Structure in phase space associated with spiral and bar density waves in an $N$-body hybrid galactic disc

Alice C. Quillen, Jamie Dougherty, Micaela B. Bagley, Ivan Minchev, and Justin Comparetta

1 Department of Physics and Astronomy, University of Rochester, Rochester, NY 14627, USA
2 Steward Observatory, 933 N. Cherry Street, University of Arizona, AZ 85721, USA
3 Observatoire Astronomique, Université de Strasbourg, CNRS UMR 7550, 67000 Strasbourg, France
4 Astrophysikalisches Institut Potsdam, An der Sternwarte 16, D-14482, Potsdam, Germany

Accepted 2011 June 23. Received 2011 May 24; in original form 2010 October 25

ABSTRACT

An $N$-body hybrid simulation, integrating both massive and tracer particles, of a Galactic disc is used to study the stellar phase-space distribution or velocity distributions in different local neighbourhoods. Pattern speeds identified in Fourier spectrograms suggest that two- and three-armed spiral density waves, a bar and a lopsided motion are coupled in this simulation, with resonances of one pattern lying near resonances of other patterns. We construct radial and tangential $(u, v)$ velocity distributions from particles in different local neighbourhoods. More than one clump is common in these local velocity distributions regardless of the position in the disc. Features in the velocity distribution observed at one galactic radius are also seen in nearby neighbourhoods (at larger and smaller radii) but with shifted mean $v$ values. This is expected if the $v$ velocity component of a clump sets the mean orbital galactic radius of its stars. We find that gaps in the velocity distribution are associated with the radii of kinks or discontinuities in the spiral arms. These gaps also seem to be associated with Lindblad resonances with spiral density waves and so denote boundaries between different dominant patterns in the disc. We discuss implications for interpretations of the Milky Way disc based on local velocity distributions. Velocity distributions created from regions just outside the bar’s outer Lindblad resonance and with the bar oriented at $45^\circ$ from the Sun-Galactic centre line more closely resemble that seen in the solar neighbourhood (triangular in shape at lower $u$ and with a Hercules-like stream) when there is a strong nearby spiral arm, consistent with the observed Centaurus Arm tangent, just interior to the solar neighbourhood.

Key words: Galaxy: kinematics and dynamics – galaxies: kinematics and dynamics – galaxies: spiral.

1 INTRODUCTION

The velocity distribution of stars in the solar neighbourhood contains structure that has been particularly clearly revealed from Hipparcos observations (Dehnen & Binney 1998; Famaey et al. 2005). Much of this structure was previously associated with moving groups (Eggen 1996). Moving groups are associations of stars that are kinematically similar or have similar space motions. Young, early-type stars can be moving together because they carry the kinematic signature of their birth (Eggen 1996). However, a number of the kinematic clumps identified through the kinematic studies also contain later type and older stars (Dehnen & Binney 1996; Nordstrom et al. 2004; Arifyanto & Fuchs 2006; Famaey, Siebert & Jorissen 2008).

In a small region or neighbourhood of a galaxy, structure in the stellar velocity distribution can be caused by different dynamical processes. Bar or spiral patterns can induce structure in the phase-space distribution, particularly in the vicinity of Lindblad resonances (LRs; Yuan & Kuo 1997; Dehnen 1999; Fux 2001; Quillen & Minchev 2005; Minchev & Quillen 2006; Antoja et al. 2009; Gardner et al. 2010; Lepine et al. 2011). Mergers and orbiting satellite subhaloes can leave behind stellar streams (e.g. Bekki & Freeman 2003; Meza et al. 2005; Helmi et al. 2006; Gomez et al. 2010). In a local neighbourhood, these could be seen as increases in the stellar number density at a particular velocity.

The velocity vector of a star in the solar neighbourhood can be described in Galactocentric coordinates with components $(u, v, w)$. Here $u$ is the radial component with positive $u$ for motion towards...
the Galactic centre. The $v$ component is the tangential velocity component subtracted by the velocity of a particle in a circular orbit computed from the azimuthally averaged mid-plane gravitational potential. The $v$ component is positive for a particle with tangential component larger than the circular velocity that is moving in the same direction as disc rotation. The $w$ component is the velocity in the $z$ direction.

Quillen & Minchev (2005) showed that LRs associated with a spiral pattern, given by their approximate $v$ velocity component, were likely locations for deficits in the velocity distribution in the solar neighbourhood (also see Fux 2001; Lepine et al. 2011). On either side of the resonance the shape of orbits changes. For example closed or periodic orbits can shift orientation so they are parallel to a bar on one side of the resonance and perpendicular on the other side. On resonance there may be no nearly circular orbits. As a consequence in a specific local neighbourhood there can be a particular velocity associated with resonance that corresponds only to orbits with high epicyclic variations. These may not be well populated by stars leading to a deficit of stars in the local velocity distribution at this particular velocity. This picture is also consistent with the division between the Hercules stream and the thin disc stars that is likely caused by the $m = 2$ outer Lindblad resonance (OLR) with the Galactic bar (Dehnen 2000; Minchev, Nordhaus & Quillen 2007; Gardner et al. 2010).

Structure seen in the velocity distribution can also be caused by recent large-scale perturbations. Perturbations on the Galactic disc caused by bar formation or a galactic encounter can perturb the action angle distribution. Subsequent phase wrapping can induce structure in the velocity distribution (Minchev et al. 2009; Quillen et al. 2009). Transient spiral density waves can also leave behind structure in the local phase-space distribution (De Simone, Wu & Tremaine 2004). The local velocity distributions can be particularly complex when both spiral and bar perturbations are present or when multiple spiral patterns are present (Quillen 2003; Minchev & Quillen 2006; Chakrabarty & Sideris 2008; Antoja et al. 2009; Minchev & Famaey 2010; Minchev et al. 2010, 2011).

The variety of explanations for structure in the regional phase-space distribution presents a challenge for interpretation of current and forthcoming radial velocity and proper motion surveys. We may better understand the imprint of different dynamical structures in the phase-space distribution by probing simplified numerical models. Most numerical studies of structure in localized regions of the galactic disc phase-space distribution have been done using test particle integrations (e.g. Dehnen 2000; De Simone et al. 2004; Quillen & Minchev 2005; Chakrabarty & Sideris 2008; Antoja et al. 2009; Minchev et al. 2009; Quillen et al. 2009; Gardner et al. 2010). Few studies have probed the local phase-space distribution in an $N$-body simulation of a galactic disc (but see Fux 2000) because of the large number of particles required to resolve structure in both physical and velocity space. In this paper we use an $N$-body hybrid simulation, integrating both massive and tracer particles, that exhibits both spiral and bar structures to probe the velocity distribution in different regions of a simulated disc galaxy. Our goal is to relate structure in the local phase-space distribution to bar and spiral patterns that we can directly measure in the simulation.

2 SIMULATIONS

In this section we describe our $N$-body hybrid simulations and their initial conditions. We also describe our procedure for making spectrograms of the Fourier components so that we can identify pattern speeds in the simulations.

2.1 The $N$-body hybrid code

The $N$-body integrator used is a direct-summation code called $\phi$-GRAPE (Harfst et al. 2007) that employs a fourth-order Hermite integration scheme with hierarchical commensurate block times steps (Makino & Aarseth 1992). Instead of using special-purpose GRAPE hardware we use the Sapporo subroutine library (Gaburov, Harfst & Portegies Zwart 2009) that closely matches the GRAPE-6 subroutine library (Makino et al. 2003) but allows the force computations to be done on graphics processing units (GPUs). The integrator has been modified so that massless tracer particles can simultaneously be integrated along with the massive particles (Comparetta & Quillen 2011). The tracer particles help us resolve structure in phase space without compromising on the capability of carrying out a self-consistent $N$-body simulation. The simulations were run on a node with 2 NVIDIA Quadro FX 5800 GPUs. The cards are capable of performing double-precision floating point computations; however the computations were optimized on these GPUs to achieve double-precision accuracy with single-precision computations using a corrector (Gaburov et al. 2009). The Quadro FX cards were designed to target the Computer-Aided Design and Digital Content Creation audience and so were slower but more accurate than other similar video cards even though they did not implement error correction code.

2.2 Initial conditions for the integration

The initial conditions for a model Milky Way galaxy were generated using numerical phase-phase distribution functions using the method discussed by Widrow, Pyn & Dubinski (2008) and their numerical routines which are described by Widrow et al. (2008), Kuijken & Dubinski (1995) and Widrow & Dubinski (2005). This code, called GalacticICS, computes a gravitational potential for bulge, disc and halo components, and then computes the distribution function for each component. Particle initial conditions are then computed for each component. The galactic bulge is consistent with a Sérsic law for the projected density. The halo density profile is described by five parameters, a halo scalelength, a cusp exponent, a velocity dispersion, and an inner truncation radius and truncation smoothness, so it is more general than an NFW profile. The disc falls off exponentially with radius and as a sech$^2$ with vertical height. We have adopted the parameters for the Milky Way model listed in table 2 in Widrow et al. (2008). A suite of observational constraints are used to constrain the parameters of this model, including the Oort constants, the bulge dispersion, the local velocity ellipsoid, the outer rotation curve, vertical forces above the disc and the disc surface brightness profile.

The number of massive particles we simulated for the halo, bulge and disc is 50,000, 150,000 and 800,000, respectively. The halo is live. The number of test (massless) particles in the disc is 3 million and exceeds the number of massive particles by a factor of 3. The total mass in disc, bulge and halo is $5.3 \times 10^{10}$, $8.3 \times 10^{9}$ and $4.6 \times 10^{11}$ $M_\odot$, respectively. The mass of halo, bulge and disc particles is $9.2 \times 10^5$, $5.5 \times 10^5$ and $6.6 \times 10^4$ $M_\odot$, respectively. Snapshots were output every 5 Myr and our simulation ran for 1.3 Gyr.

The smoothing length is 10 pc and is similar to the mean disc interparticle spacing. We have checked that the centre of mass of the simulation drifts a distance shorter than the smoothing length.
during the entire simulation. The parameters used to choose time-steps in the Hermite integrator were \( \eta = 0.1 \) and \( \eta_t = 0.01 \) (see Makino & Aarseth 1992; Harfst et al. 2007). The minimum time-step is proportional to \( \sqrt{N} \) and is estimated from the ratio of the acceleration to the jerk. Our value places the minimum possible time-step near the boundary of recommended practice.\(^1\) The total energy drops by 0.2 per cent during the simulation, at the boundary of recommended practice that suggests a maximum energy error \( \delta E/E \lesssim 1/\sqrt{N} \) (see Quinlan & Tremaine 1992 on how the shadow time-step near the boundary of recommended practice.

In our goal to well resolve the galactic disc we have purposely undersampled both halo and bulge, and so have introduced spurious numerical noise into the simulation from these populations. The product of the mass of halo particles and the halo density exceeds that of the same product for the disc implying that numerical heating from the massive particles in the disc is lower than that induced by the live halo particles. The large time-steps, relatively small smoothing length and large halo particles imply that this simulation is noisier than many discussed in the literature.

We first discuss the morphology of the simulated galaxy and then use the spectrograms constructed from the disc density as a function of time to identify patterns.

3 MORPHOLOGY

In Fig. 1 we show the disc density projected on to the mid-plane in Cartesian coordinates for a few of the simulation outputs (snapshots). Disc particles are used to create these density histograms, but bulge and halo particles are neglected. In Figs 1(a) and (b) both massive and tracer particles are shown. Fig. 1(c) is similar to Fig. 1(b) except only massive disc particles are shown. We find no significant differences between the distribution of massive and tracer disc particles, consistent with the previous study using the same hybrid code (Comparetta & Quillen 2011).

The numbers of massive particles are sufficient enough that numerical noise is present and amplified to become spiral structure. During the initial 40 snapshots (over a total time of 200 Myr) (shown in Fig. 1a) two- and three-armed spiral arms grow in the region with radius 5–12 kpc. At around a time \( T \sim 225 \) Myr an open and somewhat lopsided axisymmetric structure grows within a radius of 4 kpc that becomes the bar. At later times both two- and three-armed structure exists outside the bar (shown in Fig. 1b).

