Constraining spiral structure parameters through Galactic pencil-beam and large-scale radial velocity surveys

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ABSTRACT
We investigate the effect of spiral structure on the Galactic disc as viewed by pencil beams centred on the Sun, relevant to upcoming surveys such as ARGOS, SEGUE and GAIA. Using a steady-state spiral structure model, we create synthetic Galactic maps which we call pencil-beam maps (PBMs) of the following observables: line-of-sight velocities, the corresponding velocity dispersion and the stellar number density that are functions of distance from the observer. We show that PBMs are a powerful tool for analysing spiral patterns but the uncertainty of the Milky Way structure prevents their simple interpretation. For the case of steady-state spiral structure, we show that such maps can be used to infer spiral structure parameters, such as pattern speed, solar phase angle and number of arms. The mean line-of-sight velocity and velocity dispersion are affected by up to $\sim 35 \text{ km s}^{-1}$, which is well within the detectable limit for forthcoming radial velocity surveys. One can measure the pattern speed by searching for imprints of resonances. In the case of a two-armed spiral structure it can be inferred from the radius of a high velocity dispersion ring situated at the 2:1 inner Lindblad resonance (ILR). This information, however, must be combined with information related to the velocities and stellar number density in order to distinguish from a four-armed structure. If the pattern speed is such that the 2:1 ILR is hidden inside the Galactic bulge, the 2:1 outer Lindblad resonance will be present in the outer Galaxy and thus can equivalently be used to estimate the pattern speed. Once the pattern speed is known, the solar angle can be estimated from the line-of-sight velocities and the number density PBMs. The models presented here are preliminary as transient spiral structure models should also be considered. Forthcoming radial velocity surveys are likely to provide powerful constraints of the structure of the Milky Way disc.

Key words: stellar dynamics – surveys – Galaxy: disc – Galaxy: kinematics and dynamics – Galaxy: structure – galaxies: spiral.

1 INTRODUCTION
It has been well established by now that the Milky Way is not axisymmetric with both a central bar and spiral structure perturbing its disc. Due to our location in the Galactic plane both spiral and bar structure is impossible to observe directly. Galactic bar parameters such as orientation and pattern speed have been inferred indirectly from both asymmetries around the Galactic Centre (e.g. Blitz & Spergel 1991; Weinberg 1992) and its effect on the local velocity distribution of old stars, i.e. the Hercules stream (Dehnen 1999, 2000; Fux 2001; Minchev, Nordhaus & Quillen 2007).

Spiral structure parameters, however, are much more uncertain. Current spiral density wave models (Fux 2001; Lépine, Mishurov & Dedikov 2001; De Simone, Wu & Tremaine 2004; Quillen & Minchev 2005) strongly disagree on the strength of the spiral structure, the number of arms and the pattern speed. These models differ in their predictions of the induced velocity streaming at different angular positions in the Galaxy. For example, a four-armed density wave with velocity perturbations of $\sim 20 \text{ km s}^{-1}$ will exhibit rapidly varying radial and tangential velocity components with azimuth across distances of a few kpc, and we could expect to detect $\sim 20$–$50 \text{ km s}^{-1}$ variations in the mean line-of-sight stellar velocity as a function of the distance from the Sun. However, the strength of the spiral arm perturbation remains controversial. Based on the OGLE number counts, Paczynski et al. (1994) estimated that the Sagittarius–Carina arm has a factor of 2 increase in density compared to the underlying disc. This model is inconsistent with COBE studies which find a much smaller contrast ($\sim 15$ per cent) and show that the Perseus and Scu–Cru arms are more dominant (Drimmel & Spergel 2001).

$\text{H}_1$, CO, Cepheid and far-infrared observations suggest that the Galactic disc contains a four-armed tightly wound structure. On the
other hand, Drimmel & Spergel (2001) have shown that the near-infrared observations are consistent with a dominant two-armed structure. Lépine et al. (2001) suggests that locally the Milky Way can be modelled by the superposition of a two- and four-armed structure moving at the same pattern speed. By studying the nearby spiral arms, Naoz & Shavit (2007) find that the Sagittarius–Carina arm is a superposition of two features, moving at different pattern speeds. The effect of a two- and four-armed structure, moving at different angular velocities, on the velocity dispersion of a galactic disc has been explored numerically by Minchev & Quillen (2006).

