# DEEP INELASTIC ELECTRON SCATTERING

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## CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I.</td>
<td>Introduction</td>
<td>204</td>
</tr>
<tr>
<td>II.</td>
<td>Experimental Method</td>
<td>205</td>
</tr>
<tr>
<td></td>
<td>Equipment</td>
<td>205</td>
</tr>
<tr>
<td></td>
<td>Radiative corrections</td>
<td>207</td>
</tr>
<tr>
<td>III.</td>
<td>Proton and Deuterium Cross Sections</td>
<td>209</td>
</tr>
<tr>
<td>IV.</td>
<td>Kinematics and Variables</td>
<td>213</td>
</tr>
<tr>
<td>V.</td>
<td>Scale Invariance and Scaling Variables</td>
<td>218</td>
</tr>
<tr>
<td>VI.</td>
<td>Separation of $\sigma_\pi$ and $\sigma_1$ for the Proton and Deuteron</td>
<td>219</td>
</tr>
<tr>
<td>VII.</td>
<td>Extraction of Neutron Structure Functions</td>
<td>222</td>
</tr>
<tr>
<td>VIII.</td>
<td>Validity of Scale Invariance</td>
<td>225</td>
</tr>
<tr>
<td></td>
<td>Proton</td>
<td>225</td>
</tr>
<tr>
<td></td>
<td>Deuteron</td>
<td>229</td>
</tr>
<tr>
<td></td>
<td>Neutron</td>
<td>231</td>
</tr>
<tr>
<td>IX.</td>
<td>Comparison of Electron and Muon Scattering Results</td>
<td>235</td>
</tr>
<tr>
<td>X.</td>
<td>Comparison with Theoretical Models</td>
<td>235</td>
</tr>
<tr>
<td></td>
<td>The parton model</td>
<td>235</td>
</tr>
<tr>
<td></td>
<td>Sum rules</td>
<td>238</td>
</tr>
<tr>
<td></td>
<td>Vector dominance models</td>
<td>240</td>
</tr>
<tr>
<td></td>
<td>Regge models</td>
<td>242</td>
</tr>
<tr>
<td></td>
<td>Duality</td>
<td>244</td>
</tr>
<tr>
<td></td>
<td>Resonance models</td>
<td>246</td>
</tr>
<tr>
<td></td>
<td>Models that predict deviations from scaling</td>
<td>246</td>
</tr>
<tr>
<td>XI.</td>
<td>Related Physics</td>
<td>247</td>
</tr>
<tr>
<td></td>
<td>Neutrino-hadron scattering</td>
<td>247</td>
</tr>
<tr>
<td></td>
<td>Electron-positron colliding beams</td>
<td>248</td>
</tr>
<tr>
<td></td>
<td>Inelastic Compton scattering</td>
<td>248</td>
</tr>
<tr>
<td></td>
<td>Polarization effects</td>
<td>248</td>
</tr>
<tr>
<td></td>
<td>Lepton pairs from hadronic interactions</td>
<td>249</td>
</tr>
<tr>
<td></td>
<td>Electromagnetic mass differences</td>
<td>249</td>
</tr>
<tr>
<td></td>
<td>Relationships between inelastic e-p and p-p scattering</td>
<td>249</td>
</tr>
<tr>
<td></td>
<td>Photoproduction</td>
<td>250</td>
</tr>
<tr>
<td>XII.</td>
<td>Conclusions</td>
<td>250</td>
</tr>
</tbody>
</table>

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I. INTRODUCTION

Electron scattering has been a powerful tool in the study of nucleon and nuclear structure. Because the interaction of the electron is well understood in terms of quantum-electro-dynamics (1), the scattering of electrons can be used to probe unknown structure of hadronic systems. In this review we will concern ourselves exclusively with electron-nucleon scattering. Historically, elastic scattering produced the first extensive body of information about the structure of the nucleon, starting with the work of Hofstadter & collaborators (2). Since then substantial additional programmatic work on elastic and inelastic scattering has been carried out at various accelerators. The inelastic studies concentrated mainly on measurements of electroproduction of nucleon resonances, a topic initially investigated by Panofsky & Allton (3). With the advent of the two-mile electron accelerator at the Stanford Linear Accelerator Center (SLAC) (4), with its higher energies and larger intensities, a new region of inelastic scattering—commonly referred to as the “deep inelastic” region—could be extensively investigated. This region corresponds to the excitation of the continuum well beyond the nucleon resonances.

Studies of elastic electron-proton scattering, electroproduction of resonances, and deep inelastic scattering from the proton and neutron have been carried out by an MIT-SLAC collaboration since 1967, using electrons of energies up to 21 GeV from the SLAC accelerator. Initial measurements of deep inelastic electron-proton scattering (5) showed that the continuum cross sections were larger than had been previously expected.

There were theoretical conjectures, prior to the first experimental measurements, that suggested that the inelastic spectra might decrease rapidly with increasing four-momentum transfer squared, $q^2$, perhaps as rapidly as elastic electron-proton scattering. The first measurements established that the inelastic cross sections had only a weak dependence on $q^2$ in addition to the $q^{-4}$ dependence arising from the photon propagator. The measurements established, in addition, that the scattering could be characterized in a particularly simple manner, now known as “scaling” or scale invariance.

The MIT-SLAC collaboration program of inelastic electron scattering has covered a wide range of four-momentum transfer and missing mass of the recoiling hadronic system. In these experiments only the scattered electron was detected. The reactions that have been studied are inelastic electron scattering from the proton, deuteron, and a number of different nuclei (6–8). Experimental work on inelastic electron scattering has also been carried out at DESY [See (9) and earlier references therein], and there has been a program of inelastic muon scattering at SLAC (10).

The purpose of this article is to review and bring up to date the large amount of experimental information from singles measurements of deep inelastic scattering from the proton and neutron. Some of the results on the electroproduction of nucleon resonances will be briefly discussed, but only insofar as the results provide a comparison of interest with the deep inelastic data. The range and validity of scaling for the neutron and proton will be treated in detail. Here, we will em-
phasize primarily the experimental measurements from the MIT-SLAC collaboration.2

The inelastic electron-nucleon scattering results have generated an extensive amount of theoretical work. While this article is not intended to provide a detailed review of the theory, there will be some discussion of the main thrusts of the theoretical effort. The theoretical advances have led to useful and important insights into the experimental results even though there is presently only a partial understanding of the implications of the measurements. Our discussion of the theoretical effort is aimed both at giving a sense of the theoretical understanding of the results and at pointing out the relevance of the measurements to the present theories.

In the remainder of this review we will give a brief summary of the experimental equipment and techniques, followed by a qualitative description of the electron-proton and electron-deuteron scattering results. The scattering formalism, scaling variables, and the problems of separating the transverse and longitudinal contributions to the measured cross sections and of determining the electron-neutron cross sections are then treated. This is followed by a discussion of the range and validity of scaling behavior for the proton, deuteron, and neutron. The next sections are devoted respectively to theoretical models, a comparison of muon and electron scattering, sum rules results, and collateral experiments related to deep inelastic electron scattering.

II. EXPERIMENTAL METHOD

Before proceeding to the results, a brief description of the experimental method (11) is in order. A relatively monochromatic electron beam from the linear accelerator was passed through a liquid hydrogen or deuterium target and then through a series of beam monitors. The scattered electrons were momentum analysed by one or the other of a pair of magnetic spectrometers installed at the scattering site (see Figure 1). In separate experiments the SLAC 20 GeV and 8 GeV spectrometer were used to cover different kinematic regions. Downstream of the magnetic elements of the spectrometers were placed scintillation counter hodoscopes which registered the momentum and scattering angle of each scattered electron. In conjunction with the hodoscopes there were particle identification counters which were employed to identify electrons amid a background of pions. These consisted of a gas Cerenkov counter, a total absorption counter for electromagnetic cascades, and a few counters used to sample early shower development in the total absorption counter.

Equipment.—This section outlines the principal elements of the SLAC scattering facility (4).

The primary beam of energy in the range 4.5–20 GeV, with $\Delta E/E$ typically

FIGURE 1. Plan view of the SLAC experimental area in which the MIT-SLAC deep inelastic electron scattering measurements were carried out.

from 0.002 to 0.005, was focussed on liquid hydrogen or deuterium condensation targets, 3 inch thick. The beam was pulsed for approximately 1.6 μsec at repetition rates up to 360 cps.

The targets were vertical cylinders with aluminum or stainless steel windows. In the later experiments forced circulation of the liquid gave assurance that beam-induced target-density changes were negligibly small. In addition, recoil proton yields were measured in a 1.6 GeV spectrometer, otherwise unused in the program, to give a direct measure of relative target density, and in all experiments series of measurements at different beam currents were made to establish the effects of beam heating. Residual uncertainties in target densities were about ±1%. Target temperatures were monitored with hydrogen vapor-pressure thermometers.

Beam monitoring was carried out by a pair of toroid beam monitors upstream of the target. Throughout the experimental program, their calibrations were frequently checked against a Faraday cup. The calibrations agreed in general to better than 1%.

The two spectrometers were capable of determining the momenta of particles up to 20 GeV and 8 GeV. Because of the amount of data at constant angle required to correct the measured cross sections for radiative effects, only a limited number of angles were employed in the inelastic program. The 20 GeV spectrometer was used to measure the scattering at 6° and 10°, and the other instrument, with its larger solid angle, was employed in the determination of the generally smaller cross sections at angles of 18°, 26°, and 34°. Both spectrometers employed hodoscopes that determined 40 momentum intervals each of fractional mo-
mentum acceptance of $10^{-3}$. There were 55 and 32 angle counters in the 8 GeV and 20 GeV spectrometers which spanned angular acceptances of 16 mr and 7 mr respectively. Additional hodoscopes were employed in the 20 GeV spectrometer in a portion of the program to determine the azimuthal scattering angle and the longitudinal position in the target of the scattering event. Precise values of the momentum resolutions and solid angle acceptances for the instruments were determined both from precision measurements of the fields of the magnetic elements and by electron-beam studies of the assembled spectrometers. The solid angle acceptances were approximately 0.75 and 0.06 millisteradians for the 8 GeV and 20 GeV instruments respectively. Cross calibrations were carried out using measurements of elastic scattering. The uncertainties in the solid angles were typically about ±2%.

At many kinematic points, especially at lower scattered electron energies, appreciable pion contamination of the scattered electron yields was observed. Pion rejection was accomplished primarily through the use of lead-lucite total electromagnetic shower absorbers, useful at all but the lowest particle energies, and by measurements of the incipient shower formed in the first radiation length of the total absorption detector. A threshold gas Čerenkov counter operated near atmospheric pressure served to reject pions of momenta below about 4 GeV.

The only background which could not be rejected by the particle discrimination arrays arose from electrons from the Dalitz decay of photoproduced or electroproduced neutral pions. Electron yields from this source increased sharply as the detected particle energies were decreased. The increasing yields were one of the major factors that set lower limits to the scattered electron energies that could be usefully measured. Dalitz decays produce equal numbers of positrons and electrons of the same energy. The Dalitz electron contaminations were determined by making appropriate positron measurements.