As our halo is live (and not fixed) lopsided motions are not prevented in the simulation. Lopsided motions and three-armed structure have previously been observed in N-body simulations of isolated discs (e.g. see the simulations by Chilingarian et al. 2010). The bulge does not remain fixed during the simulation as a lopsided perturbation develops that moves its centre slowly up to a distance of 0.5 kpc away from the initial origin. This distance is larger than the distance our centre of mass drifts during the simulation (less than 10 pc). The lopsided motion may be related to the development of three-armed structures in the disc and asymmetries in the bar. Lopsidedness is common in spiral galaxies (see Jog & Combes 2004; Saha, Combes & Jog 2007). If the pattern speed is slow, a lopsided mode can be long lasting (Ideta 2002). There is no single strong frequency or amplitude associated with the lopsided motion in our simulation, though the motion is primarily counter-clockwise and moving with the direction of galactic rotation. We have checked that the centre of mass of the simulation drifts less than the smoothing length of 10 pc so the lopsided motion is not due to an unphysical drift in the total momentum in the simulation. However, the growth of the lopsided motion could be due to noise associated with our small smoothing length and large halo mass particles. Simulations of a disc residing in a rigid halo would not exhibit lopsided motions. However, as most galaxies display lopsidedness we can consider it a realistic characteristic of a galactic disc. We will discuss this motion later on when we discuss the \( m = 1 \) Fourier component spectrograms.

We also show the disc density in polar coordinates in Figs 2 and 3. These figures show differential density histograms as a function of radius and azimuthal angle \( (r, \phi) \) (or in cylindrical coordinates again projected on to the mid-plane). The bins to create these two plots are logarithmically spaced in radius. The disc density \( \Sigma(\phi, r) \) in these two plots has been normalized at each radius. These histogram show \( \langle \Sigma(\phi, r) \rangle = \Sigma(r) \) where \( \Sigma(r) \) is the mean density at radius \( r \) averaged over the angle \( \phi \). A logarithmic spiral would give high densities along a straight line in these plots, with slope set by the spiral arm pitch angle. Two-armed structures would correspond to two linear but sloped features, each separated by 180°. Trailing spiral structures have negative slopes. A bar on these diagrams would correspond to two vertical features separated by 180°. The centroid of the galaxy bulge is subtracted prior to computing the density histograms shown in Figs 2 and 3 in polar coordinates but not in Fig. 1 that is in Cartesian coordinates. The times of the snapshots in Myr are shown in the upper-right corners of each panel shown in Figs 1–3.

In Fig. 2 the differential surface density in polar coordinates is shown at three different times early in the simulation and during the early phase of spiral arm growth. The nearly vertical strip in the leftmost panel tilts and moves to the right at later times. In the rightmost panel it is a stronger perturbation and with a larger, trailing pitch angle. The spiral arm shown in the centre of the leftmost panel becomes more tightly wound, and stronger (as seen on the right) as would be expected from a swing amplification mechanism. After about 50 Myr the spiral patterns are more slowly evolving. Other than during bar formation at about 330 Myr, and in the beginning of the simulation before 50 Myr, the spiral pitch angles do not seem to vary significantly.

In Fig. 3 the differential surface density is also shown at three different times but after bar formation. On the lower parts of the panels the bar is seen as a pair of vertical strips. We can see that the bar length increases later in the simulation. Exterior to the bar, both two- and three-armed patterns are seen and their patterns move more slowly than the bar. The winding or pitch angle of spiral features is about 24° and \( \alpha \equiv \text{d}p/\text{d}n_r \sim 2.3 \). This pitch angle exceeds many but not all estimates for spiral arm pitch angles near the solar neighbourhood (see table 1 in Vallée 2008; most pitch angle estimates are \( \sim 13° \)). Other than during bar formation and before 50 Myr, the spiral pitch angles do not seem to vary significantly.

When animated the two- and three-armed features appear to constructively and destructively add with density peaks occurring at times and positions when two patterns lie on top of one another (e.g. as discussed by Henry, Quillen & Gutermuth 2003; Meidt, Rand & Merrifield 2009). The sum of two different patterns can cause individual arm peaks to shift position and temporarily appear more or less tightly wound.

---

\(^1\) N-Body Simulation Techniques: There is a right way and a wrong way, by Katz N., http://supernova.lbl.gov/~evlinder/umass/somulid/com8lx.ps

\(^2\) http://www.ifa.hawaii.edu/~barnes/ast626_09/NBody.pdf
Figure 1. Disc stellar densities projected into the $xy$ plane during early spiral arm and bar growth. The time of each snapshot is shown in Myr on the lower right of each panel. (a) Here we show the earlier part of the simulation. During this time we see initial stochastic spiral arm growth followed by the onset of bar instability. (b) Here we show snapshots from the later part of the simulation. We can see that the bar is not symmetrical; it can be lopsided. At times the galaxy appears to have three arms (e.g. at $T = 750$ Myr). Kinks and discontinuities are commonly seen in the spiral structure. (c) The same as (b) except only massive disc particles are shown. There is no significant difference between the distributions of massive and massless disc particles.

Kinks or changes in pitch angle of the spiral arms seen in the density images in Cartesian coordinates (in Fig. 1b) correspond to places where the spiral arms are discontinuous or change slope in the density distribution as seen in the polar plots (e.g. Fig. 3). For example, the spiral pattern just outside the bar has a lower pattern speed than the bar. Consequently, the phase of the spiral arms varies with respect to the bar. At times spiral arms appear to be linked with the bar and at other times they appear separated from the bar. Spiral arms in the outer galaxy have slower pattern speeds than those in the inner galaxy. At times the spiral arms appear to be connected, and at other times, armlets or kinks are seen.

4 SPIRAL AND BAR PATTERNS IN THE SIMULATION

Before we discuss velocity distributions extracted from local neighbourhoods we first discuss patterns of waves measured from the simulations.
Differential surface density as a function of the log of the radius $r$ and azimuthal angle $\phi$. Here, we show the surface density at three different times early in the simulation and during spiral arm growth. The times of the snapshots in Myr are shown in the upper-right corners of each plot. Patterns move to the right with time and trailing structures have negative slopes. A logarithmic spiral arm has high densities along a line on this plot with pitch angle given by the slope of the line. The vertical feature in the leftmost panel tilts and moves to the right in the centre and then rightmost panel. In the rightmost panel it is a stronger perturbation and with a larger and trailing pitch angle. The spiral arm shown in the centre of the leftmost panel becomes more tightly wound, and stronger, as would be expected from a swing amplification mechanism.

**Figure 2.**

Similar to Fig. 2 except showing the differential surface density after bar formation. On the lower parts of these panels the vertical features are the bar. There are two vertical lines per panel as would be expected from an $m = 2$ bar-like structure. Exterior to the bar, both two-armed and three-armed patterns are seen and their patterns move more slowly than the bar. On an animated version of this plot it is clear that patterns in the inner galaxy are faster than those in the outer galaxy. Interference between the patterns sometimes causes armlets to appear and disappear.

**Figure 3.**

<table>
<thead>
<tr>
<th>280</th>
<th>390</th>
<th>525</th>
<th>700</th>
</tr>
</thead>
<tbody>
<tr>
<td>log$_{10}$r (kpc)</td>
<td>log$_{10}$r (kpc)</td>
<td>log$_{10}$r (kpc)</td>
<td>log$_{10}$r (kpc)</td>
</tr>
<tr>
<td>-150-0</td>
<td>-150-0</td>
<td>-150-0</td>
<td>-150-0</td>
</tr>
<tr>
<td>50 50 50 50</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25 40 60 100</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2 0.2 0.2 0.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1 0.1 0.1 0.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2 0.2 0.2 0.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0 0.0 0.0 0.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1 0.1 0.1 0.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2 0.2 0.2 0.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0 0.0 0.0 0.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1 0.1 0.1 0.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2 0.2 0.2 0.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0 0.0 0.0 0.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1 0.1 0.1 0.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2 0.2 0.2 0.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0 0.0 0.0 0.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1 0.1 0.1 0.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2 0.2 0.2 0.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0 0.0 0.0 0.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1 0.1 0.1 0.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2 0.2 0.2 0.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0 0.0 0.0 0.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1 0.1 0.1 0.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2 0.2 0.2 0.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0 0.0 0.0 0.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1 0.1 0.1 0.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2 0.2 0.2 0.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0 0.0 0.0 0.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1 0.1 0.1 0.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2 0.2 0.2 0.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0 0.0 0.0 0.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1 0.1 0.1 0.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2 0.2 0.2 0.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0 0.0 0.0 0.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1 0.1 0.1 0.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2 0.2 0.2 0.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0 0.0 0.0 0.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1 0.1 0.1 0.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2 0.2 0.2 0.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0 0.0 0.0 0.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1 0.1 0.1 0.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2 0.2 0.2 0.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0 0.0 0.0 0.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1 0.1 0.1 0.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2 0.2 0.2 0.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0 0.0 0.0 0.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1 0.1 0.1 0.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2 0.2 0.2 0.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0 0.0 0.0 0.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1 0.1 0.1 0.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2 0.2 0.2 0.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0 0.0 0.0 0.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1 0.1 0.1 0.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2 0.2 0.2 0.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0 0.0 0.0 0.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1 0.1 0.1 0.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2 0.2 0.2 0.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0 0.0 0.0 0.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1 0.1 0.1 0.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2 0.2 0.2 0.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0 0.0 0.0 0.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1 0.1 0.1 0.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2 0.2 0.2 0.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0 0.0 0.0 0.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1 0.1 0.1 0.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2 0.2 0.2 0.2</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**4.1 Constructing spectrograms**

We constructed spectrograms using the procedure outlined by Meidt et al. (2008) in their section 3.2 (also see section 3.4 in Masset & Tagger 1997). In each radial bin, defined by a binning function $b(r)$, and at each snapshot time $t$ we compute

$$W_C(r, t) = \sum_i \cos(m \theta) b(r)$$

$$W_m(r, t) = \sum_i \sin(m \theta) b(r)$$

(1)

where the sum is over all disc particles in the radial bin (both massive and tracer) and the integer, $m$, is the azimuthal wavenumber. Bulge and halo particles are ignored. We use logarithmically spaced radial bins. We compute the Fourier transform of the complex function composed of $W_C(r, t)$ and $W_m(r, t)$ or

$$\tilde{W}(\omega, r) = \int_{T_1}^{T_2} e^{i\omega t} h(t) \left[W_C(r, t) + i W_m(r, t)\right] dt$$

(2)

where $h(t)$ is a Hanning function spanning our time window $[T_1$, $T_2]$ between snapshot time $T_1$ and $T_2$. In our spectrogram figures we show the amplitude of the complex Fourier components or $|\tilde{W}(\omega, r)|$. In our spectrograms the angular frequency, $\omega$, increases along the y-axis and $\log_{10} r$ increases along the x-axis.

A bar or two-armed spiral structure has strong $m = 2$ Fourier components in its density distribution (calculated with equation 1). The pattern speed of the wave is related to its angular frequency by $\Omega_0 \equiv \omega/m$. Pattern speeds are found by dividing the angular frequencies of horizontal features in the spectrograms by the integer $m$.

During an $N$-body simulation with a non-stationary or live halo, lopsided modes or waves may develop and the bulge and central disc of the galaxy may not remain at a fixed position. If the Fourier components are measured with respect to a radial position that is not centred on the bulge centroid then an $m = 1$ component can generate other Fourier components. This is sometimes called ‘aliasing’ or ‘mode-mixing’ (e.g. Olling & Dehnen 2003). As a consequence we subtract the position of the centroid of the galaxy bulge prior to computing the Fourier coefficients with $m > 1$. We will also discuss the $m = 1$ spectrogram computed without subtracting the bulge centroid position.

**4.2 Pattern speeds**

After $T \sim 400$ Myr the bar becomes stable and then evolves more slowly during the rest of the simulation. Well-defined frequencies are seen in spectrograms after bar formation. Fig. 4 shows spectrograms constructed from $m = 2$ Fourier components, whereas
Figure 4. Spectrogram of the $m=2$ Fourier components in a window spanning (a) from time $T=350$ to 950 Myr and (b) from time $T=750$ to 1305 Myr. The $y$-axis is angular frequency in $\text{Myr}^{-1}$ and the $x$-axis is radius in kpc but shown on a log scale. Overplotted are five lines that are twice the following angular frequencies in order of bottom to top $\Omega - \kappa/2, \Omega - \kappa/4, \Omega, \Omega + \kappa/4$ and $\Omega + \kappa/2$. The bar, once present, is stable and has a slowly decreasing pattern speed. There are at least two two-armed spiral patterns. The spiral patterns at both early and later times seem to extend between their inner $m=2$ or 4 LR out to their corotation radii.