Estimates for the pattern speed of the Milky Way spiral structure, or equivalently, the Sun’s position with respect to resonances associated with spiral structure, span a large range of values. Reviewing previous work, Shavit (2003) finds a clustering of estimates for the pattern speed of local spiral structure near \( \Omega_c \sim 20 \, \text{km} \, \text{s}^{-1} \, \text{kpc}^{-1} \), though other studies suggest \( \Omega_c \sim 13 \, \text{km} \, \text{s}^{-1} \, \text{kpc}^{-1} \). The model by Lépine et al. (2001) places the Sun near the corotation resonance \( \Omega_c \sim 28 \, \text{km} \, \text{s}^{-1} \, \text{kpc}^{-1} \), and was fit to Cepheid kinematics. The recent gas dynamical studies (Bissantz, Englmaier & Gerhard 2003; Martos et al. 2004) match the properties of the gas in nearby arms with a spiral pattern speed of \( \sim 20 \, \text{km} \, \text{s}^{-1} \, \text{kpc}^{-1} \). Martos et al. (2004) propose that a two-armed stellar structure consistent with the stellar distribution inferred from COBE could cause four arms in the gas distribution near the Sun. The gas dynamical model proposed by Bissantz et al. (2003) with a similar spiral pattern speed matches HI and CO kinematics. The pattern speed of a spiral density wave can be tightly constrained from the location of its resonances. For example, Quillen & Minchev (2005) associated stellar streams in the solar neighbourhood with the 4:1 inner Lindblad resonances (ILR) of a two-armed pattern and were then able to tightly constrain the pattern speed of the driving spiral density wave to within 5 per cent. Independent constraints on the pattern speed come from recent surveys of nearby open clusters (e.g. Dias & Lépine 2005) where the older clusters are found to have drifted further from their original density wave location. These authors concluded that the Sun is located near the corotation resonance (CR). A solar circle near the CR is also favoured by Lépine et al. (2001) and Naoz & Shavit (2007).

In this paper we investigate how spiral structure parameters can be inferred from velocity and density maps resulting from pencil-beam and large-scale surveys of the Galaxy. At present the influence of spiral arms on the observed kinematic properties of the Galactic disc is very poorly understood. With the advent of future Galactic all-sky (GAIA, SEGUE) and pencil-beam (ARGOS, BRAVA) radial velocity surveys, large amounts of kinematic data will be collected. The type of dynamical constraints made possible with these new data sets is not currently known. We address that issue here with synthetic models for the purpose of exploring how spiral structure might be constrained from these data.

## 2 A STEADY-STATE SPIRAL STRUCTURE MODEL AS A FIRST APPROXIMATION

At present the spiral structure model for the Galaxy is not well understood. Though it is generally accepted that spirals are density waves there exist two competing theories: (1) transient/recurrent spirals and (2) steady-state spirals.

Recurrent spiral instabilities, i.e. spirals with a changing pattern speed, strength and pitch angle, have been reported by Sellwood & Carlberg (1984) and Sellwood & Lin (1989) in their simulations of self-gravitating discs. It was argued by Toomre & Kalnajs (1991) that these transient spiral density waves are due to the swing-amplification mechanism as first formulated by Toomre (1981).

Alternatively, the concept of quasi-stationary density waves was developed by Lin, Yuan & Shu (1969) and culminated in the work by Bertin et al. (1989a,b) and Lowe et al. (1994). By introducing an inner Q-barrier to shield the 2:1 ILR, N-body simulations have been constructed to yield long-lived spiral density waves lasting for over five rotation periods (Thomasson et al. 1990; Elmegreen & Thomasson 1993; Donner & Thomasson 1994).

In this work we consider steady-state spirals for the reason that it is the model with the lowest number of free parameters. A thorough understanding of the steady-state model is needed before a much higher dimension space (that of time variability) is explored and used to interpret the observations. Pattern speeds are estimated primarily from resonances—their measured pattern is generally less than a few rotation periods, as predicted by theory and seen in simulations. N-body and smoothed particle hydrodynamics (SPH) studies have illustrated strong features associated with resonances even when spiral structure is relatively short lived (Patsis & Kaufmann 1999, for example). Thus pattern speeds should be measurable from surveys when mimicked with steady-state models. It should be kept in mind that measurements using steady-state model are likely to only be approximate if the system is evolving. Future work should consider time variable models, as well as estimate errors caused by using a steady-state model (Merrifield, Rand & Meidt 2006).