Data from the spectrometer detectors were processed in an array of fast electronic equipment, and candidate events were read into an SDS-9300 on-line computer and onto magnetic tape for subsequent analysis. A sample of the events were processed on-line to provide immediate information on the operation of the apparatus and to allow evaluations of the progress of the runs.

A detailed description of the off-line analysis developed in connection with the MIT-SLAC inelastic electron-proton scattering has been given by Breidenbach (12) and Miller (13).

**Radiative corrections.**—Corrections must be applied to the measured cross sections to eliminate the effects of the radiation of photons which occurs during the scattering process and during traversals of material before and after scattering. These corrections also remove higher order electrodynamical contributions to the electron-photon vertex and the photon propagator. The largest correction has to be made for the radiation during scattering, described by diagrams (a) and (b) in Figure 2. A photon of energy $k$ is emitted in (a) after the virtual photon is exchanged, and in (b) before the exchange. Diagram (c) is the cross section which is to be recovered after appropriate corrections for (a) and (b) have been
A measured cross section at fixed \( E, E', \) and \( \theta \) will have contributions from (a) and (b) for all values of \( k \) which are kinematically allowed. The lowest value of \( k \) is zero, and the largest occurs in (b) for elastic scattering of the virtual electron from the target particle. Thus, to correct a measured cross section at given values of \( E \) and \( E' \), one must know values of the cross section over a range of incident and scattered energies. To an excellent approximation, the information necessary to correct a cross section at an angle \( \theta \) may all be gathered at the same value of \( \theta \). Diagram (d) of Figure 2 shows the kinematic range in \( E \) and \( E' \) of cross sections which can contribute by radiative processes to the cross section at point \( S \), for fixed \( \theta \). The range is the same for contributions from bremsstrahlung processes of the incident and scattered electrons. For single hard photon emission, the cross section at point \( S \) will have contributions from elastic scattering at points \( U \) and \( L \), and from inelastic scattering along the lines \( SL \) and \( SU \), starting at inelastic threshold. If two or more photons are radiated, contributions can arise from line \( LU \) and the inelastic region bounded by lines \( SL \) and \( SU \). The cross sections needed for these corrections must themselves have been corrected for radiative effects. However, if uncorrected cross sections are available over the whole of the “triangle” \( LUS \), then a one-pass radiative correction procedure may be employed, assuming the peaking approximation (14), which will develop the approximately corrected cross sections over the entire triangle, including the point \( S \).

The application of radiative corrections requires the solution of another
difficulty, as it is generally not possible to take measurements sufficiently closely spaced in the $E-E'$ plane to apply them directly. Typically five to ten spectra, each for a different $E$, are taken to determine the cross sections over a "triangle." Interpolation methods must be developed to supply the missing cross sections and must be tested to show that they are not the source of unexpected error.

The radiative tails from elastic electron-proton scattering in practice are subtracted from the measured spectra before the interpolations are carried out. In the MIT-SLAC radiative correction procedures, the radiative tails from elastic scattering were calculated using the formula of Tsai (15), which is exact to lowest order in $\alpha$. Included in the calculation of the tail were: the effects of radiative energy degradation of the incident and final electrons, the contributions of multiple photon processes, and radiation from the recoiling proton. After the subtraction of the elastic peak's radiative tail, the inelastic radiative tails were removed in a one pass unfolding procedure as outlined above. The particular form of the peaking approximation used was determined from a fit to an exact calculation of the inelastic tail to lowest order which incorporated a model that approximated the experimental cross sections. The formulas and the procedures are described by Miller et al (8, 13). The measured cross sections were also corrected in a separate analysis using a somewhat different set of approximations (16). Comparisons of the two gave corrected cross sections which agreed to within a few percent. Figures 3 and 4 show the relative magnitude of the radiative corrections as a function of $W$ for two typical spectra with hydrogen and deuterium targets. While radiative corrections are the largest corrections to the data, and involve a considerable amount of computation, they are understood to a confidence level of 5% to 10% and do not significantly increase the total error in the measurements.

III. Proton and Deuteron Cross Sections

The range of kinematics covered in the published MIT-SLAC measurements of electron proton scattering is given in Table 1. The range of missing mass covered was $M \leq W \leq 5.7$ GeV and the range of the square of the four-momentum transfer was $0.3 < q^2 < 20$ (GeV/c)^2. The missing mass $W$ is the mass of the unobserved final hadronic state and is given by

$$W^2 = 2M(E - E') + M^2 - q^2$$

where $E$ is the incident electron energy, $E'$ is the scattered electron energy, and $M$ is the mass of the target nucleon. The quantity $q^2$ is given by

$$q^2 = 2EE'(1 - \cos \theta)$$

where $\theta$ is the electron scattering angle. In the kinematic relations shown the electron mass is neglected.

In general, the measurements were made at closely spaced values of the scattered energy $E'$, for constant scattering angle $\theta$, and constant incident energy $E$. For each scattering angle spectra were measured at a number of incident energies in order to be able to make radiative corrections to the data.
Figure 3. Spectra of 10 GeV electrons scattered from hydrogen at $6^\circ$, as a function of the final hadronic-state energy $W$. Figure (a) shows the spectrum before radiative corrections. The elastic peak has been reduced in scale by a factor of 8.5. The computed radiative tail from the elastic peak is shown. Figure (b) shows the same spectrum with the elastic peak's radiative tail subtracted and inelastic radiative corrections applied. Figure (c) shows the ratio of the inelastic spectrum before, to the spectrum after, radiative corrections.

Typical spectra for hydrogen and deuterium are shown in Figures 3, 4, 5, and 6. Bumps in the spectra at lower $q^2$—Figures 3 and 4—are seen at the $\Delta 1236$ and $N^*$1518, and the region of the $N^*$1688; in a number of spectra there is also a small bump near the $\Delta 1920$. No clear-cut evidence is seen for the excitation of the Roper $P_{11}$ resonance. In addition to these bumps there is a broad continuum of large cross section. This continuum dominates the scattering as $q^2$ increases as shown in Figures 5 and 6. These spectra have a qualitative similarity to those observed in inelastic electron-nuclear scattering.
The general dependence of the measured spectra on laboratory energy and angle and momentum transfer squared can be seen in Figures 7, 8, and 9, in which some of the measured spectra (17) are sketched or displayed. In Figure 9 the data at \( q^2 = 0 \) were determined by extrapolation from measurements made at \( \theta = 1.5^\circ \) (18). These figures show that the excitation of discrete states is dominant for smaller angles and lower incident energies, but at larger angles and higher incident energies the continuum channels dominate the scattering. This is equivalent to the statement that the continuum becomes increasingly large compared to the observed resonance excitations as \( q^2 \) increases. The Mott cross sec-

![Graph](image)

**Figure 4.** Spectra of 10 GeV electrons scattered from deuterium at 6°. See caption of Figure 3. The quasi-elastic peak has been reduced in scale by a factor of 1.5.
TABLE 1. KINEMATIC RANGE—ELECTRON–PROTON SCATTERING DATA

<table>
<thead>
<tr>
<th>Angle</th>
<th>Range $E$</th>
<th>Range $E'$</th>
<th>Range $q^2$</th>
<th>Max. $W$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6° a</td>
<td>7.0–16.0</td>
<td>3.25–13.2</td>
<td>0.25–2.31</td>
<td>4.8</td>
</tr>
<tr>
<td>10° a</td>
<td>7.0–17.7</td>
<td>3.0–12.46</td>
<td>0.80–6.7</td>
<td>5.2</td>
</tr>
<tr>
<td>18° b</td>
<td>4.5–17.0</td>
<td>2.0–8.0</td>
<td>1.0–14.0</td>
<td>5.1</td>
</tr>
<tr>
<td>26° b</td>
<td>4.5–18.0</td>
<td>1.75–5.5</td>
<td>1.5–20.0</td>
<td>5.0</td>
</tr>
<tr>
<td>34° b</td>
<td>4.5–15.0</td>
<td>1.2–3.25</td>
<td>2.0–16.7</td>
<td>4.3</td>
</tr>
</tbody>
</table>

$\text{GeV GeV GeV (GeV/c)^2 GeV}$


The nature of electron scattering from the nucleon cannot be completely determined without studies of electron-neutron scattering. A comparison of neutron and proton cross sections is a sensitive probe for the presence of a non-diffractive component in the scattering process, since a difference in the two cross sections (without the kinematic correction for target recoil) is given for each energy and angle to serve as a scale for the scattering cross sections.

**Figure 5.** Spectra of 19.3 GeV electrons scattered from hydrogen at 10°.
See caption to Figure 3.
DEEP INELASTIC ELECTRON SCATTERING

See caption to Figure 3.

sections would establish the existence of some isotopic spin exchange. Moreover the question of scaling of the neutron results can also be investigated.

In order to determine the electron-neutron scattering in the absence of a free neutron target one is forced to employ deuterium or heavier nuclei as targets. Its simple structure and low binding energy make deuterium the material of choice.

A series of measurements of inelastic scattering from deuterium has been carried out covering the same kinematic regions as the previous work on the proton, but with some extensions both to higher incident energies and lower scattered energies (see Table 2). Measurements at 6° and 10° (6, 16, 49) were carried out using the 20 GeV spectrometer and were carried out at 18°, 26°, and 34° using the 8 GeV spectrometer. Measurements of scattering from the proton were made simultaneously in order to reduce possible systematic errors in a comparison of the scattering from the two targets. Some new but still preliminary results (19, 25) on the deuterium and hydrogen cross sections at 18°, 26°, and 34° are available although the final analyses are not complete. A number of representative deuterium spectra are shown in Figures 4 and 6.

IV. KINEMATICS AND VARIABLES

To extract the effects of the nucleon structure in the scattering process it is useful to separate out the pure Q.E.D. dependence of the scattering cross section.
Figure 7. Visual fits to spectra showing the scattering of electrons from hydrogen at 10° for primary energies, $E$, from 4.88 GeV to 17.65 GeV. The elastic peaks have been subtracted and radiative corrections applied. The cross sections are expressed in nanobarns per GeV per steradian. The spectrum $E=4.88$ GeV was taken at DESY; Bartel, W. et al 1968. Phys. Lett. B 28:148.
FIGURE 8. Visual fits to spectra showing the scattering of electrons from hydrogen at a primary energy $E$ of approximately 13.5 GeV, for scattering angles from $1.5^\circ$ to $18^\circ$. The $1.5^\circ$ curve is taken from the MIT-SLAC data, (18), used to obtain photoabsorption cross sections.
Figure 9. Spectra of electrons scattered from hydrogen at momentum transfers squared up to 4 (GeV/c)^2. The curve for q^2 = 0 represents an extrapolation to q^2 = 0 of electron scattering data acquired at θ = 1.5° (18).
### Table 2. Kinematic Range—Electron Proton and Electron Deuteron Data

<table>
<thead>
<tr>
<th>Angle</th>
<th>Range $E$</th>
<th>Range $E'$</th>
<th>Range $q^2$</th>
<th>Max. $W$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6°</td>
<td>4.5–19.5</td>
<td>2.5–17.5</td>
<td>0.10– 3.7</td>
<td>5.7</td>
</tr>
<tr>
<td>10°</td>
<td>4.9–19.3</td>
<td>2.5–14.7</td>
<td>0.30– 8.6</td>
<td>5.6</td>
</tr>
<tr>
<td>18°</td>
<td>4.5–17.0</td>
<td>1.5– 8.75</td>
<td>0.44–14.6</td>
<td>5.2</td>
</tr>
<tr>
<td>26°</td>
<td>4.5–18.0</td>
<td>1.5– 5.5</td>
<td>0.91–20.1</td>
<td>5.3</td>
</tr>
<tr>
<td>34°</td>
<td>4.5–15.0</td>
<td>1.5– 3.25</td>
<td>1.54–16.7</td>
<td>4.3</td>
</tr>
<tr>
<td></td>
<td>GeV</td>
<td>GeV</td>
<td>(GeV/c)$^2$</td>
<td>GeV</td>
</tr>
</tbody>
</table>

* Kendall, Cornell Conference (6). See also (16, 19, and 25).