Figure 5. Spectrogram of the $m=4$ Fourier component in a window spanning from (a) time $T=350$ to 950 Myr and (b) from $750$ to 1305 Myr. Similar to Fig. 4. With the exception of the bar (with $\omega = 0.016$ at early times and $0.013 \text{Myr}^{-1}$ at later times) the spiral patterns seen here are strongest between their inner and outer $m=4$ LRs (as seen via intersections with green lines showing $4\omega \pm \kappa$). Higher frequency features are seen above the bar’s angular frequency at both early and later times.

Figs 5, 6 and 7 show spectrograms constructed from $m=4, 3$ and 1 Fourier components, respectively. The spectrogram from Fig. 4(a) is measured from the middle of the simulation whereas 4(b) is measured from the second half of the simulation. The same time periods are used for Figs 5–7. We can see from a comparison of Figs 4(a) and (b) (also see Figs 1 and 3) that the bar grows longer and its rotation rate slowly decreases during the simulation. At 400 Myr the bar is only 6 kpc long (semimajor axis) but by 700 Myr its semimajor axis increases to 11 kpc.

Pattern speeds of waves can be estimated from a spectrogram by dividing the angular frequency of a strong feature by the integer $m$ used to make the spectrogram. In our spectrograms we plot the angular frequency, $\omega$, rather than the pattern speed $\Omega = \omega m$, following most previous works (e.g. Sellwood & Sparke 1988; Masset & Tagger 1997). Estimated pattern speeds for waves seen in the presented spectrograms are listed in Table 1. The strongest pattern seen in the $m=2$ spectrogram shown in Fig. 4(a) is the bar wave that has an angular frequency of $\omega_B \approx 0.080 \text{Myr}^{-1}$ corresponding to a
A. C. Quillen et al.

Figure 6. Spectrogram of the \( m = 3 \) Fourier components in a window spanning from (a) time \( T = 350 \) to 950 Myr and (b) from 750 to 1305 Myr. Similar to Fig. 4 but constructed using the \( m = 3 \) Fourier components instead of the \( m = 2 \) ones. Overplotted are seven lines that are three times the following angular frequencies in order of bottom to top: \( \Omega - \kappa/2, \Omega - \kappa/3, \Omega - \kappa/4, \Omega + \kappa/4, \Omega + \kappa/3, \Omega + \kappa/2 \). The dominant three-armed pattern extends between its \( m = 3 \) ILR and its \( m = 3 \) OLR at both early and later times. The bar itself on this plot has an angular frequency of \( \omega_b \approx 0.120 \) at early times and \( \omega_b \approx 0.105 \) Myr\(^{-1} \) at later times. There is a possible coupling between the CR of the bar and the \( m = 3 \) ILR of the three-armed spiral pattern as these two resonances are close.

Figure 7. Spectrogram of the \( m = 1 \) Fourier component in a window spanning from (a) time \( T = 350 \) to 950 Myr and (b) from 750 to 1305 Myr. Similar to Fig. 6. The bulge centroid position was not subtracted previous to measurement of this spectrogram. On the left the bright feature at \( \omega = 0.08 \) Myr\(^{-1} \) is associated with the bar. The bar can be seen in the \( m = 1 \) spectrogram because it moves with the lopsided oscillation. Low-frequency features may be symptomatic of non-linear coupling between waves.

The pattern speed of \( \Omega_b \approx 0.040 \) Myr\(^{-1} \). There are slower patterns at larger radii seen both in the middle and in later parts of the simulation. These correspond to two-armed spiral patterns. Fig. 6 showing the \( m = 3 \) spectrograms shows that there is also at least one strong three-armed pattern present in the simulation.

Figs 4 and 5 show curves corresponding to the angular frequencies \( m(\Omega \pm \kappa/2) \), \( m(\Omega \pm \kappa/4) \), \( m\Omega \) with \( m = 2 \) and \( m = 4 \), respectively, where \( m \) is that used to compute the spectrogram. Each pair of lines (when there is a \( \pm \) in the above listed frequencies) is the same colour in the figures. These lines can be used to identify inner Lindblad resonance (ILR) and OLRs. Figs 6 and 7 in addition show curves corresponding to the angular frequencies \( m(\Omega \pm \kappa/3) \), with \( m = 3 \) and \( m = 1 \), respectively. Here, \( \Omega(r) \) and \( \kappa(r) \) are the angular rotation rate and the epicyclic frequency. We computed these frequencies using a rotation curve generated from the azimuthally averaged mid-plane gravitational potential measured from the positions of all the massive particles at time \( T = 500 \) Myr.

© 2011 The Authors, MNRAS 417, 762–784
Estimated pattern speeds and locations of resonances.

<table>
<thead>
<tr>
<th>Pattern</th>
<th>Time (Myr)</th>
<th>Pattern speed (radians Myr(^{-1}))</th>
<th>(r_{\text{in}} - r_{\text{out}}) (kpc)</th>
<th>ILR(_2) (kpc)</th>
<th>ILR(_3) (kpc)</th>
<th>ILR(_4) (kpc)</th>
<th>CR (kpc)</th>
<th>OLR(_1) (kpc)</th>
<th>OLR(_2) (kpc)</th>
<th>OLR(_3) (kpc)</th>
<th>OLR(_4) (kpc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bar</td>
<td>350–950</td>
<td>0.040</td>
<td>0.5–6</td>
<td>2.4</td>
<td>3.6</td>
<td>5.1</td>
<td>7.2</td>
<td>7.6</td>
<td>9</td>
<td>10</td>
<td>11</td>
</tr>
<tr>
<td>Inner two-armed spiral</td>
<td>350–950</td>
<td>0.030</td>
<td>3–10</td>
<td>2.4</td>
<td>3.8</td>
<td>4.4</td>
<td>7.2</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>17</td>
</tr>
<tr>
<td>Outer two-armed spiral</td>
<td>350–950</td>
<td>0.015</td>
<td>6–15</td>
<td>4.7</td>
<td>7.1</td>
<td>9.1</td>
<td>13.5</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>19</td>
</tr>
<tr>
<td>Three-armed spiral</td>
<td>350–950</td>
<td>0.023</td>
<td>3–15</td>
<td>4.5</td>
<td>9.1</td>
<td>11.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bar</td>
<td>750–1305</td>
<td>0.035</td>
<td>0.5–6</td>
<td>3.6</td>
<td>3.0</td>
<td>6.0</td>
<td>8.1</td>
<td>9.1</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inner two-armed spiral</td>
<td>750–1305</td>
<td>0.022</td>
<td>3–10</td>
<td>3.2</td>
<td>4.9</td>
<td>6.0</td>
<td>9.5</td>
<td>12.0</td>
<td>13.3</td>
<td>14</td>
<td>17</td>
</tr>
<tr>
<td>Outer two-armed spiral</td>
<td>750–1305</td>
<td>0.013</td>
<td>6–20</td>
<td>6.3</td>
<td>8.9</td>
<td>10.9</td>
<td>15</td>
<td>19</td>
<td>19.5</td>
<td>22</td>
<td>22</td>
</tr>
<tr>
<td>Three-armed spiral</td>
<td>750–1305</td>
<td>0.023</td>
<td>5–15</td>
<td>4.5</td>
<td>9.1</td>
<td>12.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

These pattern speeds are measured from features seen in the spectrograms shown in Figs 4 and 6. The pattern speeds can be approximately converted to units of km s\(^{-1}\) kpc\(^{-1}\) by multiplying by 1000. The times listed in the second column correspond to the range of times used to construct these spectrograms. The radii \(r_{\text{in}}\) and \(r_{\text{out}}\) are estimated inner and outer radii in kpc of the individual patterns. Here ILR\(_2\) and OLR\(_2\) refer to the inner and outer \(m = 2\) LRs. Likewise ILR\(_3\) and ILR\(_3\) refer to the \(m = 3\) and \(m = 4\) ILRs. When a resonance from one pattern is near that of another pattern, one pattern could be coupled to or driving the other.

An LR with a pattern speed, \(\Omega_p\), and integer \(m\) occurs where

\[
m(\Omega - \Omega_p) = \pm \kappa
\]

(see Binney & Tremaine 1987). The sign in the above determines whether the resonance is an ILR or OLR. A corotation resonance (CR) occurs where \(\Omega = \Omega_p\), or the angular rotation rate equals the pattern speed. Here, we refer to each resonance by its integer, here denoted as \(m\). Other works refer to the \(m = 4\) ILR as the 4:1 ILR (e.g. Meidt et al. 2009) or the 4/1 ILR (e.g. Rautiainen & Salo 1999). We also use the notation ILR\(_2\) to refer to the \(m = 4\) ILR. The radii of the estimated Lindblad and CRs for pattern speeds estimated from the spectrograms are also shown in Table 1. Resonance locations are estimated using the rotation curve computed from the azimuthally averaged density distribution at time \(T = 500\) Myr. The bar pattern also contains non-zero Fourier components in its density distribution with \(m \neq 2\). As the bar lacks symmetry when rotated 180\(^\circ\), or when reflected about its major or minor axis, it also contains non-zero odd-\(m\) Fourier components in its density distribution.

We see in Fig. 4(a) that the bar pattern extends out to its corotation radius where the bar pattern intersects 2\(\Omega_0\) (the dark blue line) at about 6 kpc. The pattern is seen past the bar’s corotation radius in the spectrogram, consistent with previous studies (e.g. see fig. 3 in Rautiainen & Salo 1999). Between the CR and the bars \(m = 2\) outer Lindblad resonance (OLR\(_2\)) periodic orbits can be elongated and oriented perpendicular to the bar.

Recent studies of the Milky Way have revealed two bar-like structures, a long thin bar (Benjamin et al. 2005) at about 45\(^\circ\) from the Sun/Galactic centre line and half length 4.4 kpc and a shorter or traditional thicker bar (or triaxial bulge component) at about 15\(^\circ\) (van der Marel et al. 1983). Estimates of the bar’s pattern speed are in the range 50–55 km s\(^{-1}\) kpc\(^{-1}\) or \(~0.055\) Myr\(^{-1}\) (Dehnen 2000; Minchev et al. 2007; Gardner et al. 2010). As shown by Gardner et al. (2010) the Hercules stream is consistent with either long or short bar as long as the pattern speed places the solar neighbourhood just outside the bar’s OLR\(_2\). Our simulated bar has half length \(~5\) kpc and so is longer than the Milky Way’s long bar, but this is consistent with its slower pattern speed at 0.040 Myr\(^{-1}\) or 40 km s\(^{-1}\) kpc\(^{-1}\). In our simulation a position corresponding to that of the solar neighbourhood would be at a galactocentric radius of about 10 kpc, just outside the bar’s OLR.

At both earlier and later times, features at lower pattern speeds are also seen in the \(m = 2\) spectrograms. A peak just past the end of the bar is seen with a pattern speed of \(\Omega_2 \sim 0.30\) Myr\(^{-1}\) at earlier times (Fig. 4a) and at \(\Omega_3 \sim 0.022\) at later times (Fig. 4b). This would correspond to an inner two-armed spiral pattern. This pattern appears to end within its own corotation radius at 7 or 9 kpc (early and late, respectively). There is also a slower outer two-armed spiral pattern primarily seen at early times with a pattern speed \(\Omega_2 \sim 0.015\) (though possibly also at later times with \(\Omega_3 \sim 0.010\)). This pattern also ends near its own CR.

There is a weak and narrow higher frequency peak in the \(m = 2\) spectrograms near the end of the bar at an angular rotation frequency of \(\omega \sim 0.13\) and \(\omega \sim 0.11\) Myr\(^{-1}\) at early and late times, respectively. Unlike the slower frequency in the \(m = 2\) spectrograms that ends near their corotation radii, this one extends between its corotation radius and its OLR. There is a similar higher frequency feature in the \(m = 4\) spectrograms (Fig. 5) at \(\omega \sim 0.23\) and 0.17 Myr\(^{-1}\) at early and later times, respectively.

The \(m = 4\) spectrograms shown in Fig. 5 look remarkably similar to the \(m = 2\) spectrograms (Fig. 4). Two-armed and oval density distributions often contain \(m = 4\) components, so this is not necessarily surprising. The higher frequency features near the end of the bar are also seen in the \(m = 4\) spectrograms. With the exception of the bar itself, the peaks in the \(m = 4\) spectrum extend between their inner and outer \(m = 4\) LRs (shown as green lines). This differs from the two-armed spiral patterns that end within their corotation radii. In the \(m = 4\) spectrogram the inner spiral pattern appears split into two different peaks. We may be able to better measure angular frequencies in the \(m = 4\) spectrogram than the \(m = 2\) spectrogram because the angular frequencies are higher and so less affected by the precision limit set from the width of the time window used to compute the spectrograms.

