

PHY 504

Problem Set #7

due 24 October 2008

1. Derivation 4.14. Use these formulas to obtain expressions involving dot products for $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}$, $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$, and $(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{a} \times \mathbf{b})$.
2. The two-dimensional Euclidean group is the symmetry group of the 2d plane. It is generated by 2d rotations and translations.

- (a) Show that a rotation about the origin followed by a translation can be represented as a 3×3 matrix of the form

$$\begin{pmatrix} \cos \theta & \sin \theta & a_x \\ -\sin \theta & \cos \theta & a_y \\ 0 & 0 & 1 \end{pmatrix} \text{ acting on column vectors } \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}.$$

Explain why any element of the Euclidean group can be written in this form.

- (b) Show that these transformations form a non-commutative group.
3. A rotation of 60° about the x -axis is followed by a rotation of 45° about the z -axis.
 - (a) Find the resulting rotation matrix. Find the angle and axis of rotation to which this matrix corresponds.
 - (b) Repeat for a rotation of 45° about the z -axis followed by a rotation of 60° about the x -axis.
4. Derivation 4.17.
5. A girl riding on a platform rotating at π rad/s drops a ball from a height of 1 m, while standing a distance of 1 m from the center of the platform. How far from her feet does the ball hit the platform?