In the assumption of one photon exchange, the laboratory differential cross section (20) for unpolarized electrons scattering from unpolarized nucleons, with only the scattered electron detected, is:

$$
\frac{d^5\sigma}{d\Omega dE'} = \left(\frac{d\sigma}{d\Omega}\right)_M \left[ W_2(v, q^2) + 2W_1(v, q^2) \tan^2 \theta / 2 \right]
$$

where

$$
\left(\frac{d\sigma}{d\Omega}\right)_M = \frac{e^4 \cos^2 \theta / 2}{4E^2 \sin^4 \theta / 2}
$$

The functions $W_1$ and $W_2$ depend on the properties of the target nucleon and can be represented as functions of two invariants, $q^2$ and $v = E - E'$. The above expression is the analog of the Rosenbluth cross section. There is another expression (21) that is often used to describe inelastic electron scattering and which is the analog of photoproduction. In this description the cross section for inelastic scattering is given by:

$$
\frac{d^2\sigma}{d\Omega dE'} = \Gamma_t \left[ \sigma_t(v, q^2) + \epsilon \sigma_e(v, q^2) \right]
$$

where $\sigma_t$ and $\sigma_e$ are the absorption cross sections for virtual photons with transverse and longitudinal polarization components respectively, and the virtual photon spectrum $\Gamma_t$ is given by:

$$
\Gamma_t = \frac{\alpha}{4\pi^2} \frac{K}{q^2} \left[ \frac{2}{E} \right]
$$

The quantity $K$ equals $(W^2 - M^2)/2M$ where $M$ is the nucleon rest mass and $\epsilon$ equals

$$
[1 + 2(1 + v^2/q^2) \tan^2 \theta / 2]^{-1}
$$

In the limit $q^2 \to 0$, gauge invariance requires that $\sigma_e \to 0$ and $\sigma_t(v, q^2) \to \sigma_{\gamma p}(v)$.
where $\sigma_{\text{ph}}(\nu)$ is the photoabsorption cross section for real photons of energy $\nu$. The two descriptions are equivalent and it follows that

$$W_1 = \frac{K}{4\pi^2 \alpha} \sigma_1, \quad W_2 = \frac{K}{4\pi^2 \alpha} \frac{q^2}{q^2 + \nu^2} (\sigma_s + \sigma_t)$$

In order to make separate determinations of $W_1$ and $W_2$ (or $\sigma_s$ and $\sigma_t$), it is necessary to measure the inelastic cross section at different angles for the same values of $q^2$ and $\nu$. Appropriate changes in the values of both $E$ and $E'$ are required.

The results of the separation are, for convenience, expressed in terms of the parameter $R = \sigma_s / \sigma_t$. The experimental values of $W_1$ and $W_2$ are given by:

$$W_1 = \frac{d^2\sigma}{d\Omega dE'} \exp \left[ \frac{(d\sigma)}{(d\Omega)} M \right]^{-1} \left\{ (1 + R) \frac{q^2}{q^2 + \nu^2} + 2 \tan^2 \theta / 2 \right\}^{-1}$$

$$W_2 = \frac{d^2\sigma}{d\Omega dE'} \exp \left[ \frac{(d\sigma)}{(d\Omega)} M \right]^{-1} \left\{ 1 + 2 \frac{1}{1 + R} \frac{q^2 + \nu^2}{q^2} \tan^2 \theta / 2 \right\}^{-1}$$

where

$$\frac{d^2\sigma}{d\Omega dE'} \exp$$

is the experimental inelastic cross section.

In practice it was convenient to determine values of $\sigma_t$ and $\sigma_s$ from straight line fits to differential cross sections as functions of $\nu$. $R$ was determined from the values of $\sigma_s$ and $\sigma_t$, and $W_1$ and $W_2$ were, as shown above, determined from $R$.

V. Scale Invariance and Scaling Variables

On the basis of investigation of models that satisfy current algebra, Bjorker (22) had conjectured that in the limit of $q^2$ and $\nu$ approaching infinity, with the ratio $\omega = 2Mv/q^2$ held fixed, the two quantities $\nu W_2$ and $W_1$ become functions of $\omega$ only. That is:

$$\nu \to \infty$$

$$q^2 \to \infty$$

$$\omega \text{ fixed}$$

$$W_1(\nu, q^2) = F_1(\omega)$$

$$\nu W_2(\nu, q^2) = F_2(\omega)$$

It is this property that is referred to as "scaling" in the variable $\omega$ in the "Bjorker limit." As we shall see, scaling holds over a substantial portion of the ranges of $\nu$ and $q^2$ that have been studied. The question naturally arises as to whether there are other scaling variables that converge to $\omega$ in the Bjorken limit, and that provide scaling behavior over a larger region in $\nu$ and $q^2$ than does the use of $\omega$. A number of such variables have been proposed. Among these are the variables proposed by suri (23).
DEEP INELASTIC ELECTRON SCATTERING

\[ \omega_\ast = \frac{W^2}{q^2} \]

by Rittenberg & Rubinstein (24)

\[ \omega_\ast = \frac{2M \nu + M^2}{q^2 + A^2} \]

by the MIT-SLAC group (8, 13)

\[ \omega' = \frac{2M \nu + a^2}{q^2} \]

and a variable by Breidenbach & Kuti (26), based on the physics of the light cone

\[ \omega_L = \frac{M}{\sqrt{\nu^2 + q^2 - \nu}} \]

The applicability of some of these variables is discussed in later sections of the text.

Since \( W_1 \) and \( W_2 \) are related by

\[ \nu W_2/W_1 = (1 + R)(1/\nu + \omega/(2M)) \]

it can be seen that scaling in \( W_1 \) accompanies scaling in \( \nu W_2 \) only if \( R \) has the proper functional form to make the right hand side of the equation a function of \( \omega \) (or some other variable). In the Bjorken limit, it is evident that \( \nu W_2 \) and \( W_1 \) will both scale if \( R \) is constant or is a function of \( \omega \).

VI. SEPARATION OF \( \sigma_8 \) AND \( \sigma_t \) FOR THE PROTON AND DEUTERON

Separation of \( \sigma_8 \) and \( \sigma_t \) for the proton was carried out (8, 13) over the kinematic region where data were available at three or more values of \( \epsilon \), either from direct measurements or from interpolation between measurements at fixed angle. Actual data points at different angles for the same values of \( q^2 \) and \( W^2 \) existed only for \( q^2 = 4 \) (GeV/c)^2 and \( W = 2, 3, \) and 4 GeV. However, interpolations of adequate precision could be carried out for fixed points in the \( q^2, W \) plane. The kinematic region where data were available and the subregion in which separation was carried out are shown in Figure 10. In this region \( W_1 \) and \( W_2 \) may be separately determined without assumptions about \( R \). The heavy line bounds all data points measured at the five angles 6°, 10°, 18°, 26°, and 34°. Determinations of \( \sigma_8 \) and \( \sigma_t \) were carried out at twenty-three points in the \( q^2, W^2 \) plane. Figure 11 shows four examples of these separations.

The assumption of one photon exchange which underlies the definition of the electromagnetic structure functions implies the linear dependence of \( d\sigma/d\Omega dE'/T_z \) on \( \epsilon \) for a particular point \( (q^2, \nu) \), as is displayed in the equation that relates \( \sigma_8 \) and \( \sigma_t \) to the measured cross sections. The data are everywhere consistent with
FIGURE 10. The kinematic plane in $q^2$ and $W^2$. The heavy line bounds all data points for scattering of electrons, described in (7) and (8), measured at 6°, 10°, 18°, 26°, and 34°. The "Separation Region" includes all points where data at three or more angles exist. Various values of $\omega$ are indicated with $\omega = \infty$ coinciding with the $q^2 = 0$ abscissa and $\omega = 1$ corresponding to elastic scattering ($W^2 = 0.88$). Region I indicates the region where the data are consistent with scaling in $\omega$. Region II indicates the extension of the scaling region if the data are plotted against $\omega'$. The ranges $A$, $B$, $C$ in the variable $\omega$ indicated in the figure are employed in the discussion in the text.

This requirement. The measured values of $R$ are in the range 0 to 0.5, and no striking kinematic variation is apparent. On the assumption that $R$ is a constant in this kinematic range, the average value of $R$ for the proton is $0.18 \pm 0.10$ where the quoted error includes an estimated systematic error of 0.06. The values of $R$ are also compatible with $R = 0.035q^2$ and with $R = q^2/p^2$. Various other forms would also be compatible with the results. In Figure 12, the measured values of $R$ are shown as a function of $q^2$. The results of the separation show that $\sigma_z$ is dominant in the kinematic region that was investigated, roughly given by $q^2$ from 1.0 to 11.0 (GeV/c)^2 and $W$ from 2.0 to about 4 GeV. The experimental uncertainties in the measurements of $R$ preclude a definite statement that $\sigma_z$ is significantly different from zero.

The new body of electron-proton data taken in conjunction with the deuterium measurements, whose kinematic range is shown in Table 2 has not yet been fully analyzed. A preliminary analysis shows that the average value of $R$ is consistent with the results just quoted. An average value of $R = 0.12 \pm 0.10$ was found (25). Again no clear dependence of $R$ on any kinematic variable was found. Preliminary results for $R$, for hydrogen, and for deuterium, as functions of $q^2$ are
Figure 11. Typical examples illustrating the separate determination of $\sigma_t$ and $\sigma_q$. The straight solid lines are best fits to Equation 2. The dashed lines indicate the one standard deviation values of the fits. The assumption of one-photon exchange made in calculating $\sigma_t$ and $\sigma_q$ implies that linear fits should be satisfactory. For the two upper graphs measured data exist at each angle. For the two lower graphs the data were interpolated. Effects of systematic errors are not included.

shown in Figure 13. The preliminary studies of the separation of $\sigma_t$ and $\sigma_q$ for deuterium suggest that the average value of $R$ is about the same as $R$ for the proton. A value of $R$ equal to $0.14 \pm 0.08$ was found (25). As for the proton, no clear dependence of $R$ on any kinematic variable was established.