Unlike the inner and outer two-armed patterns, the three-armed pattern both at early and at later times extends through its CR. This pattern (see yellow lines in the \(m = 3\) spectrograms shown in Fig. 6) extends between its \(m = 3\) ILR and OLRs. The pattern speed is difficult to measure at early times because there are two nearby features in the spectrogram. Both at early and later times the pattern speed of the three-armed spiral is about \(\Omega_3 \sim 0.023\) Myr\(^{-1}\).

As noted by Shaviv (2003) (see their table 3) pattern speeds estimated for spiral patterns in the solar neighbourhood fall into two groups. One group has pattern speed \(~27\) km s\(^{-1}\) kpc\(^{-1}\) similar to the angular rotation rate of a particle in a circular orbit, \(\Omega_{\odot}\), at the solar galactocentric radius, \(R_{\odot}\), and the other group has a pattern speed \(~14\) km s\(^{-1}\) kpc\(^{-1}\). The mock solar neighbourhood
position in our simulation lies outside the inner two-armed spiral pattern at 0.030 Myr$^{-1}$, but is affected by the three-armed spiral wave with pattern speed 0.023 Myr$^{-1}$ or $\sim 23 \text{ km s}^{-1} \text{ kpc}^{-1}$ and the outer two-armed spiral with a pattern speed of 0.015 Myr$^{-1}$ or $\sim 15 \text{ km s}^{-1} \text{ kpc}^{-1}$. If the spiral waves present in the solar neighbourhood are driven by the Galactic bar then it might be interesting to match the faster pattern speed measured in the solar neighbourhood with a three-armed pattern driven by the bar and the slower pattern speed with a two-armed pattern also possibly linked to the bar. However, with the exception of Naoz & Shaviv (2007), few studies have considered the interpretation of the solar neighbourhood in terms of two or more patterns.

Previous works have seen spiral structure in N-body or particle mesh simulations at pattern speeds that differ from that of the bar (e.g. Sellwood & Sparke 1988; Patsis & Kaufmann 1999; Rautiainen & Salo 1999; Voglis, Stavropoulos & Kalapotharakos 2006). Masset & Tagger (1997) interpreted this behaviour in terms of non-linear mode coupling (Tagger et al. 1987; Sygnet et al. 1988), whereas Sellwood & Lin (1989) interpret the differences in pattern speeds in terms of transient behaviour and a groove instability. The spiral patterns in the simulations by Sellwood & Sparke (1988) extend from their $m = 2$ ILRs to their OLRs (see their fig. 1). However, the patterns of the two spiral density waves seen by Patsis & Kaufmann (1999) in a simulation lacking a bar extend from their $m = 2$ inner to their outer $m = 4$ ILRs. Both of these situations differ from the two-armed structures in our simulations that end within their corotation radii. The simulations by Rautiainen & Salo (1999) are closest in behaviour to ours, showing multiple spiral patterns extending past the bar, with some patterns ending near or within their corotation radii.

As the spiral and bar patterns in our simulations are moderately long lived, the spiral patterns could be driven by the bar (Yuan & Kuo 1997; Masset & Tagger 1997; Rautiainen & Salo 1999). Rautiainen & Salo (1999) accounted for the ranges and speeds of spiral patterns using the non-linear wave-coupling scenario. They found that they could often associate the beginning of a slower outer pattern with the location of one of its resonances that is also a resonance with an inner faster bar or other spiral pattern. The overlap between the resonances relates one pattern speed to the other and couples one perturbation to the other. Rautiainen & Salo (1999) found examples of CR/ILR$_s$, CR/ILR$_{s'}$, OLR$_s$/CR and OLR$_s$/ILR$_s$ couplings between spiral and bar waves and between two different spiral density waves.

For our system there is a possible connection between the bar’s CR and the $m = 3$ ILR of the three-armed spiral armed wave. A comparison of resonance locations (see Table 1) shows that the bar’s CR lies close to the three-armed spiral’s $m = 3$ ILR. The radial distance between these resonances is small compared to the error from measurement in the spectrum. There is also a possible coupling between other resonances. It is difficult to differentiate between possibilities by resonance locations alone as driving could occur in a spatially broad region near a resonance. As suggested by Masset & Tagger (1997) we can test for mode or wave coupling by searching for waves at beat frequencies though Rautiainen & Salo (1999) found that expected beat waves are not always detectable.

4.3 Non-linear coupling of waves

The scenario introduced by Tagger et al. (1987) suggests that there are strong non-linearities in the stellar response when resonances coincide. Two waves (e.g. bar and spiral) couple to a third one at a beat frequency. For example, we consider azimuthal wavenumber $m_B$ associated with a bar perturbation at pattern speed $\Omega_B$ and so frequency $\omega_B = m_B\Omega_B$. We consider azimuthal wavenumber $m_s$ associated with a spiral pattern at pattern speed $\Omega_s$ and so angular frequency $\omega_s = m_s\Omega_s$. A third wave can be excited with an azimuthal wavenumber $m = m_B + m_s$ and angular frequency $\omega = \omega_B + \omega_s$ or with wavenumber $m = m_s - m_B$ and $\omega = \omega_B - \omega_s$.

An example explored by Tagger et al. (1987) is a bar and spiral mode, both with $m = 2$, coupled to a slow axisymmetric mode with $m = 0$ and a faster four-armed one with $m = 4$ (Masset & Tagger 1997). Like Masset & Tagger (1997) we see features in the $m = 4$ spectrograms at frequencies above that of the bar, and these can be considered predictions of the wave-coupling scenario. Unfortunately, the number of features present in the $m = 2$ and $m = 4$ spectrograms is high and the imprecision of measured frequencies makes it difficult to definitively determine whether there is coupling between $m = 2$, 4 waves and a slow axisymmetric mode.

As our simulation contains a bar ($m = 2$), a lopsided $m = 1$ motion and a three-armed wave we can consider coupling between these three waves. To create the $m = 2$, 3 and 4 spectrogram we first subtracted the bulge centroid; however the galaxy centre moves during the simulation. In Fig. 7, showing the $m = 1$ spectrograms but constructed in the inertial frame, we see features associated with the major patterns identified in the other spectrograms. For example we see a peak at $\omega = 0.08 \text{ Myr}^{-1}$ that coincides with the bar’s angular frequency. One interesting peak is at an angular rotation rate of $\omega \sim 0.010$ that is also seen in the $m = 3$ spectrogram at a radius of about 8 kpc ($\log_{10} r = 0.9$). At earlier times there is power at $\omega \sim 0.03$ in the $m = 1$ spectrogram that is broader than the similar feature seen in the $m = 2$ spectrogram. This suggests that there is sufficient power at low frequencies in the lopsided wave that it could couple to other waves.

Since there is power at low frequencies in lopsided motion, we can consider the possibility of a coupling between a slow lopsided mode and the bar. Here, the index and pattern speed of the lopsided wave are $m_L = 1$, $\Omega_L$ and that of the bar is $m_B = 2$, $\Omega_B$. The sum leads to a three-armed spiral pattern with angular frequency $\omega_s = \Omega_L + 2\Omega_B$ or pattern speed $\Omega_s = \frac{1}{2}(\Omega_L + 2\Omega_B)$. If we assume that this corresponds to the three-armed pattern with $\Omega_s \sim 0.028$ then we can solve for the angular frequency of the lopsided mode $\Omega_L \sim 0.01$. There may be power at this frequency as there are low-frequency features in the $m = 1$ spectrogram. Thus, a lopsided slow mode could be coupling the bar and the three-armed spiral waves.

A variety of resonant overlaps or non-linear wave couplings are possible and may account for the driving of the patterns present in the simulation. If we could more precisely measure frequencies of the patterns then we would better determine which associations are most important or likely. Unfortunately, peaks in the spectrograms are sensitive to the range (in time) of simulation data used to construct the spectrogram (e.g. compare Figs 4a to b). Longer time series are required to make more accurate frequency measurements. Since the pattern speeds are not fixed, increasing the size of the time window does not increase the precision of a frequency measurement. This is particularly the case for slow patterns that might be important for the non-linear coupling models.

In summary, we find that the spectrograms are rich in small features. The shape of the spectrograms seen at early times resembles that at later times implying that the frequencies of these smaller features are slowing along with the bar pattern. Even though there are many waves present, they are not stochastically appearing and disappearing but slowly evolving. LRs associated with one wave are likely to be coincident with resonances of another wave present

© 2011 The Authors, MNRAS 417, 762–784
in the simulation. Together these findings suggest that the waves present in the simulation are coupled due to non-linear interactions. The most convincing and strongest coupling is that between the bar, the lopsided wave and the three-armed spiral wave. The multitude of small features present in the spectrograms suggest that additional couplings are present. The presence of features in the spectrograms at frequencies above that of the bar is consistent with the wave-coupling scenario.

5 STRUCTURE IN THE UV PLANE

We first describe how we construct velocity distributions in local neighbourhoods. We then discuss possible relations between features seen in these velocity distribution and the spiral or bar patterns.

5.1 Velocity uv distribution histograms in local neighbourhoods

We construct velocity distribution histograms in different local neighbourhoods in the following manner. Consider a neighbourhood centred about a point in the disc plane with coordinates \((x_0, y_0, 0)\) or in cylindrical coordinates \((r_0, \phi_0, 0)\). We will refer to neighbourhoods by the cylindrical coordinates of their centres; \((r_0, \phi_0)\).

We consider a star or simulated particle in the neighbourhood of this centre point if its coordinates \((x, y, z)\) are distance in the plane, \(R = \sqrt{(x - x_0)^2 + (y - y_0)^2} < r_0 f\) where \(f\) is a dimensionless scaling factor setting the size of the neighbourhood. We compute the radial and tangential components of the velocity of each star with respect to the vector to the galactic centre defined by the central coordinates of the neighbourhood, \((x_0, y_0, 0)\). The \(r\) component is \(-1\) times the radial velocity component in this galactic coordinate system. The \(v\) component is the tangential component in this coordinate system minus the circular velocity estimated from the azimuthally averaged mid-plane rotation curve computed at time \(T = 500\) Myr.

Histograms showing the \(uv\) velocity distribution are computed from the disc particles in each neighbourhood. The computed histograms are effectively coarse-grained phase-space distributions. The larger the neighbourhood, the larger is number of stars in the neighbourhood. If there are too few stars in the neighbourhood then the velocity distribution is noisy. If the neighbourhood is large then structure present on small spatial scales may be masked or smoothed by the coarse graining. An extreme example would be if the neighbourhood were so large that it contained two spiral arm density peaks.

A series of velocity distributions \((uv\) histograms) are shown in Fig. 8 at simulation time \(T = 600\) Myr. Velocity distributions are extracted from neighbourhoods centred at eight equidistant angles and at 6 radii spaced logarithmically. Each row contains histograms from neighbourhoods at a particular galactocentric radius. The lowest row has \(r_0 = 4.1\) and the top row \(r_0 = 12.5\). Each radius differs from the previous by a factor of 1.25. Each column shows neighbourhoods with a particular galactocentric angle with the leftmost column aligned with the bar. Each neighbourhood is separated in angle by 45° with angle increasing counter-clockwise in the galaxy plane. Each neighbourhood has a size set by the factor \(f\) setting the maximum distance of stars included in the neighbourhood from its centre. For the neighbourhoods shown in Fig. 8 the ratio of the neighbourhood radius to \(r_0\) is \(f = 0.1\). Positions and sizes of each neighbourhood are shown in Fig. 9 in both Cartesian and polar coordinates. At \(r_0 = 8\) kpc the radius of the neighbourhood corresponds to 0.8 kpc which is small compared to some N-body studies (see e.g. Gomez et al. 2010 who used a neighbourhood radius of 2.5 kpc) but larger than the distance of stars in solar neighbourhood samples (e.g. Siebert et al. 2011).

