SELF-GRAVITATING DISKS: GLOBAL INSTABILITIES

*global stability of gaseous disks
*global stability of stellar disks
*stellar and gaseous bars

➢Numerical models of stellar disks

Toy models, or …. what have we forgotten?

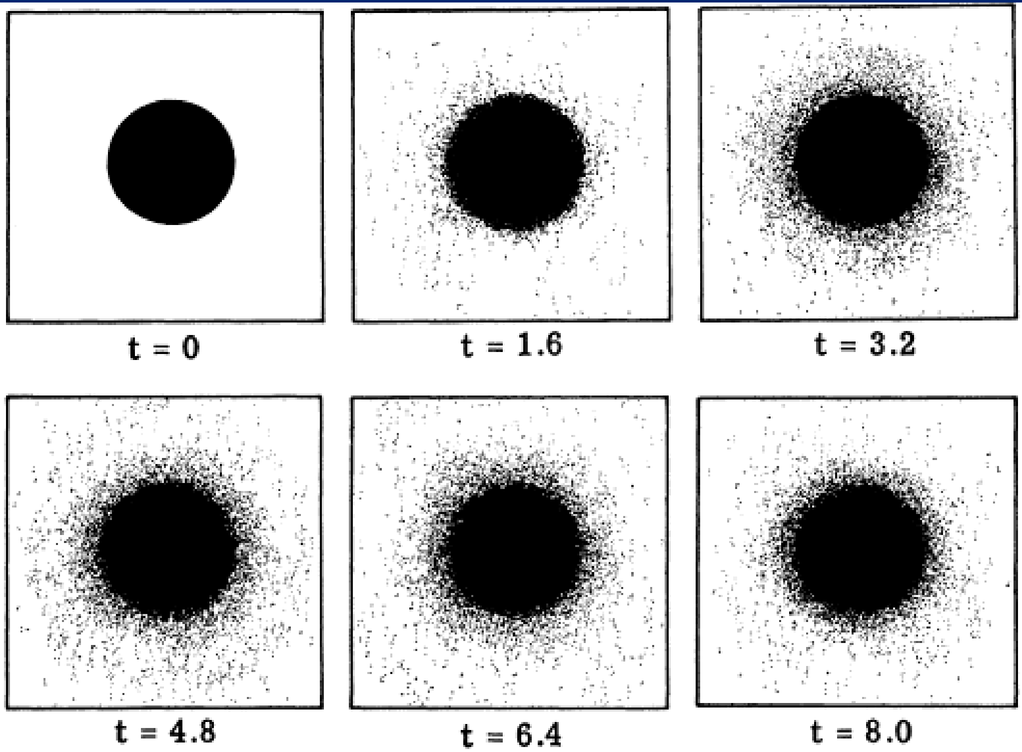


FIG. 1.—Axisymmetric evolution of an initially balanced, uniformly rotating disk of 100000 stars. The stars have an initial velocity dispersion given by Toomre's criterion, and they move under a purely radial gravitational field. Time in this and all subsequent figures is given in units of the rotational period of the cold balanced disk.

