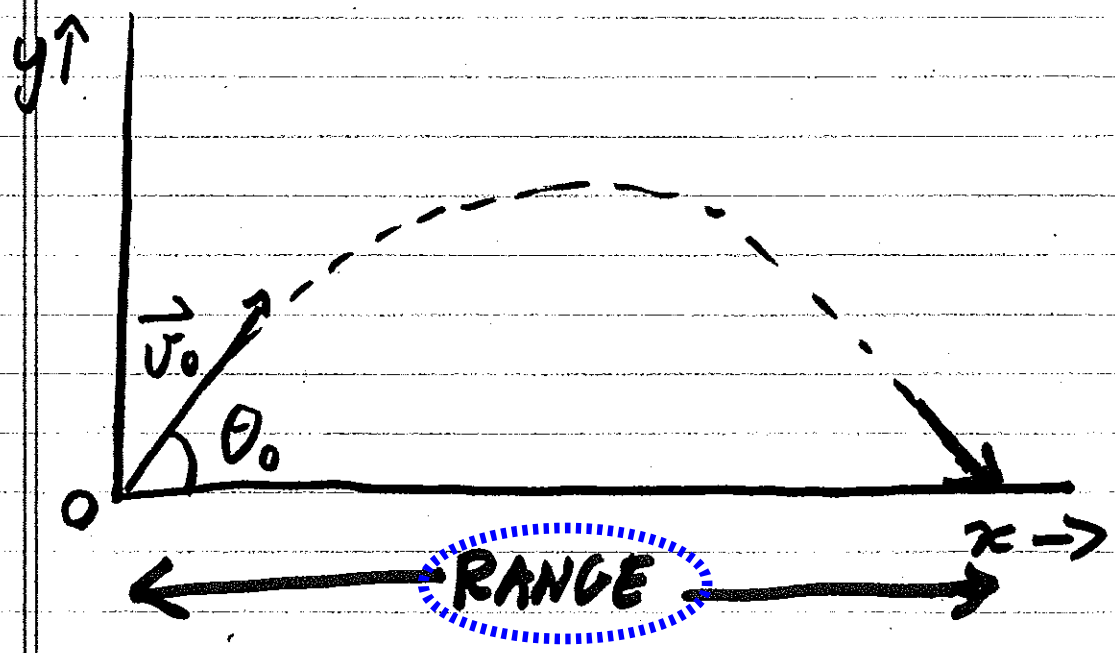
$$x = v_{0x} t = v_0 \cos\theta_0 \, t$$

VERTICAL: ($y=0$ AT $t=0$)

$$y = v_{0y} t - \frac{1}{2} g t^2$$

$$y = (v_0 \sin\theta_0) t - \frac{1}{2} g t^2$$

PROJECTILE MOTION – GROUND TO GROUND



RANGE: HORIZONTAL DISTANCE TRAVELED BEFORE HITTING GROUND

GIVEN BY x WHEN $y=0$!

$$y = 0 = v_0 \sin\theta_0 t - \frac{1}{2} g t^2$$

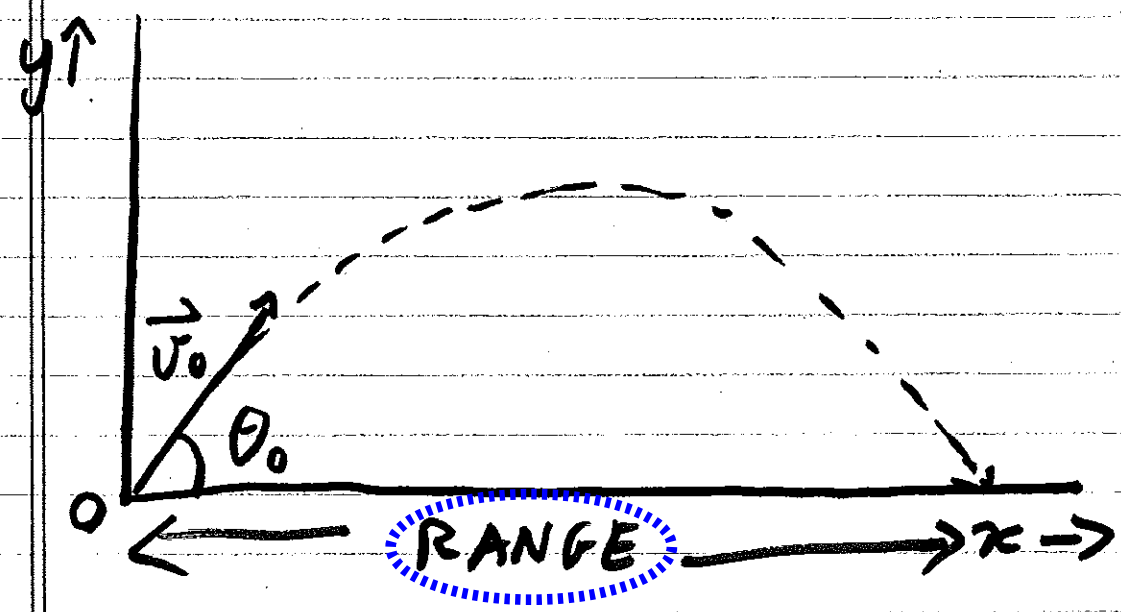
IE: $\boxed{\frac{1}{2} g t^2 = v_0 \sin\theta_0 t}$

