PHYSICS 211——PRACTICE EXAM 3

NAME (printed) ____________________

SIGNATURE ____________________

Student Number (SSN) ____________________

SECTION ____________________

INSTRUCTIONS

1) Wait for oral instructions before starting the test.

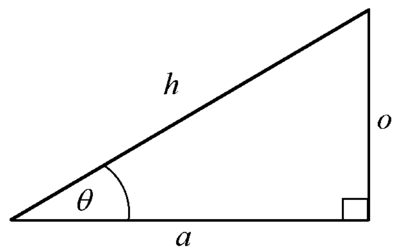
2) Remember to justify (in English) as many steps as possible for partial credit.

3) **No calculators or other aids permitted.**

For the graders:

1. ________
2. ________
3. ________
4. ________

TOTAL ________



$$\sin\theta = o/h$$
$$\cos\theta = a/h$$
$$\tan\theta = o/a$$

$$\sin(90^\circ - \theta) = \cos\theta \qquad \cos(90^\circ - \theta) = \sin\theta$$

$$\begin{aligned} \sin 0 &= 0 & \cos 0 &= 1 \\ \sin 90^\circ &= 1 & \cos 90^\circ &= 0 \\ \sin 30^\circ &= 1/2 & \cos 60^\circ &= 1/2 \end{aligned}$$

$$2\sin\theta\cos\theta = \sin 2\theta$$

1. **Circular motion I**

a) i) A circle has radius r. If the arc length of a certain segment of the circle is $2r$, how many radians is the segment angle? *[2 points]*

ii) How many radians is one complete revolution? *[2 points]*

b) Write down the rotational equivalent of the linear-motion displacent equation $v = v_0 + at$. *[4 points]*

c) Write down the rotational equivalent of the linear-motion displacent equation $x = v_0 t + \frac{1}{2}at^2$. *[4 points]*

1. d) A disc, starting from rest, undergoes a constant angular acceleration for a time t_1 and reaches an angular velocity of ω_1. It then undergoes a constant negative acceleration and comes to rest in a time t_2. Find an expression, in terms of the quantities t_1, t_2, and ω_1 for the total number of revolutions of the disc in the time $t_1 + t_2$. *[13 points]*

2. **Circular motion II and Gravity**

a) An object of mass m is travelling at constant speed v in a circle of radius r. Write down an expression for the magnitude of the force required for this circular motion. *[4 points]*

b) Write down the force F due to the gravitational attraction between two point masses m_1 and m_2 separated by a distance r (use the symbol G for the universal constant). *[4 points]*

c) The answer to (b) is still valid if the point masses are replaced by spheres. From where to where is r measured in this case? *[4 points]*

2. d) A non-rotating spherical planet has radius R and uniform density ρ (mass per unit volume). A small rock is orbiting the planet just above the surface (ie at R; there is no atmosphere to slow it down!). Find an expression for the orbital period T of the rock. Express your answer in terms of ρ, and G. (Your answer will not contain R). *[13 points]* [*HINTS*: Volume of a sphere of radius r is $\frac{4}{3}\pi r^3$; Mass = volume $\times$ density]

3. **Equilibrium**.

a) Write down the torque τ about an axis A, if a force F acts at a point B a distance d from the axis. The force acts in a direction *perpendicular* to the line AB. *[4 points]*

a) Write down the torque τ about an axis A, if a force F acts at a point B a distance L from the axis. The force acts in a direction θ with respect to the line AB. *[4 points]*

c) Write down (in mathematical notation) the two conditions for equilibrium. *[2 points each]*

3. d) A *non-uniform* rod, of mass M and length ℓ, has its center of gravity located a distance a from the left end. Find expressions for the forces F_L and F_R that need to be applied at the left and right hand ends of the rod in order to keep the rod horizontal and above the ground. *[13 points]*

4. **Rotational Dynamics**.

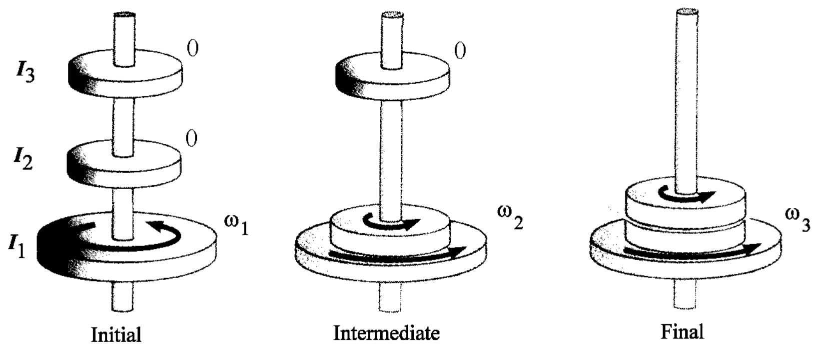
a) Write down the form of Newton's second law of motion applicable to rotational motion (in the form $\tau = \cdots$), and define each symbol in words. *[4 points]*

b) A massless rod of length $L = 6$m has a mass $m = 10$kg fixed at one end and an equal mass $m = 10$kg fixed at the other end. What is the moment of inertia about an axis perpendicular to the rod and through its center? *[4 points]*

c) A massless rod of length $L = 6$m has a mass $m = 10$kg fixed at one end and an equal mass $m = 10$kg fixed at the other end. What is the moment of inertia about an axis perpendicular to the rod and through one of its ends? *[4 points]*

4. d) **Rotational Dynamics**.

A cylinder with moment of inertia $I_1 = 1$ kg.m^2 rotates with angular velocity $\omega_1 = 6$ rad/s about a frictionless vertical axle. See "initial" diagram below. A second cylinder, with moment of inertia $I_2 = 2$ kg.m^2, initially not rotating, drops onto the first cylinder. Since the surfaces are rough, the two eventually reach the same angular velocity ω_2, as shown in "intermediate" below. Next, a third cylinder, with moment of inertia $I_3 = 3$ kg.m^2, initially not rotating, drops onto the first two to produce the "final" result shown below, with all three rotating at the same angular velocity ω_3.



(i) What is the moment of inertia of the "final" system? *[3 points]*

(ii) What are the "intermediate" and "final" angular velocities, ω_2 and ω_3? *[5 points]*

(iii) What is the ratio of the total initial to the total final kinetic energy? *[5 points]*