Figure 14 shows the cross sections $\sigma_t$ and $\sigma_q$ for the proton (8,13) plotted for constant $q^2$ as functions of $W^2$. The dashed lines indicate the $W^2$ dependence of $\sigma_{tp}$. For $q^2 < 3$ (GeV/c)$^2$ the cross sections are consistent with a constant or a slowly falling energy dependence similar to the behavior of $\sigma_q$. For larger $q^2$, $\sigma_t$ shows rising energy dependence resembling a threshold-type behavior. This rising behavior of $\sigma_t$ at high energy is unique among the various total cross sections that
have been measured. The $q^4$ dependence of $\sigma_t$ shown in Figure 15 displays no pure power law behavior, but varies in the region of the present data between $1/q^2$ and $1/q^6$, as indicated by the straight lines. The energy and $q^2$ dependences of $\sigma_t$ may be correlated with the behavior of $\nu W_2$ that is described in detail in Sec. VIII.

**VIII. Validity of Scale Invariance**

The impulse approximation is used to extract the neutron cross section from the deuteron cross section. Thus, to a first approximation, the scattering from the neutron may be found by subtracting from the deuteron scattering the scattering from the proton; however a number of corrections have to be considered. Especially important are corrections that must be made before the subtraction is carried out to account for the internal motion of the constituent particles within the deuteron. These corrections have come to be known as "smearing" corrections. Other effects are the Glauber correction [See correction to photoproduction in (27)] which is estimated to be less than one percent, the effects of mesonic currents in the deuteron which are assumed on the basis of elastic electron-deuteron studies (28) to be small, and the effects of a final state interaction which likewise are assumed to be small. There are also presumably small uncertainties due to possible off-mass shell effects in the nucleon structure functions. None of the latter three effects can be well estimated at present.

The corrections for internal motion have been studied by West (29) who in-
FIGURE 13. Ratio \( R = \sigma_e / \sigma_n \) for scattering of electrons from hydrogen and from deuterium. Unpublished preliminary data from (16) and (25). Data from a range of \( W^2 \) are averaged for each value of \( q^2 \).

included effects arising from the requirement of gauge invariance in the interaction with the deuteron. The variation of the "smearing" correction for different well-known deuteron wavefunctions has been found to be small, provided that the wavefunctions are consistent with the observed elastic electromagnetic form factor of the deuteron (28). The variation in the correction for such consistent wavefunctions is less than a few percent.

In the initial procedure employed to extract the neutron cross sections, the internal motion effects were not taken out of the deuteron cross section, but were instead inserted into the proton cross section, since this is a good deal easier to do. The smearing procedures of West (29, 130, 131) with small modifications were employed to calculate \( H_n \), the hydrogen cross sections which include
FIGURE 14. Values of $\sigma_s$ and $\sigma_t$ for hydrogen from (7) and (8) shown at constant $q^2$ as a function of $W^2$ (and $\nu$) for $q^2 = 1.5, 3, 5, \text{and } 8 \text{ (GeV/c)}^2$. The dotted curves show the $\nu$-dependence of the total photoabsorption cross section $\sigma_{\gamma p}(\nu)$.

the internal motion effects characteristic of the deuteron. In the evaluation of the data carried out so far, the quantity $D - H_n$, the deuterium cross section minus the "smeared" hydrogen cross section, will be assumed to represent the smeared neutron cross section. Values of $(D/H_n) - 1$ represent, with the above limitations, values of the smeared neutron cross section divided by the smeared proton cross section. The data for $(D/H_n) - 1$ corrected for smearing in this way accordingly have a value at a given $\omega$ which is the ratio of the cross sections averaged over a
mall region of ω around the given value. Computational models show that the difference between the ratio of the smeared cross sections and true ratio increases as ω approaches unity and is sensitive to the ω dependence of n/p. A computational program is presently underway to remove the effects of the internal motion from the deuteron data.

The quantity \((2 - D/H_o) (H_o/H) \nu W^P\) represents the quantity \(\nu(W^P - W^N)\) smeared on the assumption that the values of \(R_n\) equal those of \(R_p\). This difference is used as a first approximation to represent \(\nu(W^P - W^N)\) in comparison with theoretical models. The measured values of \((D/H_o) - 1\) and \(\nu(W^P - W^N)\) smeared) are displayed and discussed in the next section. Included is some additional information on the size of the smearing correction to \((D/H_o) - 1\) as a function of ω.

VIII. VALIDITY OF SCALE INVARIANCE

Proton.—The validity of scaling of \(\nu W_2\) for the proton has recently been studied (8,13) at laboratory angles of 6°, 10°, 18°, 26°, and 34° over the kinematic range available to the SLAC accelerator. Over a portion of this range (shown as the separation region in Figure 10) it was possible to determine separately the contributions of \(\sigma_t\) and \(\sigma_t\). In this region the scale invariance of the structure functions can be tested. Outside the indicated separation region, only consistency with scaling can be studied, as a test of scale invariance requires values of \(R\) that can only be found by extrapolation of the measured values. The conclusions that have been drawn regarding consistency with scaling for \(\nu W_2\) are independent of
the choice for extrapolation among the three parameterizations of $R$ mentioned above.

To test for scaling it is useful to plot $\nu W_2$ for fixed $\omega$ as a function of $q^2$. Constant scaling behavior is exhibited in such a plot if $\nu W_2$ is independent of $q^2$. Values of $\nu W_2$ for various ranges of $\omega$ are shown as functions of $q^2$ in Figure 16 calculated from interpolations at each angle of radiatively corrected spectra at 6°, 10°, and in Figure 17 at 6°, 10°, 18°, and 26°. In Figure 18 experimental values of $\nu W_2$ are plotted as functions of $\omega$ for various ranges of $q^2$. A constant value $R=0.18$ has been assumed in calculating $\nu W_2$. Scaling behavior is not expected where there are observable resonances, because resonances occur at fixed $W$, not at fixed $\omega$, nor is it expected for small $q^2$, because $\nu W_2$ cannot depend solely on $q^2$ in this limit. An inspection of a number of similar plots leads to a group of conclusions regarding the validity of scaling in several kinematic regions:

1. For $4<\omega<12$ (Region B of Figure 10): For $W>2.0$ GeV and $q^2>1.0$ (GeV/c)^2, $\nu W_2$ is a constant within experimental errors and hence “scales” in $\omega$ (or, indeed, in any other variable). The range of kinematics of the measurements included in this test covers $q^2$ from 1 to 7 (GeV/c)^2 and values of $W$ between 2 and 5 GeV.

2. For $\omega<4$ (Region A of Figure 10): The experimental values of $\nu W_2$ scale for $W>2.6$ GeV, but $\nu W_2$ appears to increase as $W$ decreases below 2.6 GeV. This region covers kinematic ranges of $W$ between 2.6 GeV and 4.9 GeV, and of $q^2$ between 2 (GeV/c)^2 and 20 (GeV/c)^2.

3. For $\omega>12$ (Region C of Figure 10): There are relatively few points above $q^2=1$ (GeV/c)^2 and no points above $q^2=2$ (GeV/c)^2, making it difficult to determine any variation of $\nu W_2$ with changing $q^2$. There are no measurements of $\nu W_2$ in this region, and the values of $\nu W_2$ are especially sensitive to variations of $R$. The large angle data have a maximum $\omega$ of 9 and influence the values of $\nu W_2$ for $\omega=9$.

![Figure 16](image)

**Figure 16.** The quantity $\nu W_2$ for the proton for various ranges of $\omega$ plotted against $q^2$. $\nu W_2$ is kinematically constrained to zero for $q^2=0$. Results from (7).
DEEP INELASTIC ELECTRON SCATTERING

\[ \nu W_2 \]

\[ q^2 \text{ (GeV/c)}^2 \]

\[ \omega = 4 \]

**Figure 17.** \( \nu W_2 \) for the proton as a function of \( q^2 \) for \( W > 2 \text{ GeV} \), at \( \omega = 4 \). Results from (7, 8, and 49).

**Figure 18.** \( \nu W_2 \) for the proton for \( \omega > 10 \) and \( q^2 > 0.5 \text{ (GeV/c)}^2 \). A value of 0.18 is used for \( R \). Data from (49). The line at \( \nu W_2 = 0.30 \) is drawn to facilitate comparisons between the graphs. Data for \( q^2 \) below 1.0 (GeV/c)^2 show deviations from scale invariance.
\( \omega > 12 \) only through the values of \( R \) determined in the low \( \omega \) region. Scaling cannot be tested critically in this region since the uncertainty in \( R \) prevents the large \( \omega \) behavior of \( \nu W_2 \) from being known with assurance. If \( R = 0.18 \) is assumed when \( q^2 > 0.8 \text{ (GeV/c)}^2 \), \( \nu W_2 \) decreases slightly as \( \omega \) increases. However, for larger values of \( R \) consistent with the extrapolated values, \( \nu W_2 \) is constant. Preliminary analysis of more recent data does not resolve these questions (16, 49). A falling \( \nu W_2 \) with increasing \( \omega \) would indicate a nondiffractive component of \( \nu W_2 \).

One of the most remarkable aspects of the above results is that scaling behavior, presumed to be a property asymptotically reached in the Bjorken limit, sets in at such low values of \( q^2 \) and \( \nu \). The theoretical significance of this is presently not well understood. Tests of scaling in \( \omega' \) carried out by the MIT-SLAC experimental group have shown that the use of \( \omega' \) extends the scaling region from \( W = 2.6 \text{ GeV} \) down to \( W = 1.8 \text{ GeV} \), shown as Region II in Figure 10. The constant \( a \) was determined to be \( 0.95 \pm 0.07 \text{ (GeV/c)}^2 \) by fitting that portion of the data with \( W \geq 1.8 \text{ GeV} \) and \( q^2 > 1 \text{ (GeV/c)}^2 \). The statistical significance of \( a \) is greatly reduced in a fit to the data for \( W > 2.6 \text{ GeV} \), implying that functions of either \( \omega \) or \( \omega' \) give satisfactory statistical fits in this kinematic range. In the following discussion, the value of \( a \) will be chosen to be equal to \( M^2 = 0.88 \) which gives

\[
\omega' = \omega + \frac{M^2}{q^2} = \frac{W^2}{q^2} + 1
\]

Figure 19 shows \( 2MW_1 \) and \( \nu W_2 \) for the proton as functions of \( \omega \) for \( W > 2.6 \text{ GeV} \), and Figure 20 shows these quantities as functions of \( \omega' \) for \( W > 1.8 \text{ GeV} \). The experimental results for both structure functions scale in \( \omega \) and \( \omega' \) within the errors of measurement over a large kinematic range. We have noted that if \( \nu W_2 \) scales in the Bjorken limit then scaling of \( 2MW_1 \) is expected provided \( R \) is constant or is a function of \( \omega \). As we have seen, the measured values of \( R \) are small and are not sufficiently precise to determine any possible functional dependence of \( R \).

