PHYSICS 211——PRACTICE EXAM 3

NAME (printed) SOLUTIONS

SIGNATURE ________________

Student Number (SSN) ________________

SECTION ________________

INSTRUCTIONS

1) Wait for oral instructions before starting the test.

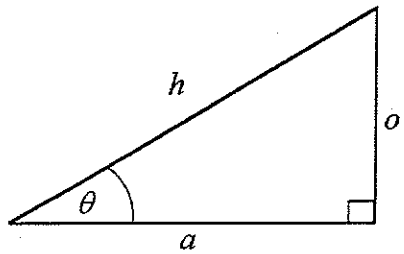
2) Remember to justify (in English) as many steps as possible for partial credit.

3) **No calculators or other aids permitted.**

For the graders:

1. ________
2. ________
3. ________
4. ________

TOTAL ________



$$\sin\theta = o/h$$
$$\cos\theta = a/h$$
$$\tan\theta = o/a$$

$$\sin(90^\circ - \theta) = \cos\theta \qquad \cos(90^\circ - \theta) = \sin\theta$$

$$\begin{aligned} \sin 0 &= 0 & \cos 0 &= 1 \\ \sin 90^\circ &= 1 & \cos 90^\circ &= 0 \\ \sin 30^\circ &= 1/2 & \cos 60^\circ &= 1/2 \end{aligned}$$

$$2\sin\theta\cos\theta = \sin 2\theta$$

1. **Circular motion I**

a) i) A circle has radius r. If the arc length of a certain segment of the circle is $2r$, how many radians is the segment angle? *[2 points]*

2

2

ii) How many radians is one complete revolution? *[2 points]*

2

2π

b) Write down the rotational equivalent of the linear-motion displacent equation $v = v_0 + at$. *[4 points]*

$\omega = \omega_0 + \alpha t$

4

c) Write down the rotational equivalent of the linear-motion displacent equation $x = v_0 t + \frac{1}{2}at^2$. *[4 points]*

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

4

1. d) A disc, starting from rest, undergoes a constant angular acceleration for a time t_1 and reaches an angular velocity of ω_1. It then undergoes a constant negative acceleration and comes to rest in a time t_2. Find an expression, in terms of the quantities t_1, t_2, and ω_1 for the total number of revolutions of the disc in the time $t_1 + t_2$. *[13 points]*

We need to find total angular displacement θ and divide by 2π to find number of revolutions..

First part :-

$$\omega_1 = 0 + \alpha_1 t_1 \quad \therefore \alpha_1 = \omega_1 / t_1$$

$$\theta_1 = 0t_1 + \tfrac{1}{2}\alpha_1 t_1^2 = \tfrac{1}{2}\frac{\omega_1}{t_1} t_1^2 = \tfrac{1}{2}\omega_1 t_1$$

$$\left[\text{Or } \theta_1 = \omega_{AVERAGE} t_1 = \frac{0+\omega_1}{2} t_1 = \tfrac{1}{2}\omega_1 t_1\right]$$

Second part:

$$0 = \omega_1 + \alpha_2 t_2 \quad \therefore \alpha_2 = -\omega_1 / t_2$$

$$\theta_2 = \omega_1 t_2 + \tfrac{1}{2}\alpha_2 t_2^2 = \omega_1 t_2 - \tfrac{1}{2}\frac{\omega_1}{t_2} t_2^2 = \tfrac{1}{2}\omega_1 t_2$$

$$\left[\text{or } \theta_2 = \omega_{AVERAGE} t_2 = \frac{\omega_1 + 0}{2} t_2 = \tfrac{1}{2}\omega_1 t_2\right]$$

$$\therefore \theta = \theta_1 + \theta_2 = \tfrac{1}{2}\omega_1 (t_1 + t_2)$$

$\therefore$ Number of revolutions $= \dfrac{\omega_1}{4\pi}(t_1 + t_2)$

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2. **Circular motion II and Gravity**

a) An object of mass m is travelling at constant speed v in a circle of radius r. Write down an expression for the magnitude of the force required for this circular motion. *[4 points]*

$$F = \frac{mv^2}{r}$$

4

b) Write down the force F due to the gravitational attraction between two point masses m_1 and m_2 separated by a distance r (use the symbol G for the universal constant). *[4 points]*

$$F = G\frac{m_1 m_2}{r^2}$$

4

c) The answer to (b) is still valid if the point masses are replaced by spheres. From where to where is r measured in this case? *[4 points]*