## 3 THE SIMULATIONS

We perform 2D test-particle simulations of an initially axisymmetric exponential galactic disc. In order to reproduce the observed kinematics of the Galactic disc, we use disc parameters consistent with observations (Table 1). The reader is referred to Minchev & Quillen (2007) for a more detailed description of our simulation set up. In all of our simulations we start with an initially warm disc, i.e. the radial velocity dispersion at \( r_0 \) is \( \sigma_v = 0.20v_0 \) where \( v_0 \) is the velocity of the local standard of rest. The background axisymmetric potential due to the disc and halo has the form \( \Phi_0(r) = v_0^2 \log(r) \), corresponding to a flat rotation curve.

We treat the spiral pattern as a small perturbation to the axisymmetric model of the galaxy by viewing it as a quasi-steady density wave in accordance with the Lin–Shu hypothesis (Lin et al. 1969). The spiral wave gravitational potential perturbation is expanded in Fourier components as

\[
\Phi_1(r, \phi, t) = \sum_m \epsilon_m \exp\left(i\alpha \ln r - m(\phi - \Omega_c t)\right). \tag{1}
\]

The parameter \( \alpha \) is related to the pitch angle of the spiral wave, \( \alpha = m \cot(p) \), negative for trailing spirals with rotation counter-clockwise, and \((r, \phi)\) are plane polar coordinates. The pattern speed is given by \( \Omega_c \) and the spiral strength by \( \epsilon_m \). For a two-armed structure the \( m = 2 \) term dominates. Upon taking the real part of

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<th>Table 1. Simulation parameters used.</th>
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<tr>
<td>Parameter</td>
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<tr>
<td>Solar neighbourhood radius</td>
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<tr>
<td>Circular velocity at ( r_0 )</td>
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<td>Radial velocity dispersion</td>
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<td>( \sigma_v ) scalelength</td>
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equation (1) the perturbation due to the two-armed spiral density wave becomes

$$\Phi_1(r, \phi, t) = \epsilon_s \cos (\alpha \ln r - 2(\phi - \Omega_s t)).$$  \hspace{1cm} (2)$$

Integrations are performed forward in time. The perturbation is grown from zero to its maximum strength in four rotation periods at $r_0$. In order to improve statistics, positions and velocities are time averaged for 10 spiral periods. We distribute particles (stars) between in inner and outer galactic radii ($r_{\text{in}}, r_{\text{out}}$) = (0.3$r_0$, 2.0$r_0$). New particles are added until the final number of outputs is $2.5 \times 10^5$. In addition, the twofold symmetry of our model galaxy is used to double this number.

We present our results by changing the spiral pattern speed, $\Omega_s$, and keeping the solar radius fixed at $r_0 = 1$. For a two-armed spiral pattern the primary resonances are the 2:1 ILR and OLR. Those are achieved when $\Omega_s/\Omega_0 = 1 + \kappa/2 \approx 0.3, 1.7$, respectively, where $\kappa$ is the epicyclic frequency. Similarly, the second-order resonances are the 4:1 Lindblad resonances (LRs) at $\Omega_s/\Omega_0 = 1 + \kappa/2 \approx 0.65, 1.35$. We examine a region of parameter space for a range of pattern speeds placing the SN between the 4:1 LRs.

4 VARIATION OF GALAXY MORPHOLOGY WITH PATTERN SPEED

Interpretation of line-of-sight velocities with Galactic longitude and distance from the Sun is not straightforward. To help out we first discuss morphology as seen by an outside viewer.

In Fig. 1 we present stellar number density contour plots for simulations of galactic discs with different pattern speeds. The background axisymmetric disc is subtracted to emphasize the spiral structure. The quantity plotted is $(\Sigma - \Sigma_{\text{axi}})/\Sigma_{\text{axi}}$, where $\Sigma$ and $\Sigma_{\text{axi}}$ are the perturbed and axisymmetric stellar number densities. Concentric circles represent the 2:1 LRs (dashed), the solar radius (solid) and the CR (dash–dotted). Darker colours correspond to lower density.

The inner 0.3$r_0$ disc is not plotted since we do not model the Galactic Centre. Each panel represents a simulation with a distinct pattern speed, $\Omega_s$, and all other parameters kept the same (see Table 1). Pattern speeds considered range approximately between the 4:1 LRs, $\Omega_s = [0.6, 1.3]\Omega_0$ in units of 0.1. The minima of the two-armed spiral potential are graphed in each panel as solid curves. Note the crowding of resonances as the pattern speed increases.