It is of interest to examine the explicit \( q^2 \) dependence of \( W_2 \) at constant \( W \) in the scaling region. Since for \( \omega > 4 \) which corresponds to \( (W^2 - M^2)/q^2 > 3 \) the quantity \( \nu W_2 \) is approximately constant, we find that

\[
W_2 \sim \frac{1}{1 + q^2/(W^2 - M^2)}
\]

which is a relatively weak dependence on \( q^2 \). For \( 1 \leq \omega \leq 2 \), \( \nu W_2 \) is found to be proportional to \( (1 - 1/\omega)^3 \) and the dependence of \( W_2 \) is

\[
W_2 \sim \left[ \frac{1}{W^2 - M^2 + q^2} \right]^3 \left[ \frac{W^2 - M^2}{q^2} \right]
\]

It is thus evident that for \( (W^2 - M^2)/q^2 \ll 1 \), \( W_2 \) has approximately the same \( q^2 \) dependence as the square of the elastic form factor. The \( q^2 \) dependence is clearly intermediate between these extremes for intermediate values of \( \omega \).
Deuteron.—The final analysis of the larger angle measurements on deuterium is not yet complete so that separations of the scalar and transverse contributions to the inelastic $e-d$ scattering are preliminary and of limited scope. However, on the assumption that $R^D = R^P$, a result consistent with the preliminary determinations of $R^D$ as discussed in Sec. VI, values of $\nu W_2^P$ may be derived from the measured results. These values are shown in Figure 21(a) and (b), where (a) displays the values for $\theta = 6^\circ$ and $10^\circ$ (16), and (b) the values for $\theta = 18^\circ$, $26^\circ$, and $34^\circ$ (19, 25), as functions of $\omega$. The values of $\nu W_2^P$ measured in the same series of experiments are included also. It appears that within the errors of measurement
**Figure 20.** $2M W_1$ and $\nu W_2$ for the proton as functions of $\omega'$ for $W > 1.80$ GeV, $q^2 > 1(\text{GeV/c})^2$ and using $R = 0.18$. Data from (8).
the preliminary deuteron results exhibit scale invariance with about the same uncertainties as the proton. The data displayed are subject to the restrictions that $W \geq 2$ GeV and $q^2 \geq 1$ (GeV/c)$^2$.

**Neutron.**—Values of $(D/H)$—1 which represent—within the limitations described in Section VII—the values of the smeared neutron cross sections divided by the smeared proton cross sections are plotted in Figure 22 against $x = 1/\omega$. The points shown are preliminary and represent data (19) having values of
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
X = 1/ω

\( \frac{D/H - 1}{\text{smeared}} \)

FIGURE 22. The quantity \( \frac{D/H - 1}{\text{smeared}} \) as a function of \( x = 1/ω \). The data are from (19), and are for \( W > 2 \text{ GeV} \) and \( q^2 > 1(\text{GeV}/c)^2 \). This quantity is approximately equal to the ratio of neutron to proton scattering cross sections. The ratios shown are averages over small intervals of \( x \). See also Figure 23.

Some of the data were reported earlier (6). Further analyses have not made significant changes in the ratio of neutron to proton cross sections except for the two smallest \( ω \) points which increased on the average by 1.5 standard deviations. These results indicate sizable differences between the neutron and proton cross sections, and thus provide evidence for a significant nondiffractive component in the scattering process. The values of the neutron-proton cross section ratio are consistent with a single function of \( ω \). Thus within the errors the neutron cross section exhibits scaling. With the expected reduction in the presently shown errors from further data analysis a more stringent test for scaling behavior of the neutron will soon be available.
Figure 23 shows the effect of the smearing correction, as calculated, on the plot of \((D/H) - 1\) versus \(\omega\). The dashed line represents a curve drawn through the corrected points shown in Figure 22. The solid curve represents the dashed curve before smearing corrections were made. The "smearing" effects for the points shown are thus smaller than the errors in the experimental points for \(\omega > 1.5\), and cannot account for the differences between the proton and neutron observed in Figure 22. The smearing correction is much larger for \(\omega < 1.5\), and the value of \(n/p\) in this region is more dependent on the assumptions in the smearing procedure. West (29) has found that, relatively independent of the deuteron model, there is no smearing correction in the neighborhood of \(\omega = 1.5\), because of a crossover of the corrected and uncorrected curves. Since there is a sizable difference in the uncorrected yields for the proton and neutron at this point, this firmly establishes the neutron-proton difference. It should be emphasized again that the corrected curve shown in Figure 23 is the ratio of the

![Figure 23](image)

**Figure 23.** This figure displays the magnitude of the correction for internal motion on the measurements of \((D/H - 1)\) as a function of \(x = 1/\omega\). The solid curve in the present figure, *Uncorrected*, represents a curve drawn through the uncorrected points of \((D/H - 1)\), i.e. \(H^* = H\). The dashed curve, *Corrected*, represents the data of Figure 22, \((D/\bar{H} - 1)\), i.e., \(H^* = \bar{H}\). The line, *Limit of Data* shows the highest value of \(x\) for which data of significant statistics exist.
neutron to proton cross sections where both include the effects of internal motion.

Figure 24 represents $\nu(W_2^p - W_2^n)$ (smeared) as a function of $x = 1/\omega$. There is a maximum of this quantity at about $x = 1/3$.

Inelastic muon scattering measurements carried out at SLAC (10) have yielded information on the ratio of the neutron and proton cross sections in the deep inelastic region, and on the comparison of electron and muon scattering. The comparison is discussed in Sec. IX. The ratio of neutron and proton cross sections was developed from measurements of the scattering of a 12 GeV/c muon beam from hydrogen, carbon, and copper targets. Measurements of inelastic scattering were made for $0.3 \leq q^2 \leq 2 \text{ (GeV/c)}^2$ and $0.6 \leq \nu \leq 6 \text{ GeV}$. In order to extract the ratio of neutron to proton cross sections, data were analysed in kinematic regions where possible shadowing effects arising from a $\rho$-domi-
nance mechanism would be minimized. In these regions, the minimum momentum transfer for $p$ production $t_{\text{min}} = (q^2 + M^2 p^2)/4p^2$ was large. With this requirement the $A$ dependence found was $A^{0.99 \pm 0.01}$ showing that there was no detectable shadowing. This data yielded an average ratio of neutron to proton cross section of $0.91 \pm 0.06$. Since the range of $\omega$ for this data is $\omega > 5$, the electron and muon scattering results are consistent.

IX. COMPARISON OF ELECTRON AND MUON SCATTERING RESULTS

The SLAC muon scattering group has made a detailed study (10) of the comparison of the muon and electron scattering results in the regions of kinematic overlap. Figure 25 shows a comparison of the results. The agreement is excellent. Since the radiative corrections for muon scattering are about a factor of four smaller than for electron scattering, this comparison is consistent with the conclusion that the radiative corrections are not introducing an appreciable error into the electron scattering results. One further conclusion arises from this comparison. The muon scattering group has used these results to put a limit on any possible difference in interactions of the muon and the electron. Using the conventional form for a breakdown parameter

$$\frac{1}{\Lambda_D^2} = \frac{1}{\Lambda_{\mu}^2} - \frac{1}{\Lambda_{e}^2}$$

they find that $\Lambda_D > 4.1$ GeV at the 98% confidence level for data with $q^2 < 4.0$ (GeV/c)$^2$ and $\nu < 9$ GeV.

X. COMPARISON WITH THEORETICAL MODELS

Many different attempts have been made to understand the experimental results in deep inelastic electron-nucleon scattering (30a-e). The main theoretical lines of approach that have been followed are the parton model, the vector dominance model, dual resonance models, Regge pole models, and studies of current commutators on the light cone.

The parton model.—In the parton model, developed by Feynman (31) and Bjorken & Paschos (32), the proton is conjectured to consist of point-like constituents, called partons, from which the electron is scattered incoherently. This model is implemented in the infinite momentum frame of reference, in which the relativistic time dilation slows down the motion of the constituents nearly to a standstill. The incoming electron thus "sees," and incoherently scatters from constituents which are essentially real and which are noninteracting with another during the time the virtual photon is exchanged. In this frame, the impulse approximation is assumed to hold, so that the scattering process is sensitive to the properties and motions of the constituents.

Consider a proton of momentum $P$, made up of partons, in a frame approaching the infinite momentum frame. The transverse momenta of any parton is negligible and the $i$th parton has the momentum $P_i = x_i P$, where $x_i$ is a fraction
Figure 25. Comparison of cross sections for $\mu p$ and $e p$ scattering as functions of $q^2$. The incident muon momentum for these measurements was 12 GeV/c. The electron data was taken from (7) and the figure and the muon data are from (10). The quantity $\rho$ is the ratio of the inelastic muon to the electron inelastic cross sections and $K$, defined in the text, equals $(W^2 - M^2)/2M$. 
DEEP INELASTIC ELECTRON SCATTERING

of the proton's momentum. Assuming the electron scatters from a point-like parton of charge $Q_i$, leaving it with the same mass and charge, the contribution to $W_2(\nu, q^2)$ from this scattering is

$$W_2^{(i)}(\nu, q^2) = Q_i^2 \delta(\nu - q^2/2Mx) = \frac{Q_i^2 x}{\nu} \delta(x - q^2/2M\nu)$$

The expression for $\nu W_2$ for a distribution of partons is given by

$$\nu W_2(\nu, q^2) = \sum_N P(N) \left( \sum_{i=1}^N Q_i^2 \right) x f_N(x) = F_2(x)$$

where

$$x = q^2/2M\nu = 1/\omega$$

and where $P(N)$ is the probability of $N$ partons occurring,

$$\sum_{i=1}^N (Q_i)^2_N$$

is the sum of the squares of the charges of the $N$ partons, and $f_N(x)$ is the distribution of the longitudinal momentum of the partons. This, with the assumption of point-like constituents in the parton model automatically gives scaling behavior. The quantity $F_2(x)/x$ gives the weighted average of $f_N(x)$, the distribution of longitudinal momenta for the $N$ parton system. The current applications of the parton model have identified partons with bare nucleons and pions (33, 34), and also with quarks (32, 35). Parton models incorporating quarks have provided the most extensive quantitative comparisons with the data. Quark parton models require strong final state interactions to account for the fact that these constituents have not been observed in the laboratory. There may be some problem (36) in making the "free" behavior of the constituents during photon absorption compatible with a strong final state interaction. This question is avoided in parton models employing bare nucleons and pions because the recoil constituents are allowed to decay into real particles when they are emitted from the nucleon.