Between their centers

4

2. d) A non-rotating spherical planet has radius R and uniform density ρ (mass per unit volume). A small rock is orbiting the planet just above the surface (ie at R; there is no atmosphere to slow it down!). Find an expression for the orbital period T of the rock. Express your answer in terms of ρ, and G. (Your answer will not contain R). *[13 points]* [*HINTS*: Volume of a sphere of radius r is $\frac{4}{3}\pi r^3$; Mass = volume × density]

Let mass of rock = m; let mass of planet = M

Gravitational attraction on rock = $F = G\frac{mM}{R^2}$

This provides centripetal force $\frac{mv^2}{R}$

$$\therefore G\frac{\cancel{m}M}{R^{\cancel{2}}} = \frac{\cancel{m}v^2}{\cancel{R}} \quad \text{or} \quad G\frac{M}{R} = v^2 \quad (1)$$

1 Orbit = $2\pi R$. Time for one orbit is $T = \frac{2\pi R}{v}$

$$\text{Thus } T^2 = \frac{(2\pi)^2 R^2}{v^2} = \frac{(2\pi)^2 R^3}{GM} \text{ from } (1).$$

Using "HINTS:" $M = \frac{4}{3}\pi R^3 \times \rho$

$$\therefore T^2 = \frac{(2\pi)^2 \cancel{R^3}}{\frac{4}{3}\pi \cancel{R^3}\rho G} = \frac{3\pi}{G\rho}$$

$$\therefore T = \sqrt{\frac{3\pi}{G\rho}}$$

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3. **Equilibrium.**

a) Write down the torque τ about an axis A, if a force F acts at a point B a distance d from the axis. The force acts in a direction *perpendicular* to the line AB. *[4 points]*

$$\tau = Fd$$

4

a) Write down the torque τ about an axis A, if a force F acts at a point B a distance L from the axis. The force acts in a direction θ with respect to the line AB. *[4 points]*

$$\tau = FL\sin\theta$$

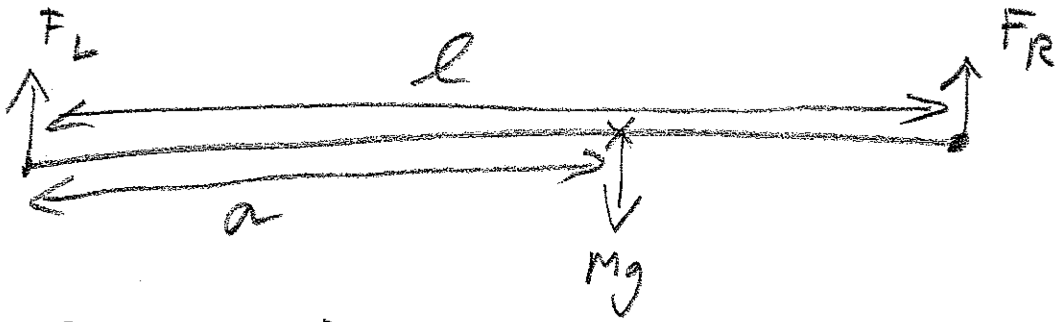
4

c) Write down (in mathematical notation) the two conditions for equilibrium. *[2 points each]*

$$\sum F = 0$$
$$\sum \tau = 0$$

4

3. d) A *non-uniform* rod, of mass M and length ℓ, has its center of gravity located a distance a from the left end. Find expressions for the forces F_L and F_R that need to be applied at the left and right hand ends of the rod in order to keep the rod horizontal and above the ground. *[13 points]*



$\Sigma F = 0$ gives $F_L + F_R = Mg$

Could take $\Sigma \tau = 0$ about (1) Left end, (2) C of G, or (3) Right end. Which is best?