We have compared the velocity distributions created with \(f = 0.1\) and with \(f = 0.05\) with smaller neighbourhoods. A comparison at time 800 Myr and at a radius of 10 kpc is shown in Fig. 10. We also show a velocity distribution constructed of massive disc particles only to illustrate that we see no evidence for significant differences between massive and massless disc particle populations. The velocity distributions in smaller neighbourhoods are noisier as would be expected since each neighbourhood contains fewer particles. The clumps in the plots are slightly sharper, otherwise the morphology is almost identical. The density distribution is relatively smooth (see Fig. 9), implying that the velocity dispersion is not low in the disc (though it is lower at larger radii than at smaller radii). A simulation of a colder disc with more tightly wound spiral arms might show features in the velocity distributions that are more strongly dependent on the size of the neighbourhood.

In Fig. 8 we see that multiple clumps exist in the local velocity distribution at all radii in the galaxy. Velocity clumps were previously seen in a local neighbourhood of the N-body simulation presented by Fux (2000). Since our simulation lacks mergers, the clumps in the velocity distribution are caused purely by dynamical phenomena in the disc (as also concluded by Fux 2000). Clumps in the velocity distribution in one neighbourhood are often also present in the nearby neighbourhood at a larger or smaller radius but at a shifted \(v\) velocity. For example at an angle of 315° there is an arc with a mean velocity of \(v \sim 50\) km s\(^{-1}\) for neighbourhood radius \(r_0 = 4.1\) that has a lower velocity of \(v \sim 10\) km s\(^{-1}\) for at \(r_0 = 5.1\). We can consider the possibility that a single-halo particle could be responsible for one of these features. The separation between the two neighbourhoods mentioned above is a kpc. However, the circular velocity at a distance of a kpc from one of our halo particles is only 2 km s\(^{-1}\). Thus a single-halo particle passing through the disc cannot account for the correlated and broad structures seen in the velocity distributions. We will discuss the relationship between \(v\) and the mean orbital or guiding radius below when we consider the relation between velocity components and orbital properties.

5.2 Velocity distributions in arm and interarm regions

When looking at the velocity distribution in different neighbourhoods it is helpful to know which neighbourhoods have a high surface density. Figs 9(a) and (b) also show the locations of neighbourhoods on the projected disc surface density used to construct the velocity histograms shown in Fig. 8. In Fig. 9(b), projected in polar coordinates, the \(x\)-axis gives the azimuthal angle with respect to the bar. Thus each neighbourhood shown on this plot directly corresponds to the position of the uv histogram in Fig. 8. This makes it possible to see how the velocity distribution varies with position and between arm and interarm regions.

A comparison between Figs 8 and 9(b) allows us to see if the velocity distribution in high-density regions differs from that at low-density regions. We find that when the density increases, the velocity dispersion is also likely to increase. The velocity distribution is wider when the neighbourhood lies on an arm or bar peak than in interarm regions. Phase-space density should be conserved as particles orbit the galaxy, so this trend is expected.

We can compare the velocity distribution on the leading (or pre-shock) side of a spiral arm to that on the trailing (or post-shock) side. Our simulated galaxy rotates counter-clockwise or with increasing \(\phi\). We denote the leading side of a spiral arm as that located at larger
Figure 8. Local velocity distributions at different positions in the outer galaxy at a simulation time $T = 600$ Myr. Velocity distributions are extracted from neighbourhoods centred at 8 equidistant angles and at 6 radii spaced logarithmically. Each panel shows the distribution in $u$ ($x$-axis) versus $v$ ($y$-axis). Each row shows neighbourhood at a particular galactocentric radius with the lowest row with $r_0 = 4.1$ kpc and the top row with $r_0 = 12.5$ kpc. Each column shows neighbourhoods with a particular galactocentric angle with the leftmost aligned with the bar. Each neighbourhood is separated in angle by 45° with angle increasing counter-clockwise in the galaxy plane. Positions and sizes of each neighbourhood are shown in Fig. 9. Here, we see that clumps exist in the velocity distribution at all radii in the galaxy. Features in the velocity distribution at one neighbourhood are often seen in the nearby neighbourhood at a larger or smaller radius but at a shifted $v$ velocity.

$\phi$ than the peak, or to the right in Fig. 9(b) (and counter-clockwise in angle from an arm peak in Fig. 9a). Observed galaxies tend to have dust lanes on the trailing side (as denoted here) of a spiral arm. This happens because the spiral pattern is usually faster than local galactic rotation (in other words the corotation radius lies interior to the pattern). In a frame moving with the gas in Fig. 9(b), the pattern moves to the right and so shocks would form on the left-hand or trailing side of each density peak.

We see that arcs in the velocity distribution with positive $u$ and $v$ are often seen on spiral arm density peaks and on the leading (or pre-shock) side of an arm. These arcs are oriented with negative slope such that $v$ increases with decreasing $u$. Moving from the leading to the trailing side at a particular radius (comparing histograms from right to left in Fig. 8) the high $v$ arcs move from positive to negative $u$. See for example $r = 8$ and $\phi = 90$, 135° or $r = 6.4$ and $\phi = 315$, 0° in Fig. 8. Interarm regions tend to lack arcs that are centred at high and positive $u$, $v$.

In interarm regions the velocity dispersion is lower and there is sometimes a clump, separated from the dominant peak, that is at negative $v$ and $v$. The clump can be elongated in $u$, resembling the Hercules stream in the solar neighbourhood’s velocity distribution (e.g. Dehnen & Binney 1998; Arifyanto & Fuchs 2006; Bovy 2010; Gardner et al. 2010).

5.3 Gaps in the $uv$ histograms

Fig. 12 shows the neighbourhoods at a radius of $r_0 = 6.4$ kpc for the same angles shown in Fig. 8 but for snapshots separated by 50 Myr between 500 and 950 Myr. The angles here are given with respect to the bar and so in a coordinate system rotating with the bar. At each angle with respect to the bar, clumps do not remain fixed as the system evolves. This implies that spiral structure influences the locations of gaps and clumps in the velocity distribution. The gaps cannot be solely due to bar perturbations.
A stream similar to the Hercules stream at $v \approx -50 \text{ km s}^{-1}$ is present in the velocity distribution at early times at angle 0 (aligned with the bar) at $r_0 = 6.4 \text{ kpc}$. Like the Hercules stream (e.g. Dehnen 1999) this arc or clump has a wide $u$ distribution, has a narrow $v$ distribution and is centred at a negative $u$ value. However, the bar’s $m = 2$ OLR (at about 9 kpc) lies outside these neighbourhoods, so this clump cannot be associated with this resonance. Similar feature low $u$, $v$ clumps in the velocity distribution are also seen at other radii such as $r_0 = 8$ and 10 kpc but appearing at different times in the simulation and at different angles with respect to the bar, consequently the bar and resonances associated with it cannot provide the only explanation for these features.

The spiral density waves have slower patterns than the bar, hence density peaks associated with them move to the left (or to a smaller $\phi$) as a function of time or moving downward in Fig. 12. At one position as a wave peak passes through the neighbourhood first a
velocity distribution typical of a leading side of an arm is seen. At a later time in the same location a velocity distribution typical of a trailing side is seen. The velocity distribution contains a positive $u$ and $v$ feature when on the leading side of a spiral feature. The feature is centred at progressively lower $u$ values as the arm passes by and the neighbourhood moves to the trailing side. See for example the rightmost column and at an angle of 315° between times 500 to 700 Myr (top five rows).

While clumps come and go in time, we find that the gaps between the clumps tend to remain at the same $v$ values. For example consider the rightmost column of Fig. 12. Two gaps are seen with different degrees of prominence at different times. We recall from Fig. 8 that gaps at one neighbourhood radius were often seen at another neighbourhood radius but shifted in $v$. To compare a gap seen in a neighbourhood at one radius to that seen in a neighbourhood at another radius we can consider motions of particle orbits.

6 INTERPRETING STRUCTURE IN THE UV PLANE

To relate structure in local velocity distributions it is helpful to recall models for orbits. We first consider epicyclic motion in the absence of perturbations, then review the orbital dynamics to the first order in the perturbation strength in the presence of perturbations. In both case we consider the relation between $uv$ velocities in a local neighbourhood and quantities describing the orbit such as a mean or guiding radius and an epicyclic amplitude.

In the absence of perturbations from spiral arms, the motion of stars in the disc of a galaxy can be described in terms of radial or epicyclic oscillations about a circular orbit (Kalnajs 1979). It is useful to specify the relation between the observed velocity components $u, v$ and the parameters describing the epicyclic motion (e.g. Fux 2000; 2001; Quillen & Minchev 2005) or the mean radius or guiding radius, $r_g$, and the epicyclic amplitude. The energy of an orbit in the plane of an axisymmetric system (neglecting perturbations from spiral structure) with a flat rotation curve is

$$E(u, v) = \frac{(v_c^2 + v^2)}{2} + \frac{u^2}{2} + v_c^2 \ln r + \text{constant},$$

where the potential energy, $\ln r$, is that appropriate for a flat rotation curve, $r$ is the Galactocentric radius and $v_c$ is the circular velocity.

With an epicyclic approximation we can write the energy

$$E = v_c^2 \left[\frac{1}{2} + \ln r_g\right] + E_{epi}$$

where the term on the left is the energy of a star in a circular orbit about a guiding radius $r_g$ and $E_{epi}$ is the energy from the epicyclic motion,

$$E_{epi} = \frac{u^2}{2} + \kappa^2 (r - r_g)^2 = \frac{\kappa^2 a^2}{2}.$$  

Here $\kappa$ is the epicyclic amplitude and $\kappa$ is the epicyclic frequency at the guiding radius $r_g$.

We now consider stars specifically in a neighbourhood of galactocentric radius $r_o$, restricting us to a specific location in the galaxy. Setting the energy equal to that written in terms of the epicyclic motion using equations (5, 6), we solve for the guiding radius, $r_g$. It is convenient to define the distance between the guiding or mean radius and the neighbourhood’s galactocentric radius, $s = r_g - r_o$. To the first order in $n$ and $v$ we find that a star in the neighbourhood with velocity components $u, v$ has a guiding radius with

$$\frac{s}{r_o} \approx \frac{n}{v_c}$$

and epicyclic amplitude $a$

$$\frac{a}{r_0} \approx v_c^{-1} \sqrt{\frac{u^2}{2} + v^2}.$$  

The factor of a half is from $\kappa^2/\Omega^2$ equivalent to 2 when the rotation curve is flat. An angle describing the phase in the epicycle is

$$\phi \sim \arctan \left(\frac{-u}{\sqrt{2}v}\right)$$

(consistent with equations 2–4, Quillen 2003, for a flat rotation curve). The epicyclic phase angle $\phi = 0$ at apocentre.

The above three relations allow us to relate the velocity components $u, v$ for stars in the solar neighbourhood, and to quantities used to describe the epicyclic motion: the guiding radius, epicyclic amplitude and phase angle. Particles with positive $v$ have $s > 0$ and so guiding radii that are larger than the radius of the neighbourhood, $r_o$. Particles with negative $v$ have $s < 0$ and so are expected to spend most of their orbits inside $r_o$. The distance from the origin, $u = v = 0$, determines the epicyclic amplitude.

It may also be useful to write

$$\frac{v}{v_c} = \frac{r_g}{r_0} - 1,$$

where we have used our definition for $s$ in equation (7). This equation shows that to the first order $v$ sets the guiding radius of a particle. This is useful because the guiding radius sets the approximate angular rotation rate and rotation period of the orbit. The guiding radius also sets the approximate epicyclic frequency (or period) of the particle’s orbit. Resonances occur where sums of integer multiples of these periods are equal to an integer multiple of a pattern speed. Thus the $v$ value of clumps or gaps in the velocity distribution may also allow us to locate resonances.

6.1 Spiral perturbations to first order in the $uv$ plane

The above approximation neglects the effect of bar or spiral perturbations. Spiral waves give a time-dependent perturbation to the gravitational potential that can be approximated by a single Fourier component depending on a single frequency. For a logarithmic spiral perturbation with $m$ arms we can consider $V(r, \theta, t) = V_0 \cos(m(\theta - \Omega_0 t + \alpha \ln r))$ where the pitch angle $\alpha = -d\Omega/d\ln r$. To the first order in the perturbation strength $V_0$, the orbit of a star in the galactic mid-plane with mean or guiding radius $r_g$

$$r(r_g, \theta, t) = r_g + a \cos(\kappa_s t + \phi_0) + C(r_g) \cos(\theta - \Omega_p t + \alpha \ln r_g + c_0))$$

(Binney & Tremaine 1987; equation 3–119a) where $a$ is an epicyclic amplitude, and $c_0, \phi_0$ are phase angles. The forced epicyclic amplitude in the WKB approximation $|C(r_g)| \approx |\frac{a}{r_g}\phi_0|$ where $\Delta_k = \kappa_s^2 - m^2(\Omega_g - \Omega_0)^2$ represents the distance to an LR and is evaluated at the guiding or mean radius, $r_g$ and $\kappa_s$ and $\Omega_0$ are the epicyclic frequency and angular rotation rate at $r_g$. The denominator $\Delta_k$ becomes small near the LR and the first-order approximation breaks down. A higher order approximation can be used to show that closed orbits exist on both sides of resonance and that epicyclic amplitudes do not become infinite on resonance (e.g. Contopoulos 1975).