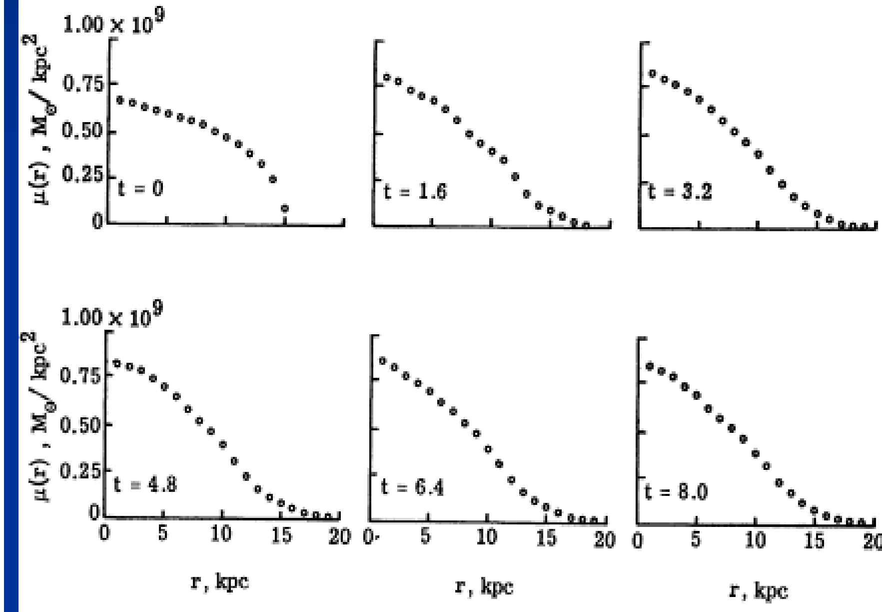
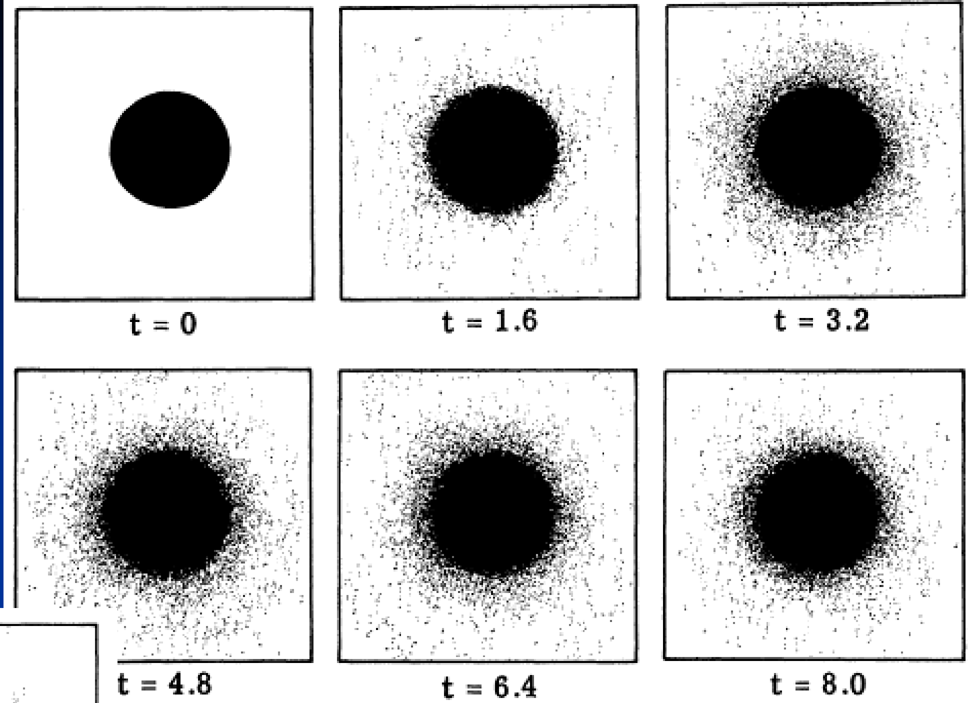