Drell, Levy & Yan (33) have derived a parton model, in which the partons are bare nucleons and pions, from a canonical field theory of pions and nucleons with the insertion of a cutoff in transverse momenta. The calculation shows that the free point-like constituents which interact with the electromagnetic current in each order of perturbation theory and to leading order in logarithms of $2M\nu/q^2$ are bare nucleons making up the proton and not the pions in the pion cloud. This result is consistent with the conclusions of Callan & Gross (37) that a small value of $R=\sigma_\pi/\sigma_\eta$ at large $q^2$ and $\nu$ requires that constituents responsible for the scattering have spin 1/2. The experimental results for $R$ most likely (25) rule out pions as constituents, but, of course, are consistent with the possibility that the proton is composed of quarks. The field theoretic parton model shows that $\nu W_2$ and $W_1$ should scale in the Bjorken limit and indicates that the original parton
model is consistent with some of the requirements of field theory. One of the results of this calculation is the suggested connection between the elastic form factor and the behavior of $\nu W_2$ in the neighborhood of $\omega = 1$. If the elastic form factor $G(q^2)$ has the dependence $(1/q^2)^{n/2}$ as $q^2 \to \infty$, then the predicted behavior for $\nu W_2$ is $\nu W_2 \to (1 - 1/\omega)^{n-1}$ as $\omega \to 1$. This relation was also pointed out by West (34) using a different approach. For either $\omega$ or $\omega' < 2$, the experimental values of $\nu W_2$ can be satisfactorily fit with a single cubic term, a result which is consistent with this prediction. The field theoretic parton model also relates deep inelastic scattering to colliding beam processes, such as $e^+e^- \to P + \text{anything}$. The original parton model has also been formally derived from the behavior of local field theory in the equal-time light cone limit (38, 39, 88, 89).

A detailed description of a quark-parton model has been given by Kuti & Weisskopf (35). Their model of the proton contains in addition to the three valence quarks, a sea of quark-antiquark pairs, and neutral gluons responsible for the binding of the quarks. The momentum distribution of the quarks corresponding to large $\omega$ is given in terms of the requirements of Regge behavior. The model is compatible with the measurements of $\nu W_2$ for the proton and neutron for values of $\omega > 2$. It has difficulty at smaller values of $\omega$ as it predicts $\nu W_2^N/\nu W_2^P \approx 2/3$ in this region whereas the measured result appears to fall to as low a value as $0.35 \pm 0.07$ at $\omega \approx 1.25$. As this model is a simple impulse approximation description, it has been pointed out that the presence of two-particle correlations could account for the observed ratio in the neighborhood of $\omega = 1$. The quark model and, more generally, the light cone algebra based on the quark model, are not compatible with a ratio less than 0.25 even with the inclusion of such correlations (40).

A recent calculation by Lee & Drell (41) provides a fully relativistic generalization of the parton model that is no longer restricted to infinite momentum frames. This theory obtains bound state solutions of the Bethe-Salpeter equation for a bare nucleon and bare mesons, and connects the observed scale invariance with the rapid decrease of the elastic electromagnetic form factors.

**Sum rules.**—Excellent tests of applications of the parton model are provided by sum rule evaluations. From weighted integrals of $\nu W_2(\omega)$ with respect to $\omega$, sum rules can be derived on the assumption that the nucleon’s momentum is, on the average, equally distributed among the partons. Two important sum rules, which can be evaluated for neutrons and protons, are:

\[
I_1 = \int_1^\infty \nu W_2(\omega) \frac{d\omega}{\omega^2} = \sum_N P(N) \frac{\left< \sum_{i=1}^N Q_{i}^2 \right>}{N}
\]

\[
I_2 = \int_1^\infty \nu W_1(\omega) \frac{d\omega}{\omega} = \sum_N P(N) \left< \sum_{i=1}^N Q_{i}^3 \right>
\]

where $I_2$ is the weighted sum of the squares of the parton charges and $I_1$ is the
mean square charge per parton. The sum $I_2$ is equivalent to a sum rule derived by Gottfried (42) who showed that for a proton which consists of three nonrelativistic point-like quarks $I_2^P$ equals 1. The experimental value of the integral when integrated over the range of the SLAC-MIT data gives:

$$I_2^P = \int_1^{20} \frac{d\omega}{\omega} \nu W_2^P = 0.78 \pm 0.04$$

where the integral is cut off for $\omega > 20$ because of insufficient information about $R$. Since the experimental values of $\nu W_2$ at large $\omega$ do not exclude a constant value, there is some suspicion that this sum might diverge. This would imply that in the quark model the scattering has to occur from an infinite sea of quark-antiquark pairs as well as from the three valence quarks. Table 3 gives a summary of the comparison of the experimental values of the sum rules with the predictions of various models. Unlike the previous sum, the experimental value of $I_1$ is not very sensitive to the behavior of $\nu W_2$ above $\omega > 20$. The experimental value is about one

**TABLE 3. Sum Rule Results—Theory and Measurement**

<table>
<thead>
<tr>
<th>Expected Value$^a$</th>
<th>Measurement $^b$</th>
<th>$\omega_m$</th>
<th>$q^2$(GeV/c)$^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_1^P$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 Quark $^c$ 3 Quark + 'Sea'</td>
<td>0.159±.005</td>
<td>20</td>
<td>1.0</td>
</tr>
<tr>
<td>1/3 2/9 + $^d$ 3$\langle N\rangle$</td>
<td>0.165±.005</td>
<td>20</td>
<td>1.0</td>
</tr>
<tr>
<td>$I_1^N$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2/9</td>
<td>2/9</td>
<td>0.120±.008</td>
<td>20</td>
</tr>
<tr>
<td>0.115±.008</td>
<td>20</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>0.107±.009</td>
<td>12</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>$I_2^P$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 1/3 + 2$\langle N\rangle$/9</td>
<td>0.739±.029</td>
<td>20</td>
<td>1.0</td>
</tr>
<tr>
<td>0.761±.027</td>
<td>20</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>0.780±.04</td>
<td>20</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>0.607±.021</td>
<td>12</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>$I_2^N$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2/3 2$\langle N\rangle$/9</td>
<td>0.592±.051</td>
<td>20</td>
<td>1.0</td>
</tr>
<tr>
<td>0.584±.050</td>
<td>20</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>0.429±.036</td>
<td>12</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>$I_2^P - I_2^N$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/3</td>
<td>0.147±.059</td>
<td>20</td>
<td>1.0</td>
</tr>
<tr>
<td>0.177±.057</td>
<td>20</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>0.178±.042</td>
<td>12</td>
<td>2.0</td>
<td></td>
</tr>
</tbody>
</table>

$^a$ $\langle N\rangle$ expectation value of number of quarks, $N$.

$^b$ Data from Kendall (6), and (16) and (19), except where noted.

half the value predicted on the basis of the simple three-quark model of the proton, and it is also too small for a proton having three valence quarks in a sea of quark-antiquark pairs. The Kuti-Weisskopf (35) model which includes neutral gluons in addition to the valence quarks and the sea of quark-antiquark pairs predicts a value of $I_2^P$ that is compatible with this experimental result.

The difference $I_2^P - I_2^N$ is of great interest because it is presumed to be sensitive only to the valence quarks in the proton and the neutron. The values of neither $I_2^P$ nor $I_2^N$ depend on the properties of gluons, and assuming that the quark-antiquark sea is an isotopic scalar, the effects of the sea cancel out in the above difference, giving $I_2^P - I_2^N = 1/3$. Unfortunately it is difficult to extract the correct value from the data because of the importance of the behavior of $\nu W_2$ at large $\omega$. If one extrapolates $\nu W_2^P - \nu W_2^N$ toward $\omega \to \infty$ for $\omega > 12$ with the asymptotic dependence expected on the basis of Regge theory $(1/\omega)^{1/2}$, one obtains a rough estimate of $I_2^P - I_2^N = 0.22 \pm 0.07$. This is not incompatible with the expected value. However, it is not clear that the form of extrapolation is correct nor at what value of $\omega$ the leading Regge term becomes dominant if it is correct. A light cone model by suri & Yennie (43) predicts the difference $\nu W_2^P - \nu W_2^N$ to be proportional to $\omega^{-1}$. Such a dependence would not permit $I_2^P - I_2^N$ to reach $1/3$.

On the basis of a formal study of quark models, Llewellyn-Smith (44) has shown that when the interactions between quarks are due to scalar or pseudoscalar fields there is an upper bound

$$I_2^P + I_2^N \leq \frac{5}{9}$$

This upper bound appears to be experimentally satisfied unless some unexpected behavior occurs in unexplored regions of $\omega$.

Bjorken (45) pointed out that the Adler sum rule for inelastic neutrino scattering derived from the commutator of the two time components of the weak current leads to the following constant $q^2$ inequality for inelastic electron scattering.

$$\int_{q^2/2M}^{\infty} d\nu [W_2^P(\nu, q^2) + W_2^N(\nu, q^2)] \geq \frac{1}{2}$$

This inequality is satisfied at $\omega \approx 5$.

**Vector dominance models.**—An early dynamical description of the process was given by Sakurai (46) in terms of the vector dominance model (VDM). This model gives for large $\nu$:

$$\sigma_t(q^2, W) = \left[ \frac{M_e^2}{M_e^2 + q^2} \right]^2 \sigma_{\gamma p}(W)$$

and

$$R = \frac{\sigma_s(q^2, W)}{\sigma_t(q^2, W)} = \left( \frac{q^2}{M_e} \right) \left[ 1 - \frac{q^2}{2M_\nu} \right] \xi(W)$$
where \( M_v \) is the mass of the vector meson, taken to be the \( \rho \) meson; \( \xi(W) \) is the ratio of the total cross sections for vector mesons with helicity states 0 and \( \pm 1 \) respectively incident on nucleons. This result implies scale invariance for \( \nu W_2 \) in the large \( \nu \), fixed \( \omega \) limit and gives the characteristic diffractive limit that \( \nu W_2 \) equals a constant as \( \omega \to \infty \). This prediction of a strong dependence of \( R \) on \( q^2 \) is in gross disagreement with experiment in the kinematic region in which \( \sigma_{\gamma} \) and \( \sigma_f \) have been separately determined, as shown in Figures 12 and 13. This difficulty would disappear if \( \xi(W) \) were zero; however, such a constraint produces the wrong \( q^2 \) dependence for \( \nu W_2 \) and would be surprising from general considerations. Cho & Sakurai (47) have suggested that this model should only have validity in the pure diffractive region, say for \( \omega \geq 10 \), where \( R \) is not yet measured. However, \( R \) has been measured up to about \( \omega \approx 9 \) with no \( q^2 \) dependence apparent.

The role of \( \rho \) dominance in inelastic electron scattering has been further studied by an experimental investigation of the \( A \) dependence of the process by the MIT-SLAC group (48). Electron scattering from Be, Cu, and Au was studied for \( \theta = 6^\circ \) at incident electron energies \( E = 4.5, 7, 10, 13.5, 16, \) and 19.5 GeV, for values of \( E' \) at and above 2.5 GeV. Over this span of data \( \nu \) varied from 0.1 GeV to 17 GeV, and \( q^2 \) varied from 0.10 (GeV/c)^2 to 3.7 (GeV/c)^2. Very general considerations of VMD suggests that at low \( q^2 \) (here meaning \( q^2 \lesssim M^2 \)), and large \( \nu \), the "hadronic" cloud associated with a physical photon should manifest itself as a cross-section dependence on \( A \) less than linear.