In fact Left end is easiest:–

$\Sigma \tau = 0$: $Mg\,a = F_R\,\ell$

$$\boxed{F_R = \frac{Mg\,a}{\ell}}$$

Then $F_L = Mg - F_R =$

gives

$$\boxed{F_L = Mg\left(1 - \frac{a}{\ell}\right)}$$

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4. Rotational Dynamics.

a) Write down the form of Newton's second law of motion applicable to rotational motion (in the form $\tau = \cdots$), and define each symbol in words. *[4 points]*

$$\tau = I\alpha$$

τ = Torque I = Moment of Inertia

α = angular acceleration.

4

b) A massless rod of length $L = 6\text{m}$ has a mass $m = 10\text{kg}$ fixed at one end and an equal mass $m = 10\text{kg}$ fixed at the other end. What is the moment of inertia about an axis perpendicular to the rod and through its center? *[4 points]*

$$I = \sum mr^2 = 2 \times 10 \times 3^2 = 20 \times 9 = \underline{180\ \text{kg m}^2}$$

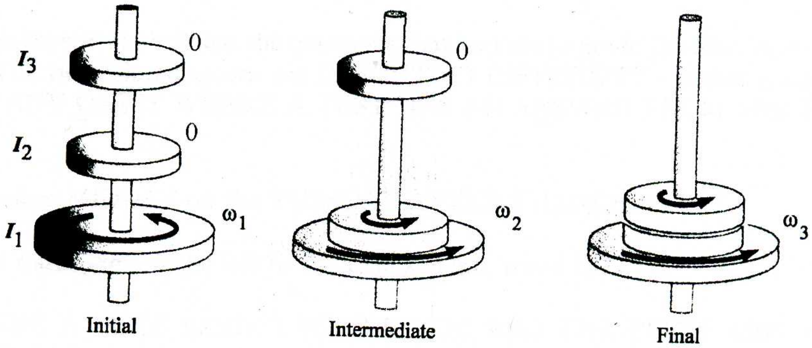
4

c) A massless rod of length $L = 6\text{m}$ has a mass $m = 10\text{kg}$ fixed at one end and an equal mass $m = 10\text{kg}$ fixed at the other end. What is the moment of inertia about an axis perpendicular to the rod and through one of its ends? *[4 points]*

$$I = 10 \times 6^2 = \underline{360\ \text{kg m}^2}$$

4. d) **Rotational Dynamics.**

A cylinder with moment of inertia $I_1 = 1$ kg.m^2 rotates with angular velocity $\omega_1 = 6$ rad/s about a frictionless vertical axle. See "initial" diagram below. A second cylinder, with moment of inertia $I_2 = 2$ kg.m^2, initially not rotating, drops onto the first cylinder. Since the surfaces are rough, the two eventually reach the same angular velocity ω_2, as shown in "intermediate" below. Next, a third cylinder, with moment of inertia $I_3 = 3$ kg.m^2, initially not rotating, drops onto the first two to produce the "final" result shown below, with all three rotating at the same angular velocity ω_3.



(i) What is the moment of inertia of the "final" system? *[3 points]*

$I = I_1 + I_2 + I_3 = 1 + 2 + 3 = 6 \text{ kg m}^2$ 3

(ii) What are the "intermediate" and "final" angular velocities, ω_2 and ω_3? *[5 points]*

$I_1 \omega_1 = (I_1 + I_2)\omega_2$ so $\boxed{\omega_2} = \frac{I_1}{I_1+I_2}\omega_1 = \frac{1}{1+2} 6 = \boxed{2 \text{ rad/s}}$

$\omega_3 = \frac{I_1}{I_1 + I_2 + I_3} \omega_1 = \boxed{1 \text{ rad/s}}$

(iii) What is the ratio of the total initial to the total final kinetic energy? *[5 points]*

$KE_i = \frac{1}{2} I \omega_1^2 = \frac{1}{2} \times 1 \times 6^2 = 18 J$

$KE_f = \frac{1}{2}(I_1 + I_2 + I_3)\omega_3^2 = \frac{1}{2} 6 \times 1^2 = 3 J$

$\frac{KE_i}{KE_f} = 6 \quad \left(\text{or } \frac{KE_f}{KE_i} = \frac{1}{6}\right)$