FIG. 2.—Evolution of the density for the disk shown in Fig. 1



Axisymmetric evolution

Non-axisymmetric evolution

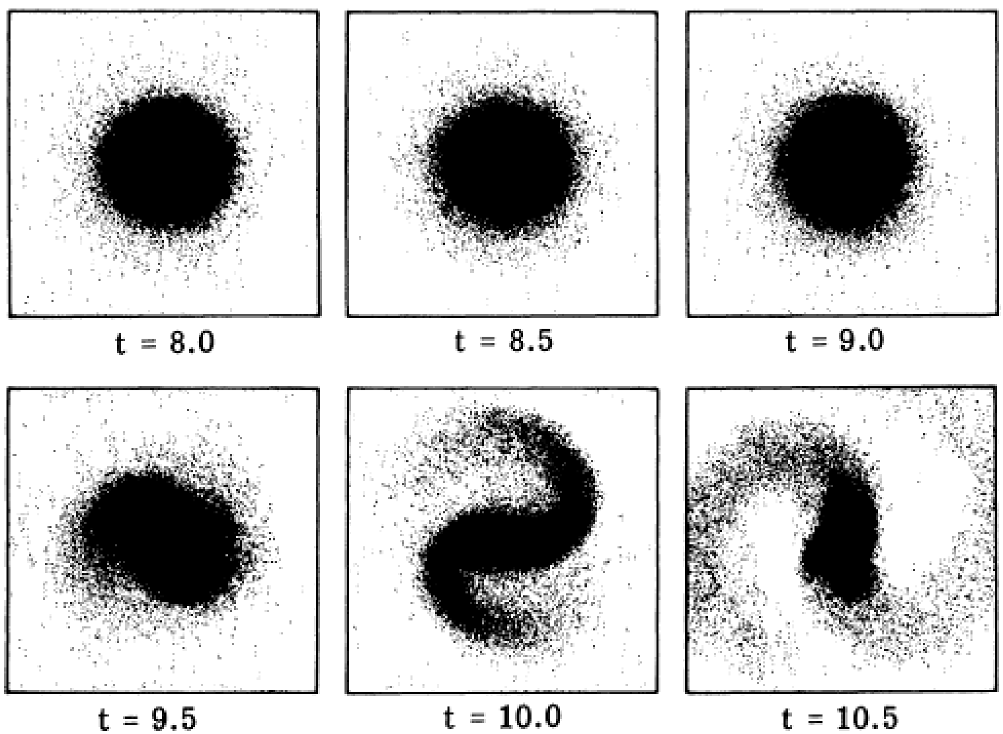


FIG. 3.—Nonaxisymmetric evolution of the disk with an initial condition taken as the disk in Fig. 1 at $t = 8$.

Doing the same:
but from the scratch

BAR INSTABILITY!

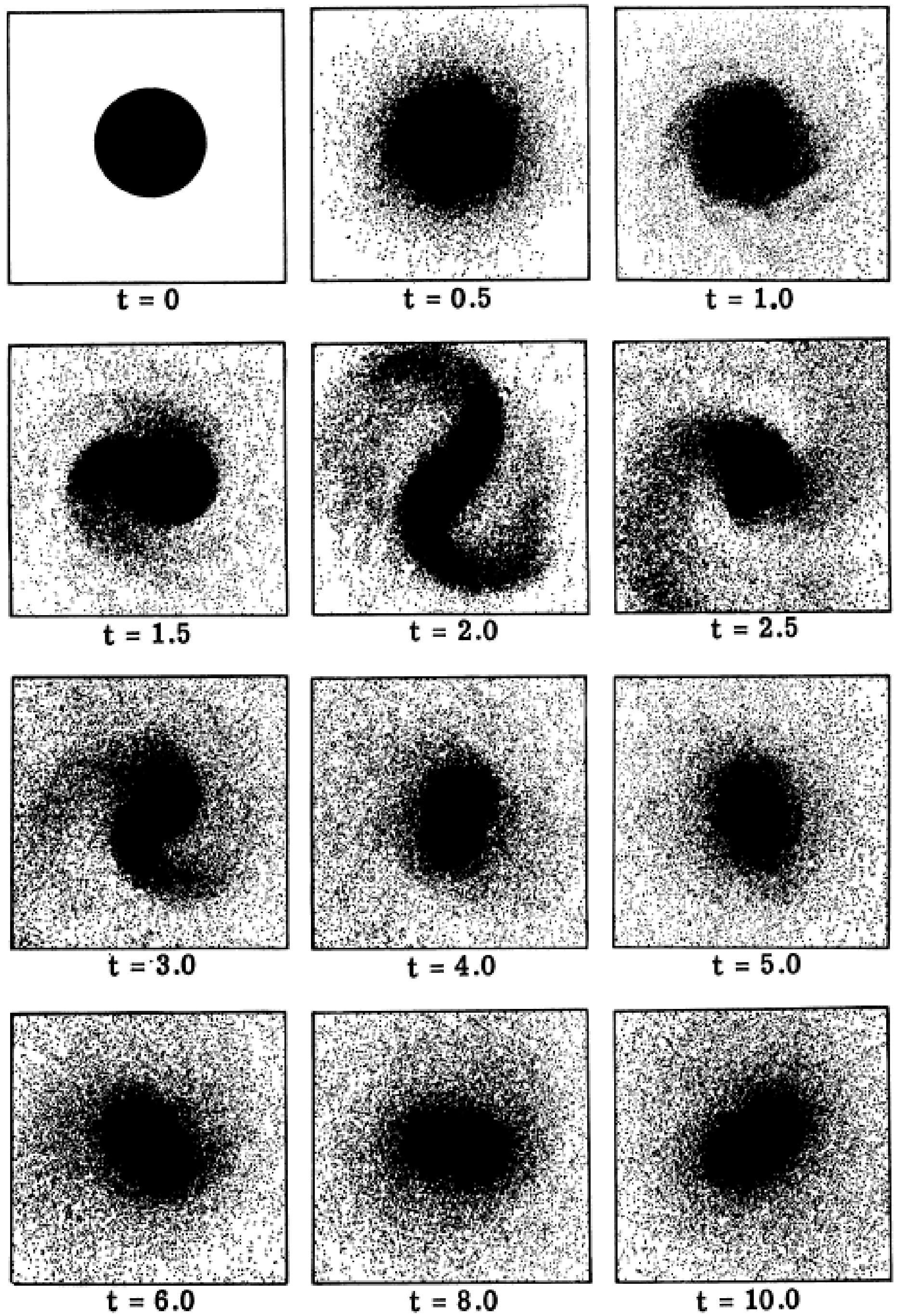


Fig. 4.—Unconstrained evolution of the initially balanced uniformly rotating disk of 100000 stars. The stars have an initial velocity dispersion given by Toomre's criterion.