The orbital motion (equation 11) consists of an epicyclic motion with amplitude $a$ and a phase $\phi_0$, and a forced motion aligned with the perturbation perturbation and moving with it. A population of stars would have a distribution in both $a$ and $\phi_0$. When $a = 0$ and $m = 2$
the orbits are ellipses. The orbit with \( a = 0 \) is periodic or closed in the frame rotating with the spiral pattern. The mean value, \( \bar{a} \), sets the velocity dispersion in the disc. When \( \bar{a} < |C| \) the orbits are nearly closed in the frame rotating with the potential perturbation and we can consider the disc to be cold.

The radial and tangential velocity components can be computed as

\[
v_r(r, \theta, t) = -a \kappa \sin(\kappa t + \phi_0) - C(r) m(\Omega_g - \Omega_p) \sin(m(\theta - \Omega_p t + \alpha \ln r_g + \rho))
\]

\[
v_{\theta}(r, \theta, t) = r \Omega_g - a \Omega_g \cos(\kappa t + \phi_0) - C(r) \Omega_g \cos(m(\theta - \Omega_p t + \alpha \ln r_g + \rho)).
\]

(12)

(13)

(from equation 3.117 by Binney & Tremaine 1987). When the rotation curve is flat \( r \Omega_g \) is independent of radius and the \( uv \) velocity components \( u = -v_\theta \) and \( v = v_\theta - r \Omega_g \). We note that the equations for \( r(r, \theta, t) \) and \( v_\theta(r, \theta, t) \) contain similar terms but with the opposite sign. A shift in \( v \) corresponds to a shift in radius and the mean orbital radius can be roughly estimated from the \( v \) component as was true in the case of pure epicyclic motion (equation 10). If the free epicycle is small \( (a = 0) \) or the disc is cold then

\[
u \left( \frac{\Omega_g}{m(\Omega_p - \Omega_g)} \right) \approx -m \nu \phi_1
\]

(14)

Figure 11. (a) A series of ellipses is shown with semimajor axes logarithmically spaced from the other. For a cold disc, to first order in the perturbation strength and for an \( m = 2 \) perturbation, orbits are approximately ellipses. The orientation of each ellipse semimajor axis is 20° counter-clockwise lower than that just inside it. The only exception to this rule is between the sixth and seventh ellipse (counting from smallest to largest) where the angular difference is larger. Spiral arm peaks (density peaks in a stellar distribution) occur where the orbits are close together or overlap. The red circle is an example of an arm peak. There are a large range of velocities or orbit orientations in a region where many elliptical orbits overlap, giving an arc in the velocity distribution. In interarm regions (e.g. green circle), the range of velocities (or angles on this plot) in a particular neighbourhood is low and so the corresponding velocity dispersion in this neighbourhood is also low. The spiral arms are trailing with galactic rotation in the counter-clockwise direction. On an arm a broad range of angles exist in the orbits crossing the neighbourhood, so the arc in the corresponding velocity space is continuous. The angular offset between the sixth and seventh ellipse, as might occur if there are different patterns in different regions of the disc, causes a discontinuity in the overlap region that manifests as a gap in the velocity distribution. (b) The inner six curves are ellipses. The outer ones are a sum of elliptical and triangle perturbations such as might occur if there are simultaneously two- and three-armed spiral density waves in different regions of the disc. The transition region exhibits discontinuities in overlap regions caused by the onset of the three-armed pattern. The blue circle is located at a radius where there is a continuity and shows that the angular distribution of orbits is bifurcated. We expect that in such a region there will be a gap in the velocity distribution.
between mean galactic radius of the ellipse and orbital angle in the neighbourhood. Thus the velocity distribution would exhibit an arc.

The green circle in Fig. 11(a) shows an interarm region where the angles of orbits only vary slightly compared to that in the red neighbourhood. In the green interarm neighbourhood we expect a narrow velocity distribution. This illustrates why we see, or expect to see, arcs in the velocity distributions on spiral arm peaks and lower velocity dispersions in interarm regions.

An arc in a local velocity distribution that has $u$ decreasing as $v$ increases can be interpreted in terms of orbital properties as a function of $r_ε$ (approximately set by $v$). The positive $v$ orbits have larger guiding radii. The change in angle or $v/u$ slope on the velocity distribution plots implies that the epicyclic angle $φ_i$ (equation (15)) decreases with increasing $r_ε$. Thus these arcs correspond to a relation between epicyclic angle and guiding radius. Consider our cartoon (Fig. 11a) showing concentric ellipses with different orientation angles. A neighbourhood on top of a spiral arm peak (such as the red one) intersects orbits with a range of epicyclic angles, corresponding on this figure to a smooth change in orientation of the ellipse. Thus we expect that an arc would be observed in a velocity distribution at the position of a spiral arm peak. The angle of the arc in the velocity distribution is likely to be affected by the winding angle of the spiral structure with a more open spiral corresponding to an arc with steeper slope. However, a neighbourhood may contain orbits with a smaller range of guiding radii if the spiral structure is less tightly wound. Model orbital distributions would be needed to use the slopes and ranges of velocities of clumps seen in velocity distributions, and their gradients, to place constraints on the spiral structure (pattern speed, pitch angle and strength) from observations of the velocity distribution alone.

We now consider the type of arc that would be seen on a trailing spiral arm. The red neighbourhood in Fig. 11(a) intersects orbits near apocentre with small guiding radii (and so negative $v$) and orbits that are near pericentre that have larger guiding radii (and so positive $v$). In between and in the neighbourhood are orbits that are moving towards the Galactic centre and so have positive $u$ values. For trailing spiral structure we expect the arc to progress from low $v$ to high $v$ passing through the top-right quadrant of the $v'v$ plane, as we seen in arm peaks in Figs 8, 12 and 13. Thus, our illustration Fig. 11(a) provides an explanation consistent with spiral structure for the arcs seen in the velocity distribution.

We can use our first-order perturbation model (equations 11–13) to consider particles with nearby guiding radii. We expand equation (11) for closed orbits ($a = 0$)

$$r(r_ε + x, θ, t) ≈ r_ε + x + C(r_ε) \cos(mθ - Ω_p t + α \ln r_ε + c_0)) - C(r_ε) \sin(mθ - Ω_p t + α \ln r_ε + c_0) \max_{r_ε} \frac{dr_ε}{dr}$$

(17)

where we have used the WKB approximation by neglecting $dC(r_ε)/dr$. The derivative of the above equation

$$\frac{dr}{dr_ε} ≈ 1 - C(r_ε) \sin(mθ - Ω_p t + α \ln r_ε + c_0) \max_{r_ε} \frac{dr_ε}{dr}$$

(18)

Consider two closed orbits separated by a small difference in guiding radius. The radial distance between these closed orbits at a particle angular location is the smallest when the above derivative is minimum. A small difference in current radius is possible when the sine term is negative. This happens midway between pericentre and apocentre. Thus the closed orbits are nearest to each other when this sine term is negative ($mφ_i = π/2$). The radial velocity is also proportional to the sine of the angle. For $α > 0$, corresponding to trailing arms, we find that the orbits are closest together when the radial velocity component is negative and the particles are approaching pericentre (positive $u$, zero $v$), as shown by the red circle in Fig. 11(a). Orbits are maximally distant when the radial velocity is positive or $90^\circ$ away from the red circle in Fig. 11(a) that shows a two-armed structure. At locations and corresponding velocities where the above derivative is high, the phase-space density would be higher and so there would be features seen in a local velocity distribution. Inclusion of the derivative $dC(r_ε)/dr$ when estimating the derivative $dr/dr_ε$ would shift the estimated angle of the minimum of the derivative. This shift may explain why arcs in the velocity distribution are primarily located in the upper-right-hand quadrant (positive $u$, $v$) and not centred at $α > 0$, $v = 0$.

The above discussion has neglected the dispersion of orbits about the closed or periodic ones shown in our cartoons (Fig. 11); however if the dispersion is neglected then the predicted velocity distribution contains particles at only a few velocities. The stellar dispersion has been used in constructing weights for orbits to populate the velocity histograms and predict velocity distributions from test particle integrations (e.g. Dehnen 2000; Quillen & Minchev 2005). We should remember that Fig. 11 only qualitatively gives us a relation between orbital and velocity distribution, however the properties of closed orbits are relevant for interpretation of these distributions (e.g. Quillen & Minchev 2005).

The above equations are given in the WKB approximation for a spiral perturbation. For a bar or two-armed open spiral perturbation to the first order the orbits are also ellipses, however the WKB approximation cannot be used. For a bar perturbation an additional term in $v_0$ should be taken into account when estimating the guiding radius and the angle $m(θ - Ω_p t)$ from $u$, $v$ (see Binney & Tremaine 1987, section 3.3). For a spiral perturbation if the WKB approximation is not used additional terms in both velocity components would be used to estimate the guiding radius, the orbit orientation and the angle with respect to it from $u$, $v$ (e.g. Contopoulos 1975).

7 GAPS AT DIFFERENT RADII

We now look at the location of gaps in the velocity distribution for the snapshot shown in Figs 8 and 9 at $T = 600$ Myr. In Fig. 8 at a radius of $r_0 = 5.1$ kpc and an angle of $φ_0 = 135^\circ$ there are two gaps seen in the velocity distribution, one centred at $v \sim 50$ and the other at $v \sim 0$ km s$^{-1}$. The one at $v \sim 0$ has a guiding radius of $r_ε \sim 5.1$ kpc, the same as the neighbourhood radius. Equation (10) can be used to predict the value of this gap observed from a neighbourhood radius of $r_0 = 4.1$ kpc instead of at 5.1 kpc. We estimate that it should lie at $v \sim 50$ km s$^{-1}$. There is an arc with a gap at about $v \sim 50$ km s$^{-1}$ in the velocity distribution at $r_0 = 4.1$ kpc and $φ_0 = 135^\circ$, as expected. The other gap in the neighbourhood with $r_0 = 5.1$, $φ_0 = 135^\circ$ at $v \sim 50$ km s$^{-1}$ has a guiding radius $r_ε \sim 6.3$ kpc (again using equation 10). A clump with this guiding radius would lie at $v \sim 0$ km s$^{-1}$ in a neighbourhood with $r_0 = 6.4$ kpc. We do not see a gap with this $v$ value at $r_0 = 6.4$ kpc and $φ_0 = 135^\circ$ but we do see one in the neighbouring position at $r_0 = 6.4$ kpc, $φ_0 = 90^\circ$. Thus we find that the relation between $v$ and guiding radius is consistent with shifts in individual features in $v$ as a function of neighbourhood radius.

We also find gaps at nearly zero $v$ values in neighbourhoods with $r_0 = 8$ and 10 kpc. In the inner galaxy there is a gap at $r_0 = 4.1$ kpc, $φ_0 = 45, 90^\circ$ with slightly negative $v$ that is also present at $v \sim -50$ km s$^{-1}$ in the neighbourhood with $r_0 = 5.1$ kpc, $φ_0 = 45^\circ$ and
Figure 12. We show \( \mu v \) velocity distribution histograms generated with neighbourhood radii of \( r_0 = 6.4 \) kpc but at different times. From top to bottom row the histograms were generated from snapshots at output times 500 to 950 Myr. Each snapshot is separated in time by 50 Myr. As in Fig. 8 the angles are given with respect to the bar and the same sequence of angles is shown. Gaps in the velocity distribution tend to remain at the same \( v \) values. Even though the positions shown are fixed in the bar’s rotating frame there are variations in the features seen in the velocity distribution that must be due to the spiral waves in the outer disc.
Figure 13. Velocity distribution histograms generated with neighbourhood radii of $r_0 = 10.0$ kpc but at different times. Similar to Fig. 12. These neighbourhoods lie just outside the bar’s $m = 2$ OLR (at about 9 kpc). A clump similar to the solar neighbourhood’s Hercules stream (with both negative $u$ and $v$) is primarily seen at angles $\phi_0 = 90^\circ$, $135^\circ$, $270^\circ$ and $315^\circ$ with respect to the bar. It is rare to see these streams at other angles with respect to the bar. Non-circular motions caused by the bar may be required to induce the large epicyclic amplitude implied by the low $v$ values in the clump.
has \( r_g \sim 3.9 \) kpc. Altogether we see gaps at the following guiding radii \( r_g \sim 3.9, 5.1, 6.3, 8 \) and 10 kpc at this time in the simulation.

