

Calculating the Lifetime of the Sun

What would you need to know in order to decide how much **time** you have before a car runs out of gas?

- How much gas you have in the tank. (gallons)
- How fast you are burning that fuel. (gallons/hour)

- Suppose you have a full tank of gas and for your car that is 12 gallons. You drive for one hour at a constant rate and after that hour you notice the gas gauge is $\frac{3}{4}$ full. This means that you are burning 3 gallons/hour.

- If you continue at this rate, what is the total possible travel time?

What is the total travel time?

1. 2 hours
2. 3 hours
3. 4 hours
4. 5 hours

- To figure out the total time you need to divide the amount of fuel by the rate of consumption.
- Total time = $(12 \text{ gallons}) / (3 \text{ gallons/hour})$
- **Total time = 4 hours.**
- Notice that the gallons cancel and when you invert and multiply the hours come up on top.

Finding the lifetime of the Sun is a very similar problem

- $4\text{H} \longrightarrow \text{He} + \text{energy}$ (Nuclear reaction)
- So four hydrogen atoms are changed into Helium atoms and energy is released.
- Once the hydrogen is gone, so is the fuel supply.

- We need to know how much fuel is in the Sun and how fast it is being used up.

To know the amount of fuel in the Sun we need to know the Sun's mass.

To know how fast the fuel is being used up we need to know the Sun's luminosity (watts).

It is also important to know the composition of the Sun: 71% Hydrogen, 27% Helium and 2% the other 90 elements.

Luminosity

- We know that the luminosity (the energy leaving the Sun every second) has to be equal to the amount of energy being produced in the core of the Sun, every second.
- Otherwise the Sun would either expand or shrink.
- So how can we find the Luminosity of the Sun?

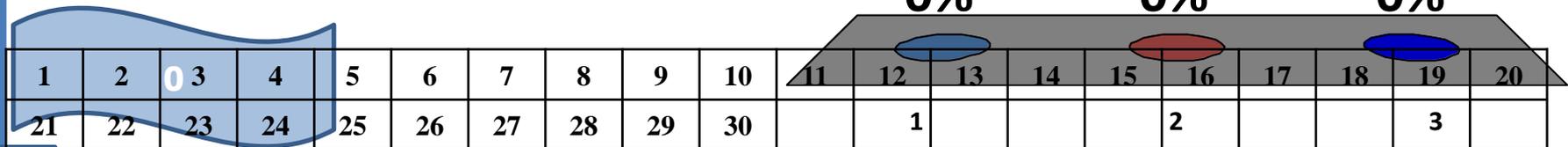
- Imagine a detector in orbit around the Earth. This detector is 1 meter^2 . The detector captures the EM radiation coming from the Sun and shows that there is 1357 watts hitting the detector.
- Is this the luminosity of the Sun?
- What would you have to do in order to determine the entire amount of energy leaving the Sun?

How can you determine the Sun's luminosity?



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1. Build enough detectors to completely surround the Sun and add up all the values.
2. Use the value for one detector and multiply it by the total area around the Sun
3. Assume that the one detector is pretty close to right.



- The Luminosity can be found by multiply the energy/second falling on the detector, by the total surface area of a sphere with a radius of the Earth's orbit.
- $L = B(4\pi d^2)$ Where B is the apparent brightness in units of watts/meter² and d is the distance to the Sun.
- $L = (1357 \text{ watts/m}^2)(4\pi)(1.5 \times 10^{11} \text{ meters})^2$
- $L = 3.8 \times 10^{26}$ watts or (joules/second)
- Total World Wide energy consumption for the year of 2005 was 5×10^{20} Joules. That is one-millionth of the energy that leaves the Sun every second.

- We know that the energy leaving the Sun every second is equal to the energy being produced in the core of the Sun every second.
- So there is 3.8×10^{26} joules of energy being produced in the core of the Sun every second.
- Remember that the nuclear reactions are what is producing this energy. So the next thing we should figure out is how many reactions are occurring every second in the core.

- To do this we need to become Albert Einstein for a few minutes.
- Mass of a proton (Hydrogen nuclei) = 1.673×10^{-27} kg.
- In the reaction, four protons are changed into one Helium atom.
- **Mass of 4 protons = 6.692×10^{-27} kg.**
- **Mass of Helium nuclei = 6.644×10^{-27} kg.**
- **There is less mass in the Helium atom: 0.048×10^{-27} kg**
- **The missing mass is turned directly into energy.**
- **$E = mc^2$**
- **$E = (0.048 \times 10^{-27}\text{kg})(3.0 \times 10^8 \text{ m/s})^2$**
- **$E = 4.32 \times 10^{-12}$ joules for every reaction**

- So now we know how much energy is produced for each nuclear reaction:
- **4.32×10^{-12} joules/reaction**
- We also know that the core produces
- **3.8×10^{26} joules/second**
- **How do we find out the number of reactions/second?**

How do you find out reactions/second?