Hohl 1971

Why this is interesting?

stars and dark matter
(only stars shown)

with gas

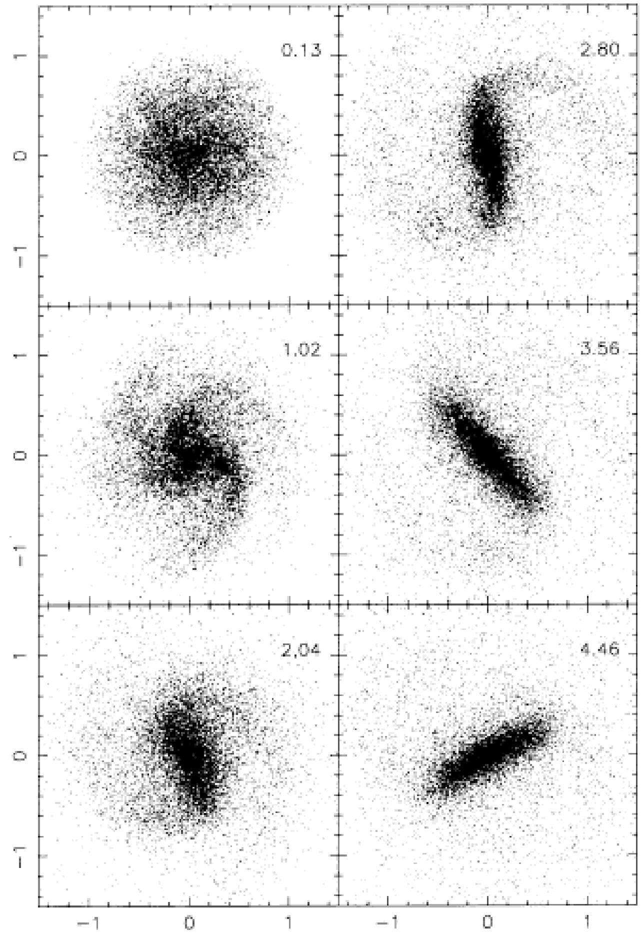


FIG. 3.—Evolution of a pure stellar disk in the bar-unstable model B(0/–) seen face-on. The time in units of τ_ϕ is given in the upper right corner. 15,000 particles are used in the disk and 10,000 in the halo. Halo particles are not plotted. Each frame is 30 kpc across.

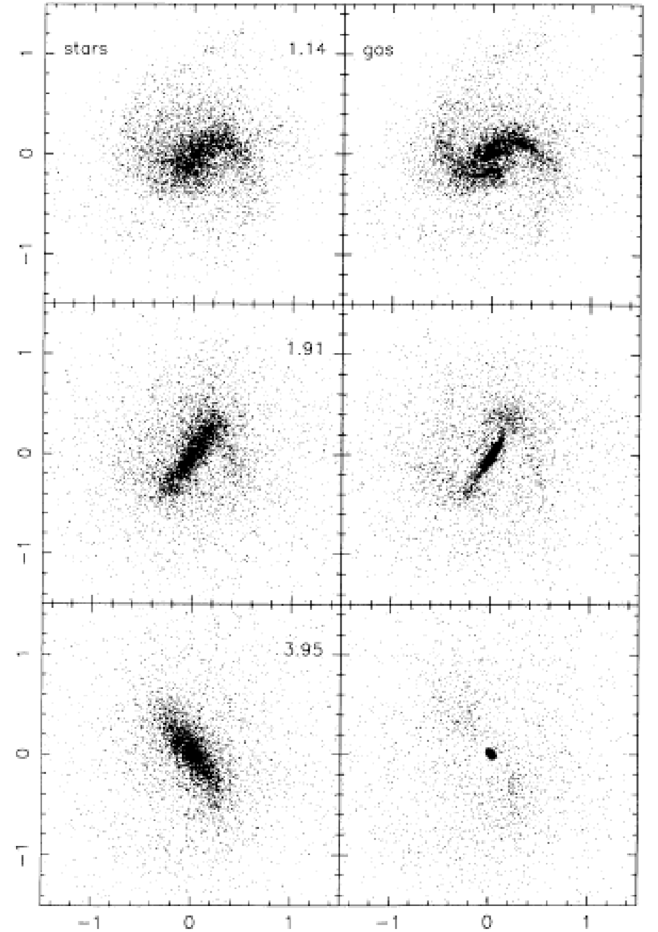


FIG. 5.—Evolution of stellar (*a*) and gas (*b*) distributions in the globally unstable galactic disk of the model B(0.01/0.25) seen face-on. The time in units of τ_ϕ is given in the upper right corners. 7500 collisional and 7500 collisionless particles are used in the disk and 10,000 in the halo. Each frame is 30 kpc across.

