

# PHY 504

## Problem Set #5

due 7 October 2009

1. For the inverse-square potential

$$V(r) = -\frac{k}{r^2}$$

- (a) show that the orbits can have the following forms, depending on the energy  $E$  and angular momentum  $l$ :

$$\begin{aligned} \frac{1}{r} &= A \cos \beta(\theta - \theta_0) & \frac{1}{r} &= A(\theta - \theta_0) \\ \frac{1}{r} &= A \cosh \beta(\theta - \theta_0) & \frac{1}{r} &= \frac{1}{r_0} e^{\pm\beta\theta} \\ \frac{1}{r} &= A \sinh \beta(\theta - \theta_0) & & \end{aligned}$$

- (b) For what  $E$  and  $l$  does each of the above types of motion occur? Express the constants  $A$  and  $\beta$  in terms of  $E$  and  $l$  for each case. Plot a typical orbit of each type using Maple.

2. A particle follows a closed orbit in a central field  $V = -ar^{n+1}$ .

- (a) What is the radius and period of a circular orbit in this potential?
- (b) Suppose the orbit is nearly circular. Find the frequency of oscillations in the radial direction as a function of its energy and angular momentum, by expanding the effective radial potential about its minimum to quadratic order in  $s = r - r_0$ . Compare to the orbital frequency found in (a).
- (c) For which values of  $n$  are closed nearly-circular orbits possible?

3. The gravitational potential of the sun takes the form

$$V(r) = -\frac{k}{r} - \frac{\varepsilon}{r^3}$$

where  $\varepsilon = kl^2/mc^2$  and  $k = GMm$ . The second term gives the leading general relativistic correction to the Newtonian potential, and contributes to the precession of the perihelion of Mercury

- (a) Find the frequency of oscillations about a nearly circular stable orbit for this potential.
- (b) Estimate the precession of the perihelion of Mercury in arc seconds per century. You can find some of the needed numbers on p. 538 of GPS.

4. Consider scattering of particles off a hard sphere of radius  $a$ . Treat the sphere as a radial potential

$$V(r) = \begin{cases} \infty & r < a \\ 0 & r > a \end{cases}$$

- (a) Describe the scattering orbits using simple geometric arguments. Determine the scattering angle as a function of impact parameter.
- (b) Calculate the differential and total cross sections, again using geometric arguments. Interpret your answer for the total cross section.