A comparison is made of the results with a simple \( \rho \)-dominance calculation by Brodsky & Pumplin (50). They assume a \( \rho \)-nucleon total cross section that is independent of energy. To facilitate the comparison a shadow factor \( S \) is defined:

\[
S = \sigma_{\text{nucleus}} / (Z \sigma_p + N \sigma_n)
\]

where the cross sections are doubly differential and evaluated at matching kinematic points. The quantity \( Z \sigma_p + N \sigma_n \) is evaluated by an appropriate combination of the measured hydrogen and deuterium cross sections so that the final comparison essentially constitutes a determination of the \( A \)-dependence of the shadowing relative to what occurs in deuterium. Deviations of \( S \) caused by smearing are less than 3% in the kinematic region considered here.

The values of \( S \), for two bands about \( q^2 = 0.5 \) and 1.0 GeV^2, are shown in Figures 26, 27, and 28 as functions of \( \nu \). The appropriate predictions of Brodsky & Pumplin are shown as solid lines. There is no consistent deviation of any of the data from \( S = 1 \) and in particular there is substantial disagreement with the predictions (50). A model by Yennie, suri & Spital (51) inserts into the calculation of \( S \) a non-VDM and VDM part each of unknown strength. A fit to the data gives a ratio of VDM to non-VDM contributions to the interaction amplitude of 0-0.10. This requires a drastic reduction in the forward \( \rho \) electroproduction amplitude predicted by a vector dominance model which fits the real photo-absorption cross section.

These results are equivalent to the statement that an evaluation of the neutron-proton cross section ratios deduced from the heavy target data with the assumptions just mentioned are the same as those displayed in Figures 5(a) and 5(b) from...
The shadowing factor $S$, defined in the text, as a function of energy loss $\nu$ for inelastic electron scattering from beryllium. A discussion of this data and of the theoretical predictions is included in the text. Data from (48).

deuterium after appropriate smearing. A direct comparison indicates this is indeed the case within the errors.

Attempts to save the vector dominance point of view have been made by Tung (52) and Wataghin (53) who have introduced generalizations of the VDM. Other approaches (54, 55) have been to apply the model with the extension of the vector meson spectral function to higher masses. While a number of different approaches can be taken to patch up the VDM theory, it is clear that the simple theory is in difficulty.

Regge models.—Another approach relates inelastic scattering to virtual forward Compton scattering which is described in terms of Regge exchange. In the Regge limit $\nu \to \infty$, with $q^2$ fixed, this gives

$$W_1(q^2, \nu) \to \beta_1(q^2)\nu^{\alpha(0)}$$

$$W_2(q^2, \nu) \to \beta_2(q^2)\nu^{\alpha(0)-2}$$

where $\alpha(t)$ is the leading trajectory. It is expected that the leading trajectory is that of the Pomeron with $\alpha(0) = 1$. Abarbanel, Goldberger & Treiman (56), and also...
Harari (57) have conjectured that the leading Regge pole at finite $q^2$ continues to dominate as $q^2 \to \infty$ with $\nu/q^2$ fixed and very much greater than unity. The requirements of scale invariance makes the following constraints:

$$\beta_1(q^2) \to (q^2)^{-\alpha(0)} \quad \text{and} \quad \beta_2(q^2) \to (q^2)^{1-\alpha(0)}$$

in this limit. Since $\alpha(0)=1$ for the leading trajectory it follows that $W_2(\omega)$ is proportional to $\omega$ and $\nu W_2(\omega)$ is a constant. An investigation of a simple model by Abarbanel, Goldberger & Treiman (56) produced residue functions which were sufficiently close to this requirement to lead to the hope that it would be obtained in a more complete model. In the detailed application of the Regge theory to this process, discussed by Moffat & Snell (58), Pagels (59), Akiba (60) and Breidenbach & Kutii (26), scale invariance is inserted into the theory by choosing the residue functions to have the required behavior. The following functional behavior is obtained for large $\omega$:

$$\nu W_2 = \sum_{p,p',A,q,\ldots} b_{i,\omega} \omega^{\alpha - 1}$$
where the sum includes all trajectories that couple to the photon, such as the Pomeron, the $p'$, and $A_2$ trajectories. The quantities $\alpha_i$ are the intercepts of the trajectories at $t=0$, and the quantities $b_i$ are unknown constants that can be determined from the measurements. The $A_2$ trajectory is required since isospin exchange must be included to account for the observed difference between structure functions of the proton and neutron at small $\omega$. An important consequence of this approach is that $\nu W_s^P - \nu W_s^N$ is predicted to be proportional to $\sqrt{1/\omega}$ for large $\omega$. Unfortunately the data are not sufficiently precise at large $\omega$ to test this prediction.

_Duality._—The observed difference between $W_s^P$ and $W_s^N$ points to the existence of significant nondiffractive contributions to the forward Compton amplitude for virtual photons. This leads to the application of the concept of duality to inelastic electron scattering. In the usual “two component” duality picture, diffractive scattering at high energies is related to the low energy background, while nondiffractive $t$-channel exchanges are connected to low energy resonances. Finite energy sum rules have been proposed by Leutwyler & Stern (61) to test this connection.

**FIGURE 28.** The shadowing factor $S$, for inelastic electron scattering from gold. See caption to Figure 26.
Bloom & Gilman (62) have pointed out that low energy resonance electroproduction has a behavior which is correlated with that of the deep inelastic scattering. Qualitatively they expect that in the scaling limit of $q^2 > 1$, the scaling function $F_2(\omega')$ should, on the average, follow the resonances and their associated low energy background, at least for values of $q^2 > 1$. Bloom & Gilman have quantitatively tested their assertion with a constant $q^2$ finite energy sum rule:

$$\frac{2M}{q^2} \int_0^{W_{\text{max}}} d\nu \nu W_2(\nu, q^2) = \int_1^{1+\frac{W_{\text{max}}^2}{q^2}} d\omega' F_2(\omega')$$

where $W_{\text{max}}$ has been chosen above the region of prominent resonances and

$$\nu_{\text{max}} = \frac{W_{\text{max}}^2 - M^2 + q^2}{2M}$$

The evaluations (25) of this sum rule with interpolated experimental values at fixed $q^2$ indicate that it is satisfied to better than 10% for a range of $q^2$ from 1 to 4 (GeV/c)^2 and for $W_{\text{max}}$ from 2.2 to 2.5 GeV. The authors, on the assumption of local duality, suggest that:

$$\frac{\nu W_2^N}{\nu W_2^P, \omega' \rightarrow 1} \left( \frac{\mu_N^0}{\mu_F^0} \right) = 0.47$$

The ratio 0.47 is not favored by the recent measurements (see Figure 22) but is also not excluded by them. In an extension of the above work, Briedenbach & Kuti (26) have found that an excellent fit to the data both inside and outside the resonance region for $q^2 > 1$ can have the form:

$$\nu W_2 = F_2(\omega') \left[ R(\omega') + BG(\omega') \right]$$

where $R(\omega')$ is the sum of Breit-Wigner forms for the prominent bumps and $BG(\omega')$ describes a smooth background term added to the resonances. For $\omega' > 2$ GeV, $R(\omega')$ approaches zero and $BG(\omega')$ approaches unity leaving $\nu W_2$ equal just to the scaling function $F_2(\omega')$. To show that the essential physics does not depend on the particular scaling variable used, the fitting procedure was also done with $\omega_L$. The fit was equally good and also leads to satisfactory finite energy sum rules.

While the Bloom & Gilman sum rule is expected to hold only in the region of $q^2 > 1$, Rittenberg & Rubinstein (24) have proposed a finite energy sum rule that is conjectured also to hold at low $q^2$ and even to the limit of real photons using a different scaling variable,

$$\omega_R = \frac{2M_\nu + M^2}{q^2 + A^2}$$

where $A^2$ is a constant chosen to maximize the goodness of fit of the data to the
scaling curve. The evaluation of this sum rule for the range of $1.0 \lesssim q^2 \lesssim 0.3 \text{ GeV}$ gives reasonably good agreement with predictions. [See especially the detailed comparison in (63)].

Resonance models.—Attempts have been made to build the structure functions from direct channel resonances. While the form factor for the excitation of any specific resonance might be expected to go to zero rapidly for large $q^2$, it has been shown that the scale invariance of $\nu W^2$ can be achieved in such a model if the total number of resonances excited increases sufficiently rapidly with $W$. Landshoff & Polkinghorne (64) have constructed a Veneziano-like model which produces scale invariance by incorporating an infinite number of resonances. When combined with the quark-parton model (65), this point of view allows a complete calculation of the nondiffractive contributions to the structure functions, essentially without free parameters. The predictions for $\nu W^2_P$ for $\omega < 3$ are in reasonable agreement with the data; however the model falls well below the data for larger $\omega$, a result which is probably due to the absence of diffractive components in the model. While the calculated difference $\nu W^2_P - \nu W^2_N$ is in fair agreement with experiment, the prediction that $\nu W^2_N/(\nu W^2_P) = 2/3$ disagrees with the measurements near $\omega = 1$ where the nondiffractive part of the amplitude might be expected to dominate. Atkinson & Contogouris (66) have extended the Landshoff & Polkinghorne model to include a diffractive part and get better agreement with measurements of $\nu W^2_P$ at large $\omega$.

A different type of resonance dominance model has been worked out by Domokos, Kovesi-Domokos & Schonberg (67) who built the structure functions from an infinite series of $N^*$ and $\Delta$ resonances. The predictions for $\nu W^2_P$ are in good agreement with experiment for values of $\omega \lesssim 7$ but fall below experiment at larger $\omega$, probably because of the absence of diffractive contributions to the structure functions. The value of $\nu W^2_N/\nu W^2_P$ near $\omega = 1$ is about 0.70–0.78, which is in disagreement with experiment.

Models that predict deviations from scaling.—A number of models predict deviations from scaling, among which are the following. On the basis of a model which postulates parton clusters, Wilson (68) has conjectured that scaling will break down somewhere in the range of $q^2$ between 8 and 30 (GeV/c)$^2$. The calculation by Cheng & Wu (69) of the high energy behavior of scattering amplitudes in quantum electrodynamics when extended to virtual photons show that scale invariance should be violated. The deviation from scaling arises from terms in $\nu W^2$ like $\ln(q^2/\lambda^2)$ where $\lambda$ is the mass of the exchanged vector mesons. If $\lambda$ is taken to be the $\rho$ mass, the sizes of these logarithmic terms are large enough to produce observable violations of scaling behavior at SLAC energies. Such violations are not observed. A super-eikonal model by Fried & Moreno (70) predicts a weak violation of scaling. The precision of the present measurements is probably not great enough to discern this predicted deviation. Weak violation predictions also resulted from the work of Fishbane & Sullivan (71) who studied $\nu W^2$ in field theory model without a transverse momentum cutoff. A general discussion
of the problems associated with scale invariance has been given by Gell-Mann (72). It should be emphasized that the current measurements are consistent with scale invariance only to within their errors which, with the inclusion of the uncertainties in the radiative corrections, range from 5 to 10%. Models which introduce a violation of scale invariance within this range constitute no contradiction with experiment and tests of their validity require measurements at higher energies.

