

PHY 504

Problem Set #5

due 1 October 2010

1. A particle of mass m moves in a uniform magnetic field of magnitude B .
 - (a) Write down a Lagrangian describing this system and the resulting equations of motion.
 - (b) What are the symmetries? What conserved quantities are associated to each?
 - (c) Use Noether's theorem to obtain expressions for the conserved energy, momentum, and angular momentum in the plane perpendicular to the magnetic field. Write these in both rectangular and polar coordinates.
 - (d) Explain how to solve for the particle's motion. Obtain the general solution for an orbit in the perpendicular plane of energy E and angular momentum l .

2. For the inverse-square potential $V(r) = -k/r^2$

- (a) show that the orbits can have the following forms, depending on the energy E and angular momentum l :

$$\begin{aligned}\frac{1}{r} &= A \cos \beta(\theta - \theta_0) & \frac{1}{r} &= A(\theta - \theta_0) \\ \frac{1}{r} &= A \cosh \beta(\theta - \theta_0) & \frac{1}{r} &= \frac{1}{r_0} e^{\pm\beta\theta} \\ \frac{1}{r} &= A \sinh \beta(\theta - \theta_0)\end{aligned}$$

- (b) For what E and l does each of the above types of motion occur? Express the constants A and β in terms of E and l for each case.
- (c) Plot a typical orbit of each type using Mathematica.

3. Consider the modified Coulomb potential

$$V(r) = -\frac{k}{r} + \frac{\varepsilon}{r^2}.$$

- (a) For which values of E and l are the orbits unbounded?
- (b) Following the analysis in Sec. 3.7, find the shape $r(\theta)$ of a bounded orbit of this potential for given energy and angular momentum.

- (c) Show that this is a precessing ellipse and determine the angular velocity of precession. Is the precession in the same or opposite direction to the orbital angular velocity?

4. Assume that the gravitational potential of the sun takes the form

$$V(r) = -\frac{k}{r} - \frac{\varepsilon}{r^n}$$

where the second term is a small correction to the usual Newtonian potential,

- (a) What is the radius and period of a circular orbit in this potential?
- (b) For a nearly circular orbit, find the frequency of oscillations in the radial direction as a function of energy and angular momentum, by expanding the effective radial potential about its minimum to quadratic order in $s = r - r_0$. Give your answer to leading nontrivial order in ε .
- (c) Compare the frequency found in (b) to the orbital frequency found in (a) and explain why this corresponds to a nearly elliptical orbit. Obtain the angular rate of precession.
- (d) The leading correction to Newtonian gravity from General Relativity takes the above form, with $n = 3$, $k = GMm$, and $\varepsilon = \frac{kl^2}{mc^2}$ (l is the angular momentum). This correction leads to nonclosed planetary orbits. Estimate the precession of the perihelion of Mercury in arc seconds per century. You may find the numbers on p. 538 of GPS useful.