PHY 504

Problem Set #4

due 25 September 2009

1. Goldstein Exercise 2.12. Show that the Lagrangian at the end of the problem differs from a more familiar Lagrangian by a term which is a total time derivative. Assuming the result of Derivation 1.8, what can you conclude?
2. A particle of mass m slides in a frictionless trough of parabolic shape $y = kx^2$, under the influence of a gravitational force $\mathbf{F} = -mg\hat{\mathbf{y}}$.
   (a) Write down a Lagrangian for the particle's motion, using the method of Lagrange multipliers.
   (b) Obtain the Lagrange equations of motion and constraint. Eliminate y and the Lagrange multiplier λ to obtain an equation just involving $x(t)$.
   (c) Solve for $x(t)$ in the limit of small x and $\dot{x}$.
3. Two particles of mass m, connected by a spring of force constant k, are constrained to move on two parallel planes separated by a distance a. Write down the Lagrangian, using Lagrange multipliers. What are the symmetries and conserved quantities? Solve for the motion, for arbitrary initial conditions.
4. Write down a Lagrangian for each of the following situations, in terms of an appropriate set of generalized coordinates. Use Lagrange multipliers as needed. What are the symmetries and corresponding conserved quantities?
   (a) A particle of mass m moving on the frictionless surface of an infinite cylinder.
   (b) Two particles of mass m moving through empty space, interacting gravitationally with each other. (Each particle experiences only the gravitational force of the other particle.)
   (c) Two particles constrained to move on the surface of a sphere, interacting gravitationally with each other.
   (d) A charged particle moving in a uniform electric field.
   (e) A charged particle moving in a uniform magnetic field.