XI. Related Physics

Deep inelastic electron scattering has been related in a number of theoretical investigations not only to other electromagnetic processes but also to weak and strong interaction physics.

Neutrino-hadron scattering.—Bjorken (73) has conjectured that the three structure functions which describe inelastic neutrino-nucleon scattering [(74) gives a summary of formalism], \( \nu+n \rightarrow \mu+ \) hadrons, also asymptotically manifest scale invariance. The doubly differential cross section is described in terms of the structure functions \( W_1, W_2, W_3 \), where the + and - correspond to neutrino and anti-neutrino interactions.

\[
\frac{d^2 \sigma}{d \Omega d E'} = \frac{G^2(E')}{2\pi^2} \left[ 2 \sin^2 \theta/2 W_1^\pm(\nu, q^2) + \cos^2 \theta/2 W_2^\pm(\nu, q^2) \right.
\]

\[
\left. + \frac{E + E'}{M} \frac{\sin^2 \theta/2 W_3^\pm(\nu, q^2)}{\sin^2 \theta} \right]
\]

In the limit in which both \( \nu \) and \( q^2 \) approach infinity, with \( \omega \) fixed, it was argued that the structure functions approach the following limiting behavior

\[
W_1^\pm(\nu, q^2) \rightarrow F_1^\pm(\omega)
\]

\[
\frac{\nu}{M^2} W_2^\pm(\nu, q^2) \rightarrow F_2^\pm(\omega)
\]

\[
\frac{\nu}{M^2} W_3^\pm(\nu, q^2) \rightarrow F_3^\pm(\omega)
\]

There have not yet been separate experimental determinations of these structure functions; however, there has been some evidence for scaling behavior. On the basis of scale invariance, the total cross section is predicted to rise linearly with energy, a prediction in agreement with current data. The data from the CERN heavy liquid bubble chamber (75) give \( \sigma = (0.52 \pm 0.13) G^2 ME/\pi \) per nucleon. A second direct consequence of scaling is that the average \( q^2 \) is predicted to be proportional to \( \nu \), verified in reference (76).

The applications of current algebra and the parton model lead to a number of relations (77) between the electromagnetic and weak structure functions in addition to relations between the weak structure functions themselves. For example,
the existence of nondiffractive contributions to the electromagnetic structure functions suggests a difference in the neutrino and anti-neutrino cross sections. In the parton model, $W_3$, which arises from the interference of the vector and axial vector currents, is a measure of the number of baryon partons minus the number of anti-baryon partons. In general, $W_3$ and $(W_{1,2}^+ - W_{1,2}^-)$ have no diffractive parts and are expected to approach zero in the limit of large $\omega$.

**Electron-positron colliding beams.**—On the basis of crossing symmetry, the process $e^- + e^+ \rightarrow p$ (or $\bar{p}$)+ hadrons has been related to deep inelastic scattering (33). A number of different approaches (73, 78, and 79) have suggested that the total cross section for $e^+ - e^-$ collisions should exhibit a point-like behavior and thus have a $1/E^2$ energy dependence. Results from the Frascati colliding beam measurements appear to confirm the predicted large cross sections (80).

**Inelastic Compton scattering.**—Inelastic Compton scattering of photons from protons, $\gamma + p \rightarrow \gamma' + \text{hadrons}$ has been studied by Bjorken & Paschos (32, 81) within the parton model. At large momentum transfers they find that this cross section is related to the inelastic electron cross section by the following expression:

$$\left( \frac{d^3\sigma}{d\Omega dE'} \right)_{\gamma p} = \frac{\nu}{EE'} \left( \frac{d^3\sigma}{d\Omega dE'} \right)_{e\nu} \left( \sum Q_i^\gamma \right) \left( \sum Q_i^\nu \right),$$

where the brackets signify mean values. Measurements of this cross section can in principle distinguish between fractionally and integrally charged constituents in a parton model.

**Polarization effects.**—A number of predictions of polarization effects expected in deep inelastic scattering of leptons have been made, some of which may be subject to test. Studies have been made by Bjorken (82), in the framework of equal time current commutators, and in the quark-parton model, by Gálfí et al (83), and by Kuti & Weisskopf (35). Nash (84) has completed similar studies in the framework of the quark-parton model of Landshoff, Polkinghorne & Short (85).

Sum rules from the light-cone structure of the spin-dependent scaling functions and studies in cutoff field theories have been reported by Gálfí et al (86). Niedermayer (87) has applied the quark light cone algebra of Fritzsch & Gell-Mann (88) [see also Cornwall & Jackiw (89)] as have Dicus, Jackiw & Teplitz (90), Wray (91), and Hey & Mandula (92).

Domokos, Kovesi-Domokos & Schonberg (93), and Close & Gilman (94) have applied the resonance model to polarized electron scattering. The Regge analysis of spin-dependent forward virtual Compton scattering can be found in (86).

Kinematic inequalities (86, 95), the application of finite energy sum rules to spin-dependent scaling phenomena [for a summary paper see (96)], and a general review of the rapidly growing field (97) is reported by Kuti et al.
Carlson & Tung (98) have shown how simultaneous measurements of the longitudinal and transverse polarization effects in inelastic lepton-hadron scattering can be used to determine the two spin-dependent structure functions which in turn may be used to test experimentally the consequences of assuming scale invariance of the leading light cone singularities.

The hyperfine splitting ($hfs$) in atomic hydrogen is an interesting bridge between the usually disconnected fields of high-energy and precision atomic physics. For an excellent survey of the literature see Brodsky & Drell (99). The size of the proton polarizability contribution to the $hfs$ from two-photon exchange ($\delta_{POL}$) was a longstanding problem subject to many discussions (99). Recently, Gnädig & Kuti (100) have shown that positivity conditions when evaluated in terms of the SLAC-MIT data put tight upper and lower bounds on the unknown part of $\delta_{POL}$. A calculation of $\delta_{POL}$ in the resonance model was carried out by Jensen, Kovesi-Domokos & Schonberg (101).

**Lepton pairs from hadronic interactions.**—The process $p+p\rightarrow l+\bar{l}+\text{hadrons}$ has been investigated in a number of different models (102). Light cone (103), quark-parton (35, 81) and dual-quark parton models (104) give a qualitatively correct description of data from measurements of $p+$nucleon$\rightarrow\mu^+\mu^-+\text{hadrons}$ (105).

**Electromagnetic mass differences of hadrons.**—The electromagnetic mass differences of hadrons are sensitive to the hadronic internal structures, and the deep inelastic results using Cottingham's dispersion integral (106) have been employed to gain some understanding of these differences. A number of theoretical difficulties which still persist have so far prevented a full understanding of the origin of these differences (107-9). For a review of the problems see (110).

**Relationships between inelastic $e-p$ and $p-p$ scattering.**—Attempts have been made to relate inelastic $ep$ and $pp$ scattering. Following the suggested relationship for elastic scattering by Wu & Yang (111), which was further developed by Abarbanel, Drell & Gilman (112), Berman & Jacob (113) studied the connection between the $ep$ and $pp$ inelastic cross sections at large momentum transfers and large energy losses, and Low & Treiman (114) and Berman, Bjorken & Kogut (115) have shown that electromagnetic processes may dominate $pp$ scattering at high transverse momentum transfers. Allaby et al (116) have also studied a possible relationship. Benecke et al (117), in proposing the hypothesis of limiting fragmentation, provide connections between the two processes, and the general ideas of constituent structure and scaling have been used by Feynman (118) and Bjorken (71) to make speculations about hadron reactions.

An application of the fragmentation model to inelastic electron or muon scattering which related hadronic cross sections and electroproduction structure functions has been carried out by Choudhury & Rajaraman (119). A fragmentation model has been studied by Nieh & Wang (120) who predict deviations from scaling, an expectation which will be checked in lepton-proton scattering studies to be carried out at the National Accelerator Laboratory (121).
Photoproduction.—Jaffe (122) has shown that parton-pair annihilation may contribute to the photoproduction of muon pairs of large invariant mass off protons. The contribution can, in principle, be distinguished from background muons from Bethe-Heitler processes and, if found to scale as is predicted, the measurements can be interpreted in terms of the photon's structure.

The production of \( W \) bosons may occur through deep inelastic scattering of electrons or muons and through photoproduction where the boson comes from a hadronic vertex. The photoproduction can occur elastically or inelastically, with a number of hadrons in the final state. The elastic production may compare favorably with neutrino induced reactions in the production of this elusive particle (123). The deep inelastic electron scattering results we have discussed suggest that the inelastic photoproduction may also contribute. A parton-quark model prediction of the cross sections has been carried out by Mikaelian (124).

XII. CONCLUSIONS

The deep inelastic electron scattering measurements have established the validity of scale invariance for the proton and neutron over a wide kinematic range, the relative smallness of \( \sigma_e \) compared to \( \sigma_p \), and the nature of the differences between the proton and neutron structure functions. These features have suggested that the nucleon may be composed of point-like constituents, and theories based on this assumption have been offered as possible explanations of the results. More fundamentally the scattering results have been related to the behavior of the product of two currents at short distances and near the light cone. Both from the point of view of constituent structure and behavior of currents near the light cone, predictions have been made for a wide variety of reactions including neutrino scattering, muon pair production, \( e^+e^- \) annihilations, and aspects of hadron-hadron interactions. Only a fraction of these predictions have been tested by comparison with experiment.

It is evident that neither the body of experimental results on deep inelastic lepton scattering nor the related theoretical developments can be considered at all complete. In particular, one wishes to know for both the proton and the neutron: (a) to what values of \( \nu \) and, independently, \( q^2 \), scale invariance will persist; (b) what is the value of \( W_{2N}/W_{2P} \) as \( x \to 1 \); (c) the details of how \( (W_{2P} - W_{2N}) \) decreases as \( \omega \to \infty \); (d) the values of \( R_P \) and \( R_N \) for large \( \nu \) and \( q^2 \); (e) the behavior of spin-dependent effects in deep inelastic scattering; and lastly, (f) what are the particles and their distributions in the final hadronic states. Answers to these experimental questions will help select between the numerous theoretical models presently available. Varied predictions for \( a \), \( b \), and \( c \) have been discussed in this review. Some experimental results have appeared [for a recent summary of the coincidence electroproduction experiments, see (125)], and a number of experiments are underway to study the nature of the final hadronic states at SLAC (126), Cornell (127), and DESY (128). Extensive discussions and reviews of hadronic final state properties have been presented by Bjorken (71) and by Llewellyn-Smith (129).
In summary, one sees that the deep inelastic results have touched a variety of the unanswered questions in the physics of the weak, electromagnetic, and hadronic interactions of hadrons and have stimulated substantial theoretical inquiry into the nature of hadronic constituent structure. Many problems have been raised by the theories which have attempted to account for the results. Clarification of many of these problems and additional insights should emerge from the programs of measurements now planned or underway.
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