Next generalisation:

\[
\frac{\partial z}{\partial \omega} = \mathbf{z}'(\omega) = A (\omega - a_1)^{q_1} (\omega - a_2)^{q_2} ... (\omega - a_n)^{q_n}
\]

\[A \neq 0, \text{ complex constant}; \quad a_i \text{ real}; \quad -1 < q_i < 1\]
\[a_1 < a_2 < ... < a_n\]

\[\Rightarrow \text{arg} \mathbf{z}'(\omega) = \text{arg} A + q_1 \text{arg} (\omega - a_1) + q_2 \text{arg} (\omega - a_2) + ... + q_n \text{arg} (\omega - a_n)\]

\[\text{Angle of mapping in } \mathbb{Z} \text{-plane:}\]
\[\text{angle on } w_i \quad \text{Angle of mapping in } \mathbb{Z} \text{-plane:}\]
\[= \text{arg} A + q_1 \pi + q_2 \pi + \ldots + q_n \pi\]

Now, since, by construction, \(\mathbf{z}(\omega)\) satisfies \(\mathbf{z}'(\omega) = A (\omega - a_1)^{q_1} ... (\omega - a_n)^{q_n}\)

\[\Rightarrow \mathbf{z}(\omega) = A \int (\omega - a_1)^{q_1} ... (\omega - a_n)^{q_n} d\omega + B \left[ \frac{\theta_i}{\pi} \right] \]

Function of this form known as Schwarz Transformations, maps: polygon in \(\mathbb{Z}\)-plane \(\Leftrightarrow\) real axis in \(\omega\)-plane.
Example: Derive a Schwarz transformation which maps the semi-infinite strip in the $z$-plane

\[ |\text{Re } z| = 1x| < \frac{1}{2} \]

\[ \text{Im } z = \frac{y}{2} > 0 \]

into the upper half-plane in the $w$-plane.

\[ \zeta \xrightarrow{\text{Schwarz}} \omega \]

Choose orientation indicated by arrows:

\[ \text{Need } "\text{left turn" } \text{at } z = 1 + i \Rightarrow \xi_1 = \xi_2 = -\frac{\pi}{\pi} = -\frac{1}{2} \]

- choose $a_1 = -1, a_2 = +1 \Rightarrow \text{not } "\text{right turn" } a_1 = -1, a_2 = +1$

\[ \Rightarrow z(\omega) = A(\omega + 1)^{-\frac{1}{2}}(\omega - 1)^{-\frac{1}{2}} \]

\[ \Rightarrow z(\omega) = \int_0^I d\omega A(\omega + 1)^{-\frac{1}{2}}(\omega - 1)^{-\frac{1}{2}} + B \]

\[ = A \int_0^1 \frac{1}{\sqrt{\omega^2 - 1}} d\omega + B = A \int_0^1 \frac{1}{\sqrt{\omega^2 - 1}} d\omega + B \]

\[ = \frac{A}{\xi} \sin^{-1} \omega + B \]

Require:

\[ \begin{cases} Z(a_1) = Z(-1) = \frac{A}{\xi} \sin^{-1}(-1) + B = Z_1 = -1 \\ Z(a_2) = Z(1) = \frac{A}{\xi} \sin^{-1}(1) + B = Z_2 = +1 \end{cases} \]

\[ -\xi A \sin^{-1}(-1) + B = -1 \]

\[ -\xi A \sin^{-1}(1) + B = 1 \]

\[ \Rightarrow B = 0 \]

\[ -\xi A \sin^{-1}(1) = -\xi \frac{\pi}{2} = +1 \Rightarrow A = \frac{2\xi}{\pi} \]

\[ \Rightarrow Z(\omega) = \frac{2\xi}{\pi} \sin^{-1} \omega \]

\[ \omega = \sin \left( \frac{\pi}{2} \zeta \right) \]