Real disk galaxies are made of **gas** and **stars**: what happens to the gas?

❖The nature of conspiracy: swing amplifier

The tight spiral approximation (WKB) approximation apparently misses an important process: **bar instability**

WHY?

The answer apparently has been provided in 1965 by Goldreich and Lynden-Bell but … nobody realized this!
It was resurrected by Alar Toomre in 1981

WKB picture of disk dynamics, because it does not apply to loosely wound structures

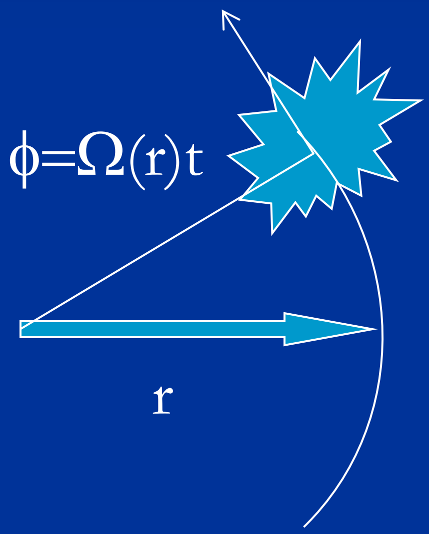
Swing amplification of density waves in differentially-rotating disks: conspiracy between shear, epicyclic oscillations and self-gravity

•Unwrapping the conspiracy

Remember: all leading spirals are sheared into trailing ones

Suppose all the relevant wavelengths and spatial scales are small enough

Follow the evolution of a small orbiting patch in a disk galaxy



Define (observable Oort) constant:

$$A = A(r) \equiv \frac{1}{2}\left(\frac{v_\varphi}{r} - \frac{dv_\varphi}{dr}\right) = 14.5 \pm 1.5 \ \text{km/s/kpc}$$

$$\frac{d}{dr}(r\Omega) = \Omega + r\Omega_r'$$

$$A = A(r) \equiv \frac{1}{2}\left(\frac{v_\varphi}{r} - \frac{dv_\varphi}{dr}\right) = -\frac{1}{2} r\,\Omega_r'$$

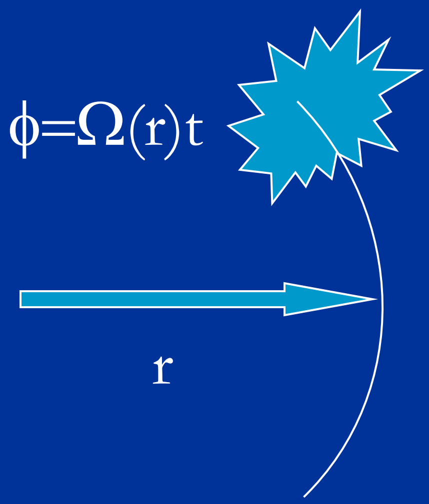
• Unwrapping the conspiracy

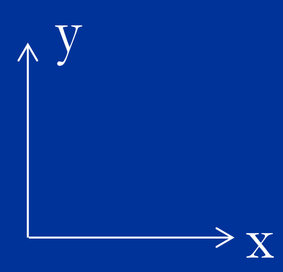
Stars obey linearized eqs.:

$$\ddot{x} - 2\Omega\dot{y} = 4\Omega A x + f_x \qquad \text{where} \qquad A = -\frac{1}{2} r \Omega'_r$$

$$\ddot{y} + 2\Omega\dot{x} = f_y$$

PROOF?





that govern small radial and tangential displacements:

