GALACTIC DYNAMICS AND THE ISM *Spring 2014*
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PROBLEM SET 1

Problem 1 (Hydrostatic Equilibrium)

A spherically-symmetric self-gravitating gas cloud is isothermal with $T(r) = const$. The equation of hydrostatic equilibrium for such a gas can be written in the form of

$$\left(\frac{1}{\rho}\right)\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{GM(r)}{r^2},$$

where $P(r)$ is the gas pressure, $\rho(r)$ is the gas density, $M(r)$ is the mass interior to the radius r, and G is the gravity constant. Assume that the gas obeys the ideal gas equation, $P = n_{\rm p}k_{\rm B}T = \rho k_{\rm B}T/m_{\rm p}$, where $n_{\rm p}(r)$ is the gas number density, $m_{\rm p}$ is the mean mass of the gas particles, and $k_{\rm B}$ is the Boltzmann constant.

(a). Develop a second-order differential equation involving ρ and r as the (only) variables.

(b). Show that $\rho(r) = \sigma^2/2\pi G r^2$ is the solution to this differential equation, where $\sigma = (k_{\rm B}T/m_{\rm p})^{1/2}$.

Problem 2 (Galaxy Luminosity Function)

Use the galaxy Luminosity Function (LF) defined in class using the Schechter parametrization, $\phi(L) = (n_*/L^*)(L/L^*)^{\alpha}\exp(-L/L^*)$, to show that

(a). The integral Schechter's LF is

$$< L >= n_* L^* \Gamma(\alpha + 2),$$

where Γ is the incomplete gamma function. Prove it! (additional reading about Schechter's LF can be found in B&M section 4.1.3).

(b). The Schechter LF is useful for estimating the amount of luminosity you fail to measure when you work with flux- or magnitude-limited samples. For example, in a nearby galaxy cluster you might be able to sample the LF 4 magnitudes fainter than M^*. While in a cluster which is 5 times further away, in order to get to the same apparent magnitude limit, you will only sample 0.5 magnitudes below M^*. So, for a faint end slope of -1.1, how much of the integrated luminosity of the above more distant cluster will be lost relative to the nearby one?

(c). An issue still hotly debated is how much luminosity is actually hiding in low luminosity/low surface brightness objects. Modern redshift surveys find flat faint end slopes but only for relatively luminous galaxies and high surface brightness. So, for Schechter functions of the same M^*, what fraction of the integrated luminosity density lies below M^* for α = -1.0, -1.25, -1.5 and -1.85?

Problem 3

Say, an observer is immersed among stars. Use stars as "standard candles" and assume each has a fixed luminosity L. Also assume that a number density of stars n is independent of position.

(a). Find how the number of stars that have a flux F larger than some value F_0 at the position of the observer, $N(F > F_0)$, scales with F.

(b). Assume that the stars are instead distributed uniformly in a very thin infinite disk. Find the distribution of N in this case.

Problem 4 (Potential Theory)

Assume that a galaxy has a flat rotation curve, $v(r) = v_{\rm c} \sim const$, out to some radius $r_{\rm max}$. For $r < r_{\rm max}$ the dominant contribution to the potential is dark matter which has a spherically-symmetric distribution. For larger radii, assume the dark matter density is zero.

Find the escape velocity from the galaxy for $r < r_{\rm max}$.

Problem 5 (Disk Galaxies)

In a disk galaxy, the bulge-to-disk ratio is the ratio of the luminosity in the bulge to that in the disk. Typically, bulge-to-disk ratios decrease with increasing Hubble's morphological type. Calculate the bulge-to-disk ratio for a typical disk galaxy with a bulge following the de Vaucouleur's law, and an exponential disk. Express your result in terms of the parameters we have defined in class (Lecture #1), i.e., h and I_0 (for the disk), and $I_{\rm e}$ and $r_{\rm e}$ (for the spheroidal bulge).

Problem 6 (Potential of a sphere)

The gravitational potential of the *Plummer sphere*

$$\Phi(r) = -\frac{Gb}{\sqrt{r^2 + a^2}}$$

approaches that of a point mass at $\mathbf{r} = 0$ when $r >> a$.

(a). Calculate the total mass of the system

(b). Show that Φ corresponds to the density

$$\rho(r) = \frac{1}{4\pi G r^2}\frac{d}{dr}\left(r^2 \frac{d\Phi}{dr}\right) = \frac{3a^2}{4\pi}\frac{M}{(r^2 + a^2)^{5/2}}.$$

(c). When the Plummer sphere is viewed from a great distance, show that the surface mass density at distance R from the center is

$$\Sigma(R) = \int_{-\infty}^{+\infty} \rho(\sqrt{R^2 + z^2})dz = \frac{Ma^2}{\pi(a^2 + R^2)^2}.$$

Check that the *core radius* r_c, where $\Sigma(R)$ drops to half its central value, is at $r_c \approx 0.644a$.