In general, changing the solar radius in a simulation with the same pattern speed is equivalent to changing the pattern speed and keeping the solar radius fixed. However, this is exactly true only if the stellar density and velocity dispersion varied linearly with radius. This is not the case in real galaxies; both of these are found to vary exponentially with radius. Lewis & Freeman (1989) estimated the number density and radial velocity dispersion scalelengths in the Milky Way to be $r_0 = 0.37 r_0$ and $r_\sigma = 0.9 r_0$, respectively. Thus we need to perform different simulation runs when changing the pattern speed.

Note the disruption of the spirals near the 2:1 LRs (dashed circles). A rapid decrease of spiral strength at the 2:1 OLR was also observed by Vorobyov & Theis (2008) in a galactic disc model consisting of solving numerically the Boltzmann moment equations. It has also been suggested by Contopoulos (1985) that strong (nonlinear) spiral structure cannot extend beyond the 4:1 ILR since at that location the stellar orbits are not in phase with the imposed spiral. As pointed out by Sellwood (1993), however, this limited extent of the spirals found by Contopoulos (1985) is probably related to the restrictive assumptions they make in order to construct self-consistent spiral structure. Another remarkable feature in the plots of Fig. 1 is the overdensity of stellar orbits just outside the 2:1 LRs (where they exist) and near the CR (dash–dotted circles). In the case of the CR, the enhancement is due to the stable Lagrange points $L_{1,3}$ associated with this resonance.

Assuming the primary spiral structure in the solar neighbourhood is two armed, the question arises: How can we determine any

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Each panel shows stellar number density contour plots of a simulation with a particular spiral pattern speed, $\Omega_s$, in units of the local circular velocity, $\Omega_0$. The axisymmetric disc is subtracted to emphasize the perturbation. The quantity plotted is $(\Sigma - \Sigma_{\text{axi}})/\Sigma_{\text{axi}}$, where $\Sigma$ and $\Sigma_{\text{axi}}$ are the perturbed and axisymmetric stellar number densities. Contours are spaced linearly with white indicating highest values. Dashed circles show 2:1 LRs, the dash–dotted line indicates the CR, and the solid circle represents the solar neighbourhood at $r/r_0 = 1$. The inner 0.3$r_0$ is not plotted since we do not simulate the central bar. We consider pattern speeds between the 4:1 ILR and the 4:1 OLR.}
\end{figure}
with known velocities, distances, etc., and constructing velocity and Galaxy? One way to do this is by collecting a large number of stars of the spiral structure (third row). The pattern speed is spiral structure parameters, given our inconvenient position in the Galactic plane by plotting these versus Galactic longitude, \( l \). (\( l \)) would look like when a two-armed spiral structure perturbs a stellar liocentric distance, \( r \). We call these maps ‘pencil-beam maps’ or do not attempt to model the reddening resulting from dust extinction.

To investigate the global structure of the Galaxy we require accurate stellar velocities and distances. In a pencil-beam spectroscopic survey line-of-sight velocities can be measured to great distances. On the other hand, proper motions are hard to measure for stars farther than about 2 kpc from the Sun. Thus those cannot be used in our investigation.

For a complete kinematics study accurate distance estimates are also needed. Due to the large distances involved in such a survey, trigonometric parallax measurements are not possible. Instead, photometric distances can be estimated, given accurate photometry. This way of computing distances, however, is hampered by the dust obscuration in the Galactic plane aside from several known windows, e.g. Baade’s Window at \( (l, b) = (0.9,\ -4) \). Another distance estimator is the use of standard candles such as Cepheids, etc. Here we do not attempt to model the reddening resulting from dust extinction but present an idealized model as a first attempt to tackle this problem. A future paper will be dedicated to a more detailed modelling. Due to this shortcoming our model can be directly applied only to the known low extinction Galactic plane windows. Like Baade’s Window, the Scutum Window at \( l = 27^\circ \) has low extinction and we can observe stars at 10 kpc distances towards the inner disc. Clump giants of the intermediate-age and older population of the disc and thick disc will be abundant in these fields. This line of sight at \( l = 27^\circ \) is tangent to the Scu–Cru spiral arm, with an AV extinction of about 3 mag at the distance of the spiral arm tangent point (~6 kpc). In the Scutum Window, the H\textsc{i} and H\textalpha\ profiles clearly show the presence of spiral arms (Madsen & Reynolds 2005).