$x = r - r_0$

$y = r_0[\phi - \Omega(r_0)t]$

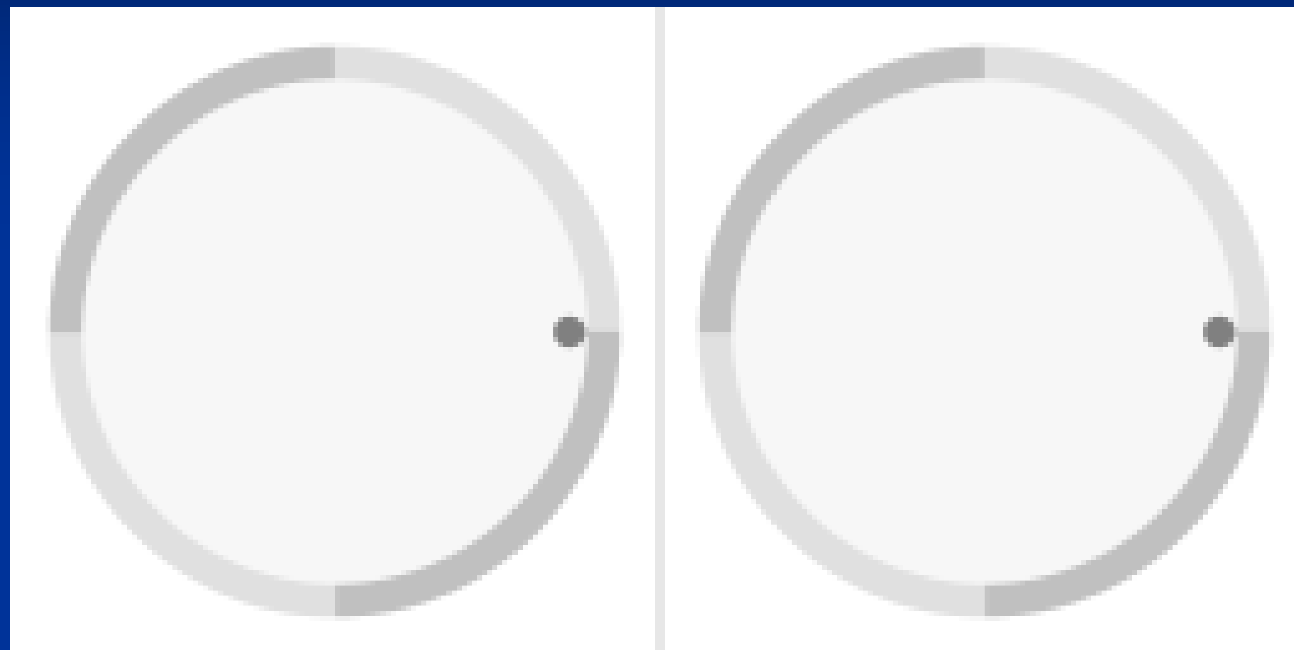
in co-moving (orbiting) coordinates

f_x and f_y – perturbing forces

•Epicycle arising from Coriolis force

view epicyclic motion in a rotating galaxy from:

non-rotating (inertial) frame (A):   rotating frame (B):



for a Keplerian potential $\Omega = \kappa$

In a frame B that rotates with constant Ω, the accelerations are given by:

radial acceleration (inertial frame)

$$\mathbf{a}_B = \mathbf{a}_A - \underbrace{\Omega \times (\Omega \times \mathbf{r}_B)}_{\text{Centrifugal}} - \underbrace{2\Omega \times \mathbf{v}_B}_{\text{Coriolis}}$$

One can describe the differential rotation in units of Oort constant (without perturbation):

$$\dot{y} = -2Ax$$

$$\kappa^2 = 4\Omega^2 - 4\Omega A$$

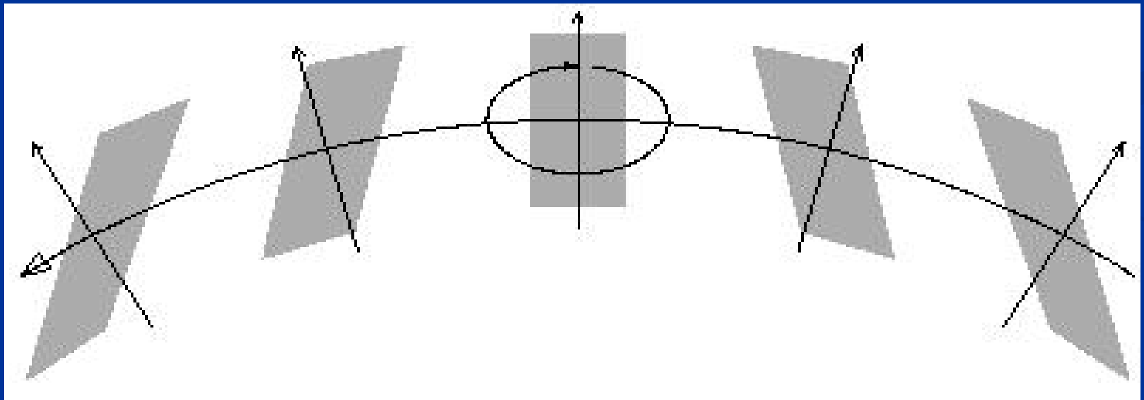
$$\dot{y} = r_0(\Omega - \Omega_0) \rightarrow r_0(d\Omega/dr)\,dr = r_0\Omega' x$$

definition of κ

2 out of 3 conspirators!

