PHY 504

Problem Set #3

due 18 September 2009

1. Derivation 1.10.

2. A simple pendulum consists of a point mass *m* suspended from a pivot by a rigid massless rod of length *L*, which pivots along a fixed axis.
   (a) Write down a Lagrangian and obtain Lagrange's equations from it.
   (b) Simplify these equations in the limit of small displacements from equilibrium and solve.
   (c) Solve the original equations numerically using Mathematica. Make three plots, for qualitatively different sets of initial conditions. Here is a sample code to numerically solve an ordinary differential equation and plot the solution:

```
solution = NDSolve[{y''[x] == -y[x], y[0] == 0, y'[0] == 1}, y, {x, 0, 10}];
Plot[y[x] /. solution, {x, 0, 10}];
```

3. Consider the double pendulum shown in Fig. 1.4. Let l_1 and l_2 be the lengths of the (massless) rods and let m_1 and m_2 be the two masses.
   (a) Write down generalized coordinates and constraints (if necessary).
   (b) Obtain a Lagrangian and equations of motion for the generalized coordinates.
   (c) Simplify these equations in the limit of small displacements from equilibrium.

4. Consider the 1-dimensional harmonic oscillator with $L = \frac{1}{2} mv^2 - \frac{1}{2} kx^2$
   (a) Derive the oscillation frequency ω.
   (b) Calculate the action for a general solution to the equations of motion which starts and ends at the point $x = 0$, over a single period $T = 2\pi/\omega$.
   (c) Now calculate the action for an *arbitrary* path between the same endpoints, which can be written as a Fourier series:

$$x(t) = \sum_{n=1}^{\infty} a_n \sin(n\omega t)$$

   Express your answer in terms of the Fourier coefficients a_n.
   (d) What sort of extremum is the path in (b)? (*i.e.* is it a maximum, minimum, or saddle point? Is it isolated?)