# PHY 504

## Problem Set #2

due 11 September 2008

1. Two particles interact via a potential $V(r_{12})$, where $r_{12} = |\mathbf{r}_1 - \mathbf{r}_2|$ is the interparticle distance. Suppose that all external torques and forces vanish.

   (a) Show that the kinetic energy $T$ can be expressed as the sum of two terms: the center-of-mass kinetic energy and a second term depending only on $\mathbf{r}_{12}$ and its time derivative. Do the same for the angular momentum $L$.

   (b) Write down $T$ and $L$ in the center-of-mass frame. Suppose that the angular momentum is in the $z$-direction. Explain why the motion is then all in the $xy$-plane. Also write the kinetic energy and angular momentum in 2D polar coordinates.

   (c) Still working in the center-of-mass frame, write down and simplify the equations expressing conservation of angular momentum and energy.

   (d) Describe the motion *in words* as completely as possible, if the two particles are subject to a rigid-body constraint, for generic initial conditions.

2. A single particle moves in the $xy$-plane, in a central potential $V(r)$.

   (a) Using Newton's 2nd Law, write down the equations of motion for the particle in polar coordinates.

   (b) Write the total energy $E$ and angular momentum $L$ in polar coordinates. Obtain two first-order equations for $r$ and $\theta$ in terms of $E$ and $L$. Eliminate $\theta$ from the equation for $r$.

   (c) Show that the equations found in (a) follow from those in (b).

   (d) Obtain integral expressions for $r(t)$ and $\theta(t)$.

3. Mathematica problem:

   (a) Use the command ParametricPlot to plot a circle parameterized by $\theta$. The option **`AspectRatio->Automatic`** may be useful.

   (b) Using Mathematica, evaluate the integral expressions found in 2(d), for the case $V(r) = \frac{1}{2}(r - r_0)^2$, to find $t(r)$.

   (c) Plot $t(r)$, for a reasonable range of values.