SO $y=0$ AT $t=0$ (WE KNEW THAT!)

OR $\frac{1}{2} g t = v_0 \sin\theta_0$

$$t = \frac{2 v_0 \sin\theta_0}{g} \quad \text{FOR } y=0$$

PROJECTILE MOTION – GROUND TO GROUND



RANGE GIVEN BY:

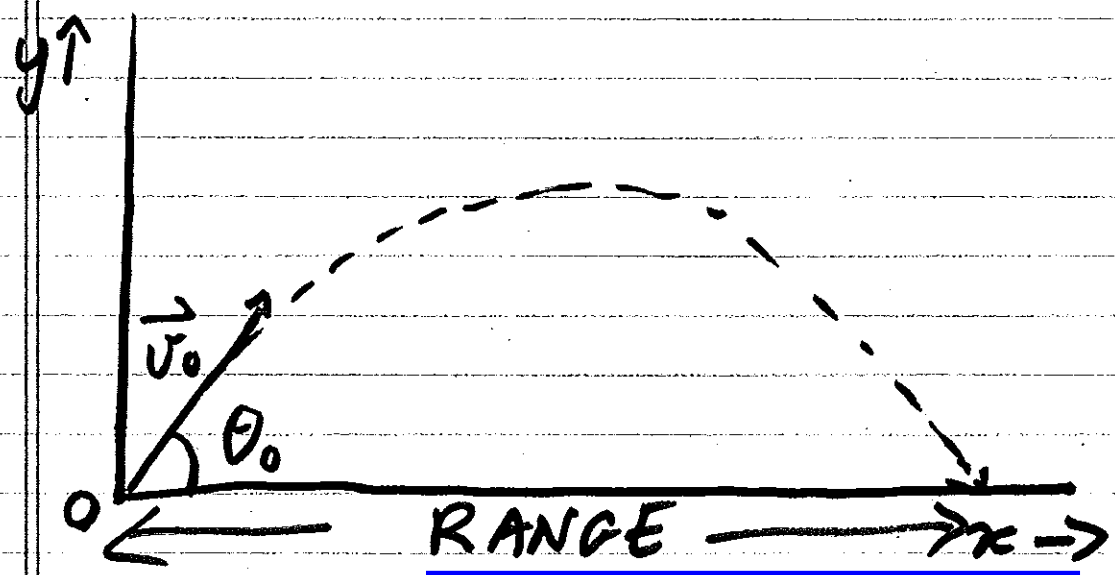
x AT $t = \dfrac{2 v_0 \sin\theta_0}{g}$

$x = v_0 \cos\theta_0 \, t$ (2 SLIDES AGO!)