Looking back at Fig. 8 we compare the location of gaps in the velocity distribution (identified by their guiding radii) and the locations of kinks (changes in winding angle) or discontinuities in the spiral arms. In Fig. 9(b) we have identified radii where there are discontinuities in the spiral structure, and changes in either winding angle (kinks) or places where spiral arms or armlets appear or disappear. These radii are shown as black circles in Fig. 9(a) and as horizontal lines in Fig. 9(b). The radii of the discontinuities on these plots are at \( r_g \sim 3.5, 5.1, 6.3, 8.7 \) and 11 kpc. A comparison between the list of discontinuities and our list of gaps identified in the velocity distributions (identified by their guiding radii) suggests that these two lists are related.

A gap in the velocity distribution occurs in a neighbourhood when there are two groups of stars each with velocity vectors in similar but different directions, but few stars with velocity vectors in between those of the two groups. In Fig. 11(a) we show concentric ellipses that increase in radius and shift in orientation angle. The difference in orientation angle between each ellipse is small and constant except between the sixth and seventh ellipse where the difference is much large and 90°. There is a discontinuity in the position of the spiral arm peak at the radius of the introduced large change in orientation angle that we can see from the shape of the orbital overlap regions. The discontinuity introduces regions such as the neighbourhood illustrated with the blue circle. This neighbourhood contains orbits with two separate groups of orbit angles. In this neighbourhood we would expect a corresponding gap in the velocity distribution. Fig. 11(a) shows that we expect gaps in the velocity distribution where there are discontinuities in the spiral structure, and suggests that gaps in the velocity distribution only occur at radii where there are discontinuities in spiral structure and at specific angles where there are spiral arm peaks. We do not see gaps in every velocity distribution in Fig. 8 but do tend to see them where the surface density is high. This is expected as density peaks can occur when multiple pattern peaks coincide.

In the previous section we introduced a first-order perturbative orbital model containing a single potential perturbation (equations (11–13)). This model can be extended for multiple patterns giving orbits with radius, radial and tangential velocity as a function of guiding or mean radius to the first order in the perturbation strengths

\[
r(r_g, \theta, t) = r_g + a \cos(k_r t + \phi_0) \\
+ C_1(r_g) \cos(m_1(\theta - \Omega_1 t + \alpha_1 \ln r_g + c_1)) \\
+ C_2(r_g) \cos(m_2(\theta - \Omega_2 t + \alpha_2 \ln r_g + c_2)). \tag{19}
\]

When \( a \) is smaller compared to \( |C_1| \) and \( |C_2| \) the orbits are nearly closed as shown in Figs 11(a) and (b).

For a particular neighbourhood with \( r, \theta \) the above equations must be inverted to solve for \( r_g, a \) and \( \phi_0 \) as a function of the velocity components. The inversion need not give a unique value of \( r_g \) so particles from separated radial regions (in guiding radius \( r_g \)) can simultaneously be present in one local neighbourhood. The similarity of terms in \( r, u, v_\theta \) implies that there is a close relation between the inverted \( r_g \) distribution and the velocity components. This situation is illustrated in Figs 11(a) and (b) where transitions between one pattern and another gives neighbourhoods containing orbits from different regions of the galaxy and a bifurcated velocity distribution in that neighbourhood. Fig. 11(a) shows a transition between an inner and an outer two-armed pattern. Fig. 11(b) illustrates a transition between a two-armed and a three-armed pattern. In this figure the inner six curves are ellipses; however the outer ones are a sum of elliptical and triangle perturbations such as might occur if there are simultaneously two-armed and three-armed spiral density waves. The elliptical and triangular perturbations vary smoothly in orientation angle from curve to curve. The transition region exhibits discontinuities in overlap regions caused by the onset of the three-armed pattern. The blue circle is located at a radius where there is a discontinuity and shows that the angular distribution of orbits is bifurcated. We expect that in such a region there will be a gap in the velocity distribution.

A neighbourhood with \( r, \theta \) is most likely to contain two separate solutions for \( r_g \) if the phases of the arguments for each perturbation have opposite signs. For example if the angle \( m_1(\theta - \Omega_1 t + \alpha_1 \ln r_g + c_1) \sim 0 \) and \( C_1 > 0 \) then \( r > r_g \) and the particles in a neighbourhood at \( r \) will have guiding radii interior to \( r \). The orbits will have \( u \sim 0 \) but \( v \) greater than the circular velocity at \( r \) (consistent with the discussion in Section 5.5). If the angle \( m_2(\theta - \Omega_2 t + \alpha_2 \ln r_g + c_2) \sim \pi \) then the opposite is true. A neighbourhood at \( r \) can simultaneously contain stars on orbits associated with an interior perturbation near apocentre and stars associated with an outer perturbation near pericentre.

Our spectrograms showed that there are both two-armed and three-armed patterns present in this simulation. The two-armed and three-armed patterns have different radial extents and different pattern speeds. Fig. 11(b) shows that the transition between a two-armed and three-armed pattern can cause a discontinuity in the spiral arm peaks. This type of discontinuity can also give neighbourhoods with two sets of orbital orientation angles and so gaps in a velocity distribution, as illustrated by the blue neighbourhood in this figure.

We now compare the list of radial discontinuities and gaps with estimated resonance locations identified from the pattern speeds in the simulations. If we only had one strong wave then we would be restricted to a few resonances. However our simulation shows multiple patterns, each with a different radial range. Keeping this in mind, perhaps it is not surprising that the density distribution shows multiple spiral arm discontinuities and the velocity distributions show multiple gaps.

If the spiral patterns are excited at resonances then we expect that they will start out and end near resonances. In Section 4, we discussed likely ranges for these patterns with likely resonances as boundaries. The innermost discontinuity, seen both as gaps in the velocity distribution and in the spiral morphology, lies at a radius of about \( r_g \sim 4 \) kpc. This lies near the bar’s \( m = 4 \) ILR and is not distant from the \( m = 2 \) ILR of the inner two-armed spiral. The discontinuity at a guiding radius, \( r_g \sim 5 \) kpc lies near the bar’s CR, the \( m = 3 \) ILR of the three-armed spiral and is not distant from the \( m = 4 \) ILR of the inner two-armed spiral and the \( m = 2 \) ILR of the outer two-armed spiral. The discontinuity at a radius \( r_g \sim 11 \) is near the \( m = 3 \) OLR of the three-armed spiral and the \( m = 2 \) OLR of the inner two-armed spiral. That at \( r_g \sim 8 \) kpc is near the bar’s \( m = 2 \) OLR and the \( m = 2 \) ILR of the outer two-armed spiral. It also may be the outer boundary of the inner two-armed spiral and near its \( m = 4 \) OLR. The division at \( r_g \sim 6.3 \) is not obviously associated with a resonance but could be due to a division between the inner and outer two-armed spirals.

The first-order approximation used here has a small divisor problem near resonance. To understand the dynamics near resonance a higher order model is required (Contopoulos 1975). Approaching resonance the epicyclic amplitude increases but does not go to infinity and closed orbits exist on both sides of resonance. Consequently, a closed orbit approximation (such as illustrated in Fig. 11) may not be invalid even quite close to resonance. As epicyclic amplitudes
7.1 Clumps like the Hercules stream

When the neighbourhood lies on a discontinuity, two sets of orbits can pass through the neighbourhood (see e.g. the blue neighbourhood in Fig. 11a). In this case the first set is those associated with the spiral arm peak and their velocities would lie on an arc passing through the top-right quadrant in the $uv$ plane. The other set of orbits comes from the inner galaxy and differs in orientation by about $90^\circ$. This difference in orientation implies that its $u$ values will have the opposite sign, and since the orbits have lower guiding radii, they should have lower $v$ values. Thus we expect that the velocities of stars on the second group of orbits can have low $v$ and the opposite sign of $u$ and so form a clump on the lower left quadrant in the $uv$ plane, similar in position to the Hercules stream at $v \sim -50\,\text{km}\,\text{s}^{-1}$ in our galaxy (Dehnen & Binney 1998).

The $m = 2$ OLR with the bar at earlier times in the simulation is at about 9 kpc. If the solar neighbourhood lies just outside this resonance with the bar in the Galaxy then the radius in this simulation most matching our position in the Galaxy (just outside the bar’s OLR) is at $r_0 \sim 10\,\text{kpc}$. We show velocity distributions in neighbourhoods with this radius and at different times in Fig. 13. At this radius, clumps similar in position to the Hercules stream are seen at angles (with respect to the bar) of $\phi_0 = 90^\circ; 135^\circ; 270^\circ$ and $315^\circ$ but not at all times. It is rare to see these streams at angles nearly aligned or perpendicular to the bar. This suggests that non-circular motions caused by the bar may be needed to produce the large epicyclic amplitudes corresponding to the strong negative $v$ values in the clump.

The velocity distribution in the solar neighbourhoi not only displays a stream at $v \sim -50\,\text{km}\,\text{s}^{-1}$ (the Hercules stream) but is also triangular shaped for velocities with low $u, v$. We show in Fig. 14 (rightmost panel) the velocity distribution of the magnitude-complete, kinematically unbiased sample of 16,682 nearby F and G dwarfs by Holmberg, Nordstrom & Andersen (2009). A triangle-shaped distribution at lower $u$, $v$ and a Hercules-stream-like feature at negative $v$ are sometimes seen in our velocity distributions at $r_0 = 10\,\text{kpc}$, near the bar’s $m = 2$ OLR, are both seen for example in the rightmost panel of Fig. 10. We have looked through our velocity distributions at this $r_0$ and with a bar orientation consistent with the $\sim 45^\circ$ angle (between bar semimajor axis and Sun the Galactic centre line) estimated from the GLIMPSE survey (Galactic Legacy Infrared Mid-Plane Survey Extraordinaire) (Benjamin et al. 2005) for other examples of velocity distributions that exhibit the gross features present in the solar neighbourhood velocity distribution. These are shown in Fig. 14 for smaller neighbourhoods with size set by $f = 0.05$ along with disc surface density at the same time-steps in Fig. 15. In Fig. 15 blue circles show the location of neighbourhoods used to extract the velocity distributions. The disc has been flipped so that rotation is clockwise rather than counter-clockwise and rotated so that the extraction neighbourhood is oriented at the top of the plot. This has been done so that these images can more readily be compared with models for the Galaxy. The location of the mock Sun is at the blue circle with a galactic longitude of $90^\circ$ viewing to the right.

At certain times and locations in our simulation the local velocity distribution is similar to that seen in the solar neighbourhood at a

Figure 14. The leftmost four panels show velocity distributions at a radius of $r_0 = 10\,\text{kpc}$ chosen because they exhibit both a Hercules-like stream and a triangular shape near the origin similar to the morphology of the solar neighbourhood’s velocity distribution (shown on the right). These velocity distributions were created with neighbourhoods with $f = 0.05$. The time (in Myr) of each snapshot is on the lower left of each panel.
For the four velocity distributions shown in Fig. 14 we show the morphology of the disc and the location (as small blue circles) of the neighbourhoods and a Hercules-like stream at low than that in the solar neighbourhood \(v\) (to the left on the plot), and approximately consistent with \(762–784\). The similarity between spectrograms at different stages in our simulations (as the bar slows down) suggests that our simulation also exhibits couplings between waves, with bar and lopsided waves likely driving spiral waves. Even though multiple waves are present, they could be coupled and slowly evolving rather than transient.

We find that arcs in the local velocity distribution are seen in neighbourhoods located on arm peaks, whereas interarm regions tend to have lower velocity dispersions. This behaviour is understood by considering a series of nested elliptical orbits, appropriate for a cold disc influenced by a single spiral perturbation. Spiral arm peaks are present where ellipses touch or overlap and these locations correspond to neighbourhoods containing orbits with a wider range of velocity vector orientations than interarm regions.