In Fig. 2 we present PBMs of the line-of-sight velocity \( v_d \) (left-hand column), the corresponding velocity dispersion \( \sigma_v \) (middle column) and the stellar number density \( \Sigma \) (right-hand column). This is a simulation of a galactic disc perturbed by a two-armed spiral density wave moving with \( \Omega_2 = 0.7\Omega_0 \). To create the contour plots in this figure we bin the disc in Galactic longitude \( l \) (x-axis), and heliocentric distance \( d/r_0 \) (y-axis) as seen from an observer at a solar orientation with respect to the concave spiral arm of \( \theta_0 = 20^\circ \). This is in contrast to Fig. 1 where we present number density plots of a face-on view. The contours in the first row show the results of an axisymmetric disc, which is indicated by the subscript ‘axi’. In the second row the disc is perturbed by a 2 spiral density wave. The third row in Fig. 2 plots contours of the difference between the perturbed and axisymmetric discs.

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1 As proposed by Joss Hawthorn, ARGOS collaboration.
for the mean velocity and its dispersion: $\Delta v_d \equiv v_d - v_{d,\text{axis}}$ and $\Delta \sigma_d \equiv \sigma_d - \sigma_{d,\text{axis}}$. The number density, on the other hand, is obtained as in Fig. 1: $\Delta \Sigma \equiv (\Sigma - \Sigma_{\text{axis}})/\Sigma_{\text{axis}}$. We showed a number density plot in the $xy$ plane of this particular simulation in Fig. 1 (panel with $\Omega_d = 0.7$). In Fig. 2, however, we plot observables from a point of view centred on the Sun, as pencil-beam surveys would see the Galaxy. The shaded contours are equally spaced with darker colour corresponding to lower density. On top of each panel we show the minimum and maximum contour values (in units of $v_0$ in the case of the velocities and the velocity dispersion). As it is commonly accepted, the Galactic longitude is zero in the direction of the Galactic Centre, with the Galactic anticentre at $l = \pm 180^\circ$. The inner $0.3r_0$ disc has been removed everywhere as in the density plots in Fig. 1. The white curve in each panel represents the projection of the solar circle in a PBM and the vertical line shows the Galactic longitude $l = 0^\circ$.

Note that in the background subtracted PBMs (third row) the spiral structure is much better pronounced compared to the raw values (first row). Knowing the global Galactic potential is imperative to extracting information from a PBM. This presents a problem since the Milky Way potential is not very well known. We could, however, use our model axisymmetric data to subtract from the observational data.

How well do we need to know the axisymmetric structure so that spiral features are not wiped out? From the third row of Fig. 2 we can estimate that distance uncertainties of $\sim 30$ per cent do not prevent detection of spiral structure. We also need line-of-sight velocity precision of $\sim 20$ km s$^{-1}$ and the axisymmetric potential must be known to within $\sim 10$ per cent.

What information about the spiral structure can we infer from PBMs such as Fig. 2? To answer this question we need to vary the parameters and look at how structure in these PBMs changes. We do this in the following sections.

5.1 Changing the spiral pattern speed

We would like to know how to infer the spiral pattern speed from structure in the PBMs. In Fig. 3 we plot the variation of PBMs with a change in the spiral pattern speed in the range $0.6 \leq \Omega_d/\Omega_d^\circ \leq 1.0$; the solar orientation with respect to the concave spiral arm is kept fixed at $\phi_d = 20^\circ$. This range places the Sun from just inside the 4:1 ILR to the CR. In contrast to Fig. 2 here all panels have the axisymmetric background subtracted to reveal the symmetry of the spiral residuals. We are now looking for features that are strong enough to be detected in an actual pencil-beam survey. The $z_{\text{min}}, z_{\text{max}}$ values indicated above each panel give the maximum error introduced by spiral structure in the otherwise axisymmetric background disc. These values for the line-of-sight velocity $v_d$ and its standard deviation $\sigma_{v_d}$, are $\sim 35$ km s$^{-1}$ for $v_d = 220$ km s$^{-1}$ (left-hand and middle columns in Fig. 3), which is well above the resolution of upcoming radial velocity surveys ($\leq 3$ km s$^{-1}$). Strong features showing marked variation with the change in pattern speed are apparent in all three observables.