$$= v_0 \cos\theta_0 \times \frac{2 v_0 \sin\theta_0}{g}$$

$$x = \frac{2 \sin\theta_0 \cos\theta_0 \, v_0^2}{g}$$

PROJECTILE MOTION – GROUND TO GROUND

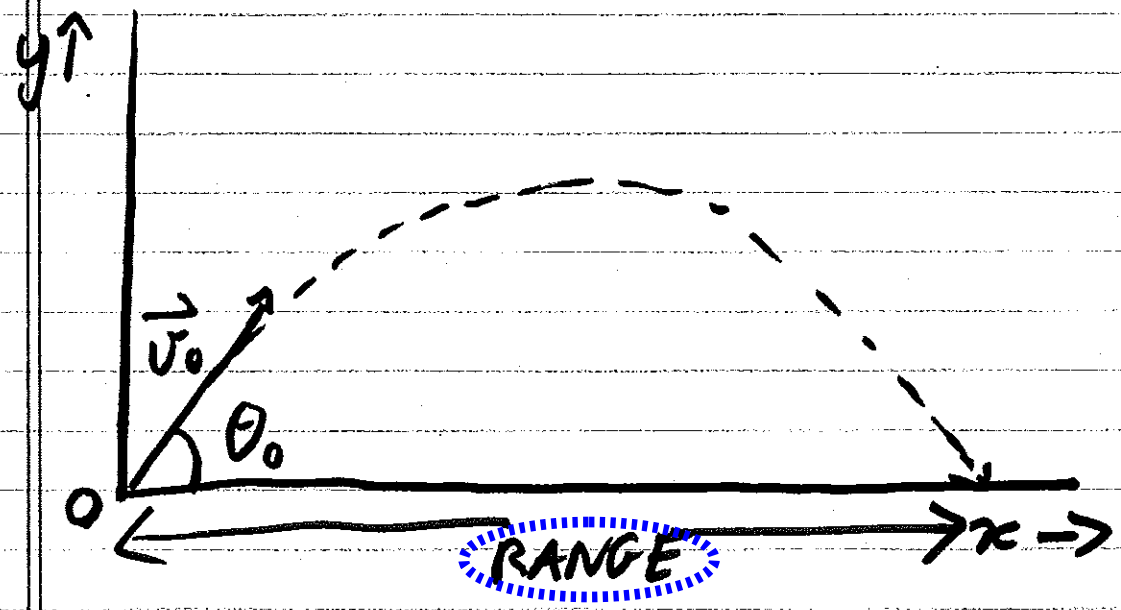


RANGE: $\boxed{\dfrac{2\sin\theta_0\cos\theta_0\, v_0^2}{g}}$

QUESTION: IF I CAN THROW A BALL WITH SOME MAXIMUM VALUE OF v_0, IS THERE AN ANGLE θ_0 FOR WHICH THE RANGE IS THE GREATEST?

$\boxed{\text{YES!}}$ WE CAN FIND IT!

PROJECTILE MOTION – GROUND TO GROUND



BEFORE WE FIND THE BEST θ_0, HOW DO I KNOW IT EXISTS?

CONSIDER $\theta_0 = 90°$: STRAIGHT UP

$\Downarrow$ RANGE = 0!

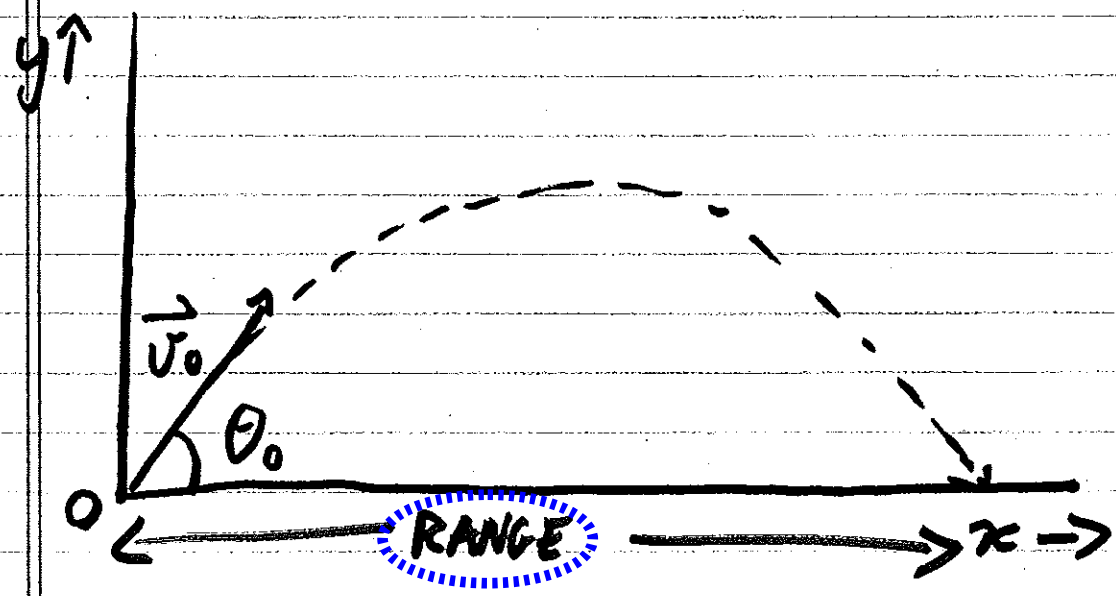
CONSIDER $\theta_0 = 0$: HITS GROUND INSTANTLY (IT NEVER LEAVES THE GROUND