Note: for “normal” disks with Ω decreasing with *r*

both shear and epicyclic motions act against the unperturbed one (to Ω)



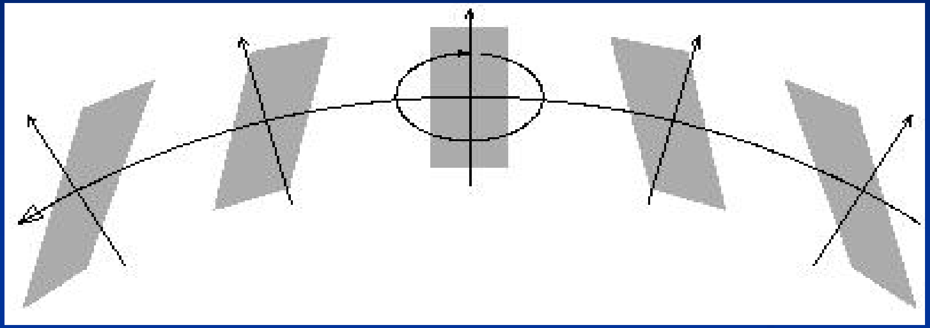
leading→trailing arm transformation by shear

timescale ~ $1/\kappa$

The above point (correspondence between shear and epicyclic motion) AND interstellar “communication” (self-gravity) allows for amplification of open spirals when they swing from leading to trailing

In other words:

- Leading arm is sheared into trailing one
- Pattern rotates in the retrograde fashion
- Its rotation timescale is ~$1/\kappa$



This means that:

The epicyclic motion approximately follows the spiral arm

Duration of the perturbation will be long → epicyclic motion is strongly amplified

The emerging trailing pattern is strongly amplified

- Amplification gain depends on **Q** and radial wavelength (λ) of the pattern

Maximum amplification ~ 100 when $\lambda \sim 1.5\, \lambda_J$

BUT: how this can explain the persistent spiral structure in galaxies?

It is not enough to have wave amplification!

After some time they will decay

Any leading arm will be sheared into trailing one and damp after some time

Need: mechanism that *generates* leading arms again and again from trailing arms

Need feedback mechanism !

•Swing Amplifier

Density contours of a leading arm unwinding to become a trailing arm. The swing amplifier boosts the arm strength considerably.