(1) In the case of $v_d$ high positive and negative velocity groups resulting from the effect of the 2:1 ILR are found at $(l, d/r_0) \approx (-30^\circ, 0.7)$ and $(l, d/r_0) \approx (25^\circ, 1.3)$, respectively, for $\Omega_d = 0.6\Omega_d^\circ$ (top left-hand panel). With the increase of pattern speed, these clumps spiral in a clockwise direction towards the Galactic Centre.

(2) The standard deviation of the line-of-sight velocities, $\sigma_{v_d}$, which can also be described as the ‘heating’ (or the random motions) of stars, peaks at a particular ring-like shape around the Galactic Centre for all pattern speeds (middle column). These rings are associated with the 2:1 ILR induced by the spiral density wave. It is clear that the radii of these rings are changing with the change of the pattern speed and thus the location of the 2:1 ILR. Beyond the CR ($\Omega_d = \Omega_d^\circ$) this resonance falls inside the inner 3 kpc or inside the Galactic bulge. Thus, using the radius of this hot ring to infer the location of the 2:1 ILR (and thus the pattern speed) is only valid if $\Omega_d$ values are in the range considered in Fig. 3.

(3) Lastly, the number density PBMs in Fig. 1 (right-hand column) are also indicative of the changing pattern speed. The disruption of the spiral arms near the 2:1 LRs and variation in spiral strength due to the encounter the second-order resonances and CR, creates a large contrast in these axisymmetric background subtracted PBMs. Many of these features can be used in addition to the information extracted from the velocities.

All of the features described above can be used to identify the location of the 2:1 ILR and thus the pattern speed. As we mentioned, however, if the solar circle is placed at or beyond the CR, the 2:1 ILR falls inside the Galactic bulge. Consequently, the features created by it disappear. Fortunately, just as this happens the 2:1 OLR enters the Galactic disc (our discs extend to a radius of $2r_0$) and similarly to the 2:1 ILR case, resonant features are created, this time in the outer parts of the disc.

5.2 Changing the Sun’s orientation with respect to a spiral arm

How can we infer the Sun’s azimuth with respect to the galactocentric line passing through the intersection of the solar circle and a concave spiral arm? To find out we plot PBMs of simulation runs with the same pattern speed and different solar phase angle. Fig. 4 shows such plots for a fixed $\Omega_d = 0.7\Omega_d^\circ$ and a solar phase angle changing from top to bottom in the range $\phi_d = [0^\circ, 40^\circ]$.

Similar to Fig. 3 we now look for strong features in the three observables that can be used to estimate $\phi_d$.

(1) Inspection of the left-hand column of Fig. 4 reveals a negative velocity stream which changes position with a change of phase angle. For $\phi_d = 0^\circ$ (top left-hand panel) this feature is centred on $l = 0^\circ$ at a heliocentric distance of $d/r_0 \approx 0.5$. As the angle is increased this steam moves to larger longitudes roughly preserving its distance from the Sun. Note that the high positive and negative features discussed in the context of $v_d$ in Fig. 3, do not vary as the angle is changed since the pattern speed is kept fixed.

(2) The line-of-sight velocity dispersion (middle column of Fig. 3) does not seem to be particularly useful for constraining the solar phase angle.

(3) Finally, the structure in the number density PBMs shows prominent variation with the change in solar angle. For example pencil-beam observations at $\pm 45^\circ$ would be drastically different depending on the phase angle.

In an actual survey we would first try to infer the position of the inner or outer LR as discussed in Section 5.1 and thus find the pattern speed.

6 FOUR-ARMED SPIRAL STRUCTURE

So far we have only discussed simulations involving a two-armed spiral density wave perturbation. In this section we show the effect of a four-armed structure, make comparison with the two-armed case and suggest a way to distinguish between the two.
Figure 3. Pencil-beam maps showing variation of structure in the observables with the change in pattern speed. The solar angle is kept fixed at \( \phi_0 = 20^\circ \) everywhere. In contrast to Fig. 2 here all panels have the axisymmetric background subtracted to reveal the symmetry of the spiral residuals. Note for the case of \( \Delta \sigma_v \), the ring-like shape around the Galactic Centre for all pattern speeds (middle column). These ‘hot’ rings are associated with the 2:1 ILR induced by the spiral density wave. As the 2:1 OLR enters the disc (around \( \Omega_1 = 0.9 \Omega_0 \), see Fig. 1) strong features in \( \Delta \sigma_v \) PBMs appear in the outer disc.