$\rightarrow$ RANGE = 0!

FOR θ_0 IN BETWEEN RANGE $\neq 0$

$\therefore$ THERE MUST BE A MAXIMUM!

PROJECTILE MOTION – GROUND TO GROUND



WE WANT θ_0 WHICH MAXIMIZES

$$\frac{2 \sin\theta_0 \cos\theta_0 \, v_0^2}{g}$$

IE WHICH MAXIMIZES

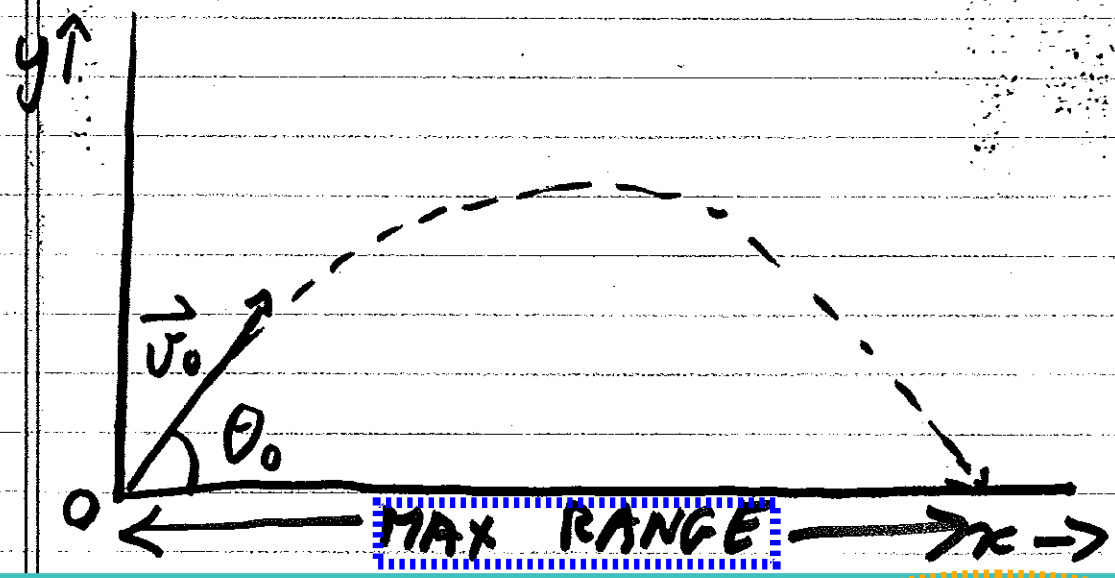
TRIG!! $2 \sin\theta_0 \cos\theta_0 = \sin 2\theta_0$

MORE TRIG! MAX VALUE OF $\sin\phi$ IS $\underline{1}$ WHEN $\phi = 90°$

$\therefore$ MAX RANGE WHEN $2\theta_0 = 90°$

OR MAX RANGE WHEN $\theta_0 = 45°$

PROJECTILE MOTION – GROUND TO GROUND



MAX RANGE WHEN $\theta_0 = 45^\circ$

(FOR FIXED v_0!)

VALUE OF MAX RANGE

$$\boxed{R = 2\sin\theta_0\cos\theta_0\,\frac{v_0^2}{g}}$$

$$= \underbrace{2\sin 45^\circ\cos 45^\circ}_{=1}\,\frac{v_0^2}{g}$$

$$R = \frac{v_0^2}{g}$$

WE IGNORED AIR RESISTANCE!

(TOO DIFFICULT)