Gaps can be seen in the velocity distributions in our simulation all over the simulated galaxy, confirming the early finding by Fux (2000). However, the \(v\) values of these gaps correspond to a small number of guiding radii that are near resonances with the bar and spiral arm patterns or where there are kinks or discontinuities in the spiral arm morphology. The relation between gaps in the velocity distribution and discontinuities in spiral structure is understood by considering a series of nested ellipses that have a discontinuity in their relative orientation angles. The discontinuity introduces neighbourhoods containing overlapping orbits that have a bifurcated distribution of orbit orientations. This bifurcated distribution corresponds to two clumps and a gap separating them in the velocity distribution. Multiple waves, each with a different pattern speed, present in the simulation introduce radii at which there are discontinuities in the spiral structure. Consequently, gaps in the velocity distributions can be seen at multiple radii in the disc.

8 SUMMARY AND DISCUSSION

Using an \(N\)-body hybrid simulation we have looked for a relationship between structure in local velocity distributions and spiral and bar density waves. Patterns for these waves are identified through Fourier spectrograms. Previous studies (e.g. Rautiainen & Salo 1999) have suggested that there are couplings between waves. The similarity between spectrograms at different stages in our simulations (as the bar slows down) suggests that our simulation also exhibits couplings between waves, with bar and lopsided waves likely driving spiral waves. Even though multiple waves are present, they could be coupled and slowly evolving rather than transient.

We find that arcs in the local velocity distribution are seen in neighbourhoods located on arm peaks, whereas interarm regions tend to have lower velocity dispersions. This behaviour is understood by considering a series of nested elliptical orbits, appropriate for a cold disc influenced by a single spiral perturbation. Spiral arm peaks are present where ellipses touch or overlap and these locations correspond to neighbourhoods containing orbits with a wider range of velocity vector orientations than interarm regions.

Gaps can be seen in the velocity distributions in our simulation all over the simulated galaxy, confirming the early finding by Fux (2000). However, the \(v\) values of these gaps correspond to a small number of guiding radii that are near resonances with the bar and spiral arm patterns or where there are kinks or discontinuities in the spiral arm morphology. The relation between gaps in the velocity distribution and discontinuities in spiral structure is understood by considering a series of nested ellipses that have a discontinuity in their relative orientation angles. The discontinuity introduces neighbourhoods containing overlapping orbits that have a bifurcated distribution of orbit orientations. This bifurcated distribution corresponds to two clumps and a gap separating them in the velocity distribution. Multiple waves, each with a different pattern speed, present in the simulation introduce radii at which there are discontinuities in the spiral structure. Consequently, gaps in the velocity distributions can be seen at multiple radii in the disc.

\(N\)-body simulations have the advantage over test particle simulations that the orbits of particles are consistent with the gravitational potential generated by the particles themselves. Additionally \(N\)-body simulations naturally contain varying or evolving bar and spiral structures that would require additional and possibly poorly constrained parameters to describe inside test particle simulations. The study here, though preliminary and based on only one hybrid simulation, can be considered to be complementary to studies carried out with test particle simulations.
8.1 Implications for studies of the Milky Way

From our simulations we have found that interarm regions tend to have lower velocity dispersions than arm peaks. They also tend to lack arcs and gaps in the velocity distribution. The Sun is likely in an interarm region (Vallée 2008). We may have a biased understanding of the Milky Way disc dynamics because the local velocity distribution does not represent that at other positions in the Galaxy. For example, the mean local velocity dispersion in the Galactic disc may have been underestimated because of our interarm location in the Galaxy. Large arcs in the local stellar velocity distribution may be present in nearby spiral arm peaks.

There remains little consensus among different studies for the pattern speed of local spiral structure (e.g. Shaviv 2003; Lepine et al. 2011; Gerhard 2011). The lack of consensus may arise because the presence of more than one wave complicates the analysis (e.g. Naoz & Shaviv 2007). The Local Spur which is thought to lie between the dominant two-armed or four-armed structures may arise because multiple waves are passing in and out of phase with each other (e.g. Henry et al. 2003; Meidt et al. 2009). Arcs in spiral arms can appear and disappear, a localized short burst of star formation may occur that does not continue to progress around the galaxy as is expected when a sole wave passes through the disc. Evidence for short isolated bursts of star formation in the disc would support interpretation of the Galactic disc in terms of multiple waves simultaneously present in the disc. We have noted that density peaks due to constructive interference of patterns tend to begin at smaller radius and progress to larger radius before fading away. Trails of recently formed clusters with age decreasing with increasing radius may be found in the Galaxy disc.

Recent studies have placed strong constraints on the pattern speed of the Galactic bar from the morphology of the Hercules stream in the local velocity distribution (Dehnen 2000; Minchev et al. 2007; Gardner et al. 2010). The association of the v value of the gap separating the Hercules stream from the rest of the stars with the bar’s m = 2 OLR tightly constrains the bar pattern speed. However, features similar to the Hercules stream are seen in our velocity distributions in locations associated with the bar’s m = 2 OLR. They are also seen at other locations in the galaxy. These clumps do not remain fixed in time and are affected by the coupled spiral structure in the outer Galaxy. Previous models of the Hercules stream have neglected spiral structure and have been based on test particle simulations, not N-body simulations (with the exception of the study by Fux 2000). If wave coupling is common then the bar’s OLR may be coincident with an LR with a spiral pattern. Future models could consider both spiral and bar patterns when interpreting the velocity distribution near the bar’s OLR.

Velocity distributions with morphology similar to that in the solar neighbourhood are seen in our simulation at location just exterior to the bar’s m = 2 OLR with bar orientation consistent with infrared studies. By similar we mean showing a Hercules-like stream and a triangular shape at lower u, v. The velocity distribution more closely resembles that seen in the solar neighbourhood when there is a strong spiral/arm, consistent with the observed Centaurus Arm tangent, just interior to the solar neighbourhood.

Like many other galaxies, the Milky Way is lopsided (Levine, Blitz & Heiles 2006). If coupling between a lopsided motion and a Galactic bar is common, then we might expect that a strong three-armed spiral wave would be present in the solar neighbourhood. However, almost all studies have assumed two- or four-armed spiral structures (Vallée 2008). If a strong three-armed wave is present in the solar neighbourhood and its ILR is coincident with the Galactic bar’s corotation radius (as likely true in our simulation), then we can estimate its pattern speed. At an ILR the pattern speed is related to the angular rotation rate by \( \Omega_{\text{p}} = \Omega_{\odot}(1 - \frac{m}{2\Omega_{\odot}}) \). If we set \( \Omega_{\odot} \) in this relation equal to the bar pattern speed (approximate 1.9 times the angular rotation rate at the solar circle \( \Omega_{\odot}; \) Gardner et al. 2010) and use a flat rotation curve with \( \frac{\Omega}{\Omega_{\odot}} = \sqrt{2} \) and \( m = 3 \), we estimate a pattern speed of \( \Omega_{\text{p}} = (1 - \sqrt{2}/3)1.9\Omega_{\odot} \approx \Omega_{\odot} \). This pattern speed for spiral structure fall into two groups, a faster set, placing the Sun near the CR, and a slower set, placing the Sun near the inner 4:1 LR (see table 3 in Shaviv 2003). A three-armed pattern with pattern speed similar to \( \Omega_{\odot} \) would have \( m = 3 \) ILR near the bar’s CR and so could be coupled to the bar. In our simulation, a two spiral pattern is also prominent near the position of the mock solar neighbourhood with ILR near the bar’s corotation rotation resonance. We can estimate the pattern speed of a two-armed wave by satisfying this condition from the bar’s pattern speed using \( m = 2 \), finding a pattern speed of \( \Omega_{\text{p}} \approx 0.55\Omega_{\odot} \) or about 14 km s\(^{-1}\) kpc\(^{-1}\) consistent with a slower group of pattern speeds estimated in a number of studies (see table 3 in Shaviv 2003).

Lepine et al. (2011) propose that a nearby kink in a spiral arm winding angle is associated with a single LR with a spiral pattern. However, our simulations suggest that kinks and discontinuities in the spiral structure not only are associated with LRs with a single pattern (as also explored by Quillen & Minchev 2005; Contopoulos & Grosbol 1986; Contopoulos 1988) but are also signatures of transitions from one pattern to another. Rather than rare, the simulation suggests that these discontinuities may exist all over the Galaxy.

The mode or wave coupling possibly present in our simulations implies that resonances between different patterns could be commonly coincidental. As shown by Minchev & Quillen (2006) this causes chaotic motions in the stellar orbital motions and so accelerated heating (or increasing velocity dispersion) near localized regions of resonance overlap. If waves are coupled in the Milky Way disc, then we might except both localized accelerated heating and radial migration (Minchev & Famaey 2010; Minchev et al. 2011) as a consequence (also see Quillen 2003; Voglis et al. 2006; Shevchenko 2011).

The waves present in our simulation are likely coupled and so not transient. We interpret the divisions seen in the velocity distribution in terms of discontinuities associated with resonances and transitions from one pattern to another rather than a model involving stochastic growth of transient spiral structure (e.g. De Simone et al. 2004).

8.2 Future work

This study only considered one single simulation, and it ran only for 1.3 Gyr. The simulation is not ideal as we have undersampled the bulge and halo in order to well resolve the disc, and the energy conservation is not excellent. However, this particular simulation is particularly rich in spiral density waves and so a good one for comparing velocity distributions to structure seen in spectrograms. While our galaxy was simulated in isolation, real galaxies suffer external perturbations. Lopsidedness in our simulation that may be an artefact of our undersampled live halo is commonly seen in external galaxies (Jog & Combes 2009). Hence, the coupling between the bar, lopsided motion and three-armed waves that we suspect is present in our simulation may be ubiquitous rather than just a rare occurrence.

© 2011 The Authors, MNRAS 417, 762–784
In our simulations, we purposely undersampled the halo and bulge particle distributions so that we could better resolve the galaxy disc. However, as a result our few and relatively massive halo particles cause spurious heating in the disc. We have measured the radial velocity dispersion in the disc at a radius of 10 kpc and found that it does not increase by more than 10 km s\(^{-1}\) across the entire simulation. This suggests that heating by massive halo particles is not excessive in this simulation. The similarity between spectrograms in the middle and later part of the simulation suggests that waves are coupled to the bar rather than randomly generated by noise associated with the granularity of our halo during the simulation. Were we to raise the number of massive particles in the disc, halo and bulge, the simulations would exhibit reduced numerical noise and so might have weaker or delayed spiral and bar structures. Future studies should not only consider more accurate simulations with larger numbers of particles but also explore simulations with differently distributed discs, bulges and haloes, the role of a cold gaseous component and perturbations due to mergers.

Here, we have only qualitatively attempted to relate spiral morphology and patterns to structure seen in the velocity distributions. However, future work could aim to bridge the gap between test particle simulations and N-body simulations by understanding the orbits in each feature seen in the velocity distributions. Here, we have focused on gross features present in the velocity distributions, such as gaps and arcs. Future studies can also aim to understand the finer features as well, which are primarily seen in smaller neighbourhoods (e.g. see Fig. 10). These studies would help interpret forthcoming pencil beam (e.g. Minchev & Quillen 2008) and large-scale velocity and proper motion surveys of stars in the Galactic disc. Here, we have interpreted gaps in the velocity distributions in terms of changes in the shapes of nearly closed orbits associated with different density waves. However, future studies could also consider alternate interpretations for structure seen in velocity distributions such as transient spiral arms (De Simone et al. 2004), bifurcations in the families of periodic orbits (Katsanikas, Patsis & Pinotsis 2011), chaotic orbits along spiral arms near a bar (Vogl et al. 2006; Harsoula & Kalapotharakos 2009), phase wrapping following an initial disturbance (Minchev et al. 2009) or individual LRs (Quillen & Minchev 2005).

ACKNOWLEDGMENTS

We thank Larry Widrow for giving us and helping us with his code GalacticICS. We thank NVIDIA for the gift of four Quadro FX 5800 and two GeForce GTX 280 video cards. We thank Evgenhii Gaburov and Stefan Harfst for making phrGRAPE and Sapporo available. Support for this work was provided by NSF through award AST-0907841.

REFERENCES


Churchwell E. et al., 2009, PASP, 121, 213


Quillen A. C., 2003, AJ, 125, 785

Quillen A. C., Quillen A. C., 2005, AJ, 130, 576


© 2011 The Authors, MNRAS 417, 762–784


Phase space structure of an N-body disc 783
Shaviv N. J., 2003, New Astron., 8, 39


This paper has been typeset from a TEX/LATEX file prepared by the author.