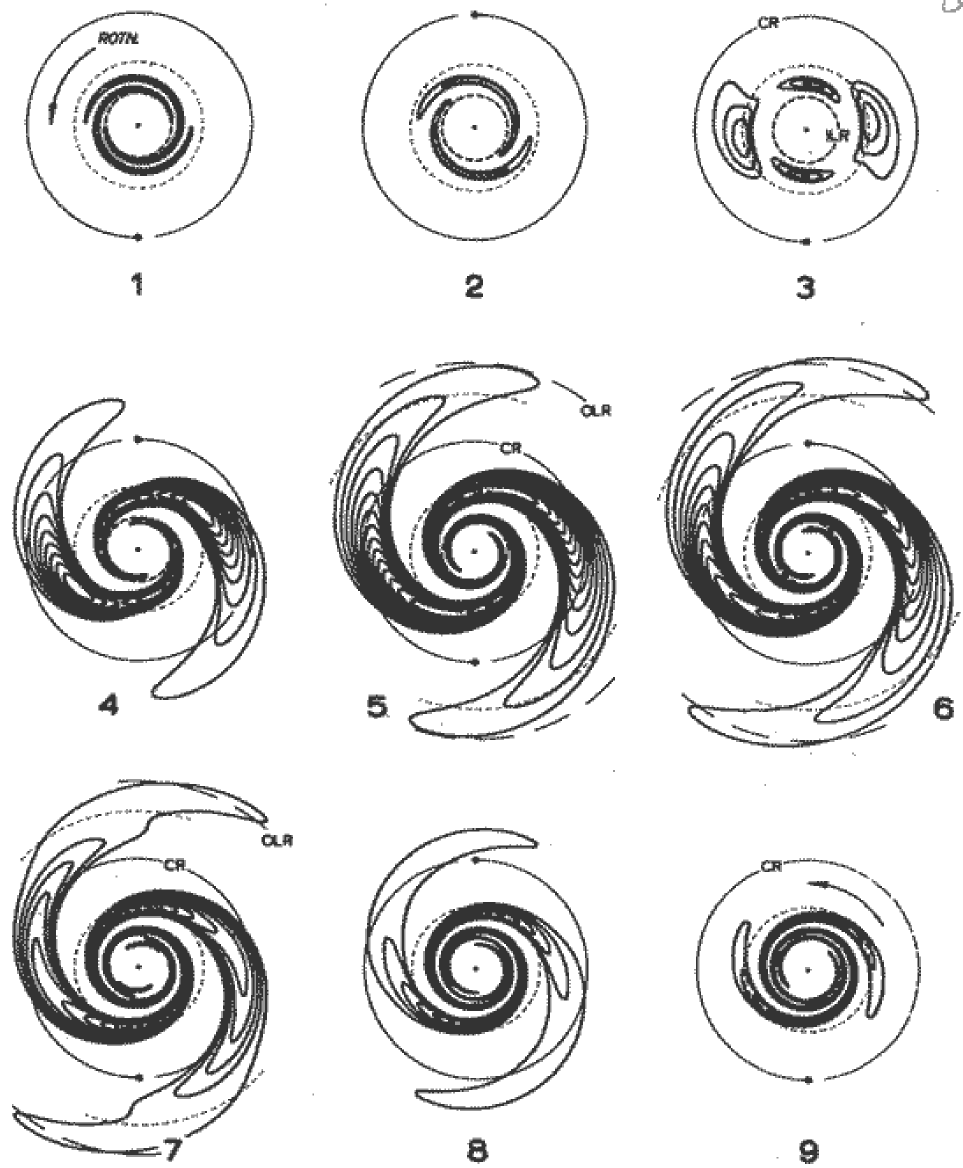


Figure 6-17. Evolution of a packet of leading waves in a stellar Mestel disk with $Q = 1.5$ and $f = 1$. Contours represent fixed fractional excess surface densities; since the calculations are based on linear perturbation theory, the amplitude normalization is arbitrary. Contours in regions of depleted surface density are not shown to minimize confusion. The time interval between diagrams is one-half of a rotation period at corotation. From Toomre (1981), by permission of Cambridge University Press.

How to generate leading arms from trailing ones?

- Reflection from the outer disk edge: phase shift of 180°

  but in real galaxies outer edges are soft: no reflection

- Trailing arms passing through galactic center: emerge as leading ones

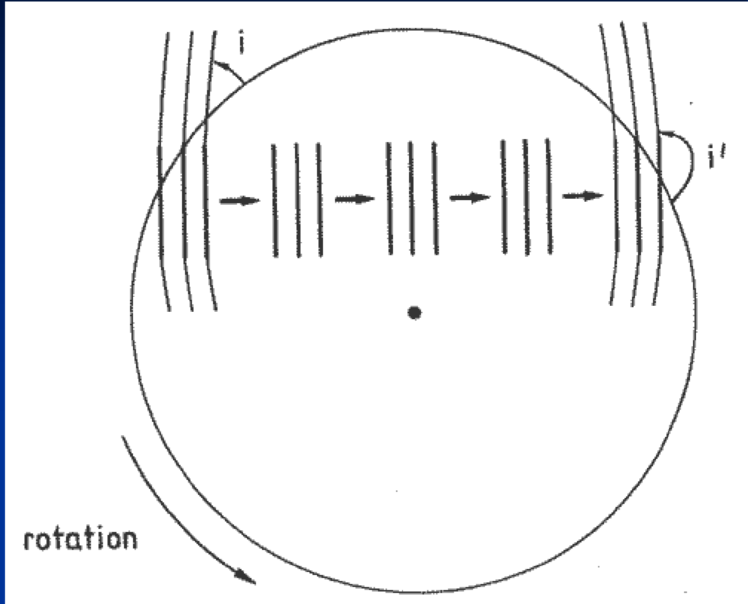


Figure 6-20. A graphical argument that suggests why trailing waves that propagate through the center of a disk emerge as leading waves. A small patch of three incoming trailing waves with inclination $i < 90°$ is shown on the left. The patch propagates through the center as a plane wave and emerges with a pitch angle $i' = 180° - i$. Since $i' > 90°$ the merging wave is leading.

Beware of ILRs!
→ will block the passage

If trailing waves can pass through the central regions of a galaxy then they emerge as leading waves on the other side. These leading waves are then subject to the swing amplification