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1. 4.32×10^{-12} J/reaction **divided by**
 3.8×10^{26} J/second
2. 4.32×10^{-12} J/reaction **multiplied**
by 3.8×10^{26} J/second
3. 3.8×10^{26} J/second **divided by**
 4.32×10^{-12} J/reaction

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30		1				2			3	

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- Reaction/second =
 $(3.8 \times 10^{26} \text{ J/second}) / (4.32 \times 10^{-12} \text{ J/reaction})$
- So there are **8.8×10^{37} reactions/second** occurring in the Sun.
- The Sun converts 564 million tons of hydrogen into 560 million tons of helium every second. That means that every second 4 million tons of hydrogen is turned into energy. ON the order of one Mount Everest per second.

- So now we know how fast we are burning our fuel. But the last thing we need to know is how big the fuel tank is.
- In an earlier class we calculated the mass of the Sun using Kepler's third law. It comes out to be $\text{Mass} = 2 \times 10^{30} \text{ kg}$.
- The mass of the Sun's core, where the reactions are occurring is about 10% of the total Sun's mass. **$M_{\text{core}} = 2 \times 10^{29} \text{ kg}$** .
- We want to use this value to discover how many reactions can occur in the Sun.

- Remember that it takes 4 hydrogen nuclei to make one helium.
- So for one reaction to occur, we need the mass of four hydrogen atoms. We found that earlier.
- Mass of 4 protons = 6.692×10^{-27} kg.
- or we could say that there is
- **6.692×10^{-27} kg/reaction**
- We know that $M_{\text{core}} = 2 \times 10^{29}$ kg.
- **So how do we find the total number of reactions possible in the Sun?**

How do we find the total number of possible reactions in the Sun?

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1. $(2 \times 10^{29} \text{ kg})$ **divided by** 6.692×10^{-27} kg/reaction
2. $(6.692 \times 10^{-27} \text{ kg/reaction})$ **divided by** $2 \times 10^{29} \text{ kg}$
3. $2 \times 10^{29} \text{ kg}$ **multiplied by** 6.692×10^{-27} kg/reaction

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30		1				2			3	

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Total # of reactions =

$$(2 \times 10^{29} \text{ kg.}) / (6.692 \times 10^{-27} \text{ kg/reaction})$$

Total # of reactions = 3×10^{55} reactions

Now we are almost finished. We know the total number of reactions that are possible

3×10^{55} reactions (the amount of fuel)

And we know the rate of the reactions

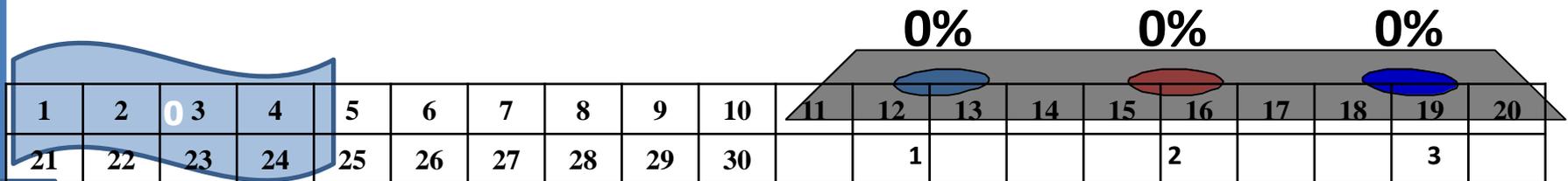
8.8×10^{37} reactions/second (rate of consumption)

- **How can you now, figure out how long the Sun's lifetime is?**

How can we figure out the Sun's lifetime?

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1. 3×10^{55} reactions **multiplied by**
 8.8×10^{37} reactions/second
2. 3×10^{55} reactions **divided by**
 8.8×10^{37} reactions/second
3. 8.8×10^{37} reactions/second
divided by 3×10^{55} reactions



- Lifetime of Sun =
(3×10^{55} reactions)/(8.8×10^{37} reactions/second)

Lifetime of Sun = 3.4×10^{17} seconds

Since there are $\pi \times 10^7$ seconds in a year.

Lifetime of Sun = 10.8 billion years.

More precise calculations find the lifetime to be more like 10 billion years.

The Sun is currently about 4.6 billion years old.

We have another 5 billion years until the fuel runs out.

There are only two assumptions made in this model

- #1. The Sun is not shrinking or expanding. This means that the emitted luminosity is equal to the energy being produced in the core every second.
- #2. The luminosity of the Sun has not changed over its lifetime. The rate of consumption is constant.
- #1 appears to be true from observations, and #2 seems to be true because temperatures have been very stable on Earth for at least 3 billion years.