Fig. 5 shows PBMs of a galactic disc, similar to Fig. 3, but perturbed by a four-armed spiral structure. As in Fig. 3 pattern speed changed from top to bottom in the range \( \Omega_1 = [0.6, 1.0] \Omega_0 \) and the solar orientation with respect to a concave arm is kept fixed at \( \phi_0 = 20^\circ \). In this case the first-order resonances are the 4:1 ILR/OLR which occur at \( \Omega_1 = 0.65 \Omega_0, 1.35 \Omega_0 \), respectively.

As expected, more structure is apparent in all PBMs in the case of the four-armed structure. Note that the hot rings in \( \Delta \sigma_v \), present in the case of the two-armed structure (Fig. 3) are also apparent in the four-armed case although not as pronounced. The reason for this is the fact that the 2:1 ILR which causes these is a second order when \( m = 2 \). These hot rings can be used in both the two- and four-armed cases to estimate \( \Omega_2 \), but are expected to be much stronger for \( m = 2 \). Inspecting \( \Delta v_d \) and \( \Delta \Sigma \) (left- and right-hand columns) in Figs 3 and 5 it is clear that pencil-beam observations along Galactic longitudes \( l = 0^\circ \) or \( l = -90^\circ \), for example, can unambiguously distinguish between \( m = 2 \) and 4 spiral structure, as the oscillation frequency doubles when \( m = 4 \).

7 CONCLUSION

Upcoming Galactic disc surveys will reveal the age, composition and phase space distribution of stars within various Galactic components. These stellar excavations will provide essential clues for understanding the structure, formation and evolution of our Galaxy. To facilitate the interpretation of the huge amounts of data resulting from these surveys, Galactic disc models, such as the one presented here, are needed to interpret the observations.
We have investigated how the Milky Way spiral structure parameters, such as pattern speed and solar phase angle, can be estimated in a deep all-sky survey. We performed a series of test-particle simulations of a warm galactic disc approximating the disc kinematics of the Milky Way. We considered both two- and four-armed spiral structure and suggested a way to distinguish between the two using velocity and number density maps.

We found that the axisymmetric potential needs to be known to \( \sim 10 \) per cent, line-of-sight velocities to \( \sim 20 \) km s\(^{-1}\) and distance uncertainties need to be less than \( \sim 30 \) per cent. The mean line-of-sight velocity and the velocity dispersion are affected by up to \( \sim 35 \) km s\(^{-1}\), which is well within the detectable limit for forthcoming radial velocity surveys. Pattern speed can be constrained by a hot ring at the 2:1 ILR in both two- and four-armed spiral structures. To distinguish between the two, however, we also need information related to the velocities and stellar number density. If the pattern speed is such that the 2:1 ILR is hidden inside the Galactic bulge the 2:1 OLR would be present in the outer Galaxy and thus can equivalently be used to estimate the pattern speed. Once the pattern speed is known the solar angle can be estimated from the number density variation with heliocentric distance; \( \phi_0 \) is also reflected in the \( v_d \) PBMs.

Future work needs to address the issue of how to obtain the axisymmetric background potential needed to subtract from the observational data as discussed at the end of Section 5. Also, it is important to know what type of tracer stars are needed that would
allow the estimation of photometric parallaxes with errors less than \( \sim 30 \) per cent, and the distribution of those stars.

While here we only considered steady-state spiral structure, other theories of spiral structure, such as transient and swing-amplified spirals, need to be investigated as well. It has also been suggested that the Galaxy harbours two sets of spiral structure moving at the same (Lépine et al. 2001) or different (Minchev & Quillen 2006; Naoz & Shaviv 2007) pattern speeds. We expect in all those cases it will be again resonant features to relate to the pattern speed (Patsis & Kaufmann 1999) and solar angle. In the case of non-steady-state spirals, however, the structure in the PBMs will vary with integration time and interpretation will become more complicated. Thus it should be kept in mind that measurements based on using a steady-state model are likely to only be approximate if the system is evolving. Future work should consider time variable models, as well as estimate errors caused by using a steady-state model.

It is also known that the Milky Way is a barred galaxy. The simulations performed here do not include the influence of the bar. This is not necessarily a shortcoming since most of the features in the PBMs we use to infer spiral structure parameters are caused by resonances and, unless a resonance overlap with the central bar exists in the same location, those would not be different when a bar is included in the simulations. Future work should also look at this problem.

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