Factors that govern $R$

- Material $\rho$  The ability to carry electric current varies drastically
- Length $L$  The longer the conductor, the greater its resistance
- Cross sectional area $A$  The thicker the conductor, the less its resistance

$$R = \rho \frac{L}{A}$$

$\rho$ is the resistivity, and depends on the material used
$\rho$ reflects intrinsic properties of the material
$\rho$ is a function of temperature

The unit of $\rho$ is $\Omega \cdot m$

The work that must be done to take $Q$ through $V$

$$W = QV$$

Since $I = \frac{Q}{t}$ or $Q = It$,
$$W = IVt$$ (Electric work)

Recall $P =$ work done/time interval $=$ $W/t$,
$$P = IV$$ (Electric Power)

Using Ohm's law $I = \frac{V}{R}$,
$$P = IV = \frac{V^2}{R}$$

The unit for Power is Watt, $1W = 1J/s$

Question:

A 10-A current is maintained in a simple circuit with a total resistance of 200 $\Omega$. What net charge passes through any point in the circuit during a 1-minute interval?

(a) 200 C  (b) 500 C  (c) 1200 C  (d) 400 C  (e) 600 C  X
**Question:** A current in a 12 Ω toaster operated at 120 V is
(a) 0.1 A
(b) 10 A
(c) 12 A
(d) 1440 A

Answer: b

**Question:** The power rating of an electric motor that draws a current of 3 A when operated at 120 V is
(a) 40 W
(b) 360 W
(c) 540 W
(d) 1080 W

Answer: b

**Question:** The temperature of a copper wire is raised. Its resistivity
(a) decreases
(b) remains the same
(c) increases
(d) any of the above, depending on the temperatures involved

Answer: c

**Question:** Which draws more current, a 100-W light bulb or a 75-W bulb? Which has the higher resistance?

Let \( P_1 = 100 \text{ W} \) and \( P_2 = 75 \text{ W} \).
Since \( P = IV \), \( V = \text{constant} \)
\[ \frac{P_1}{P_2} = \frac{I_1}{I_2} = \frac{100}{75} = \frac{4}{3} \]
The 100-W bulb draws more current

Also, \( P = \frac{V^2}{R} \), where \( V = \text{constant} \)
\[ \frac{P_1}{P_2} = \frac{R_2}{R_1} = \frac{100}{75} = \frac{4}{3} \]
The 75-W has higher resistance

**Question:** Electric power is transferred over large distances at very high voltages. Explain how the high voltage reduces power losses in the transmission lines.

Answer: The power transmitted \( P = IV \). So the higher \( V \), the smaller \( I \).
Since the power lost \( P = I^2R \), the smaller \( I \) leads to smaller \( I^2R \) loss for a given transmission line of resistance \( R \).

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**Chapter 18 DC Circuits**

Resistors in Series
Resistors in Parallel
Electromotive Force (emf)
Kirchhoff’s Rules
**Resistors in Series**

\[ V = V_1 + V_2 + V_3 \]
\[ I = I_1 = I_2 = I_3 \]

(Resistors in series have the same current)

\[ V = IR = I(R_1 + R_2 + R_3) \]
\[ R = R_1 + R_2 + R_3 \]

**Resistors in Parallel**

\[ V = V_1 = V_2 = V_3 \]
\[ I = I_1 + I_2 + I_3 \]

(Resistors in parallel have the same voltage)

\[ I = \frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = \frac{V}{R_1 + R_2 + R_3} \]
\[ I = \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \]

**QUICK QUIZ 18.1**

When a piece of wire is used to connect points \( b \) and \( c \) in this figure, the brightness of bulb \( R_1 \) (a) increases, (b) decreases, or (c) stays the same. The brightness of bulb \( R_2 \) (a) increases, (b) decreases, or (c) stays the same.

**QUICK QUIZ 18.1 ANSWER**

\( R_1 \) becomes brighter. Connecting a wire from \( b \) to \( c \) provides a nearly zero resistance path from \( b \) to \( c \) and decreases the total resistance of the circuit from \( R_1 + R_2 \) to just \( R_1 \). Ignoring internal resistance, the potential difference maintained by the battery is unchanged while the resistance of the circuit has decreased. The current passing through bulb \( R_1 \) increases, causing this bulb to glow brighter. Bulb \( R_2 \) goes out because essentially all of the current now passes through the wire connecting \( b \) and \( c \) and bypasses the filament of Bulb \( R_2 \).

**QUICK QUIZ 18.2**

With the switch in this circuit (figure a) closed, no current exists in \( R_2 \) because the current has an alternate zero-resistance path through the switch. Current does exist in \( R_1 \) and this current is measured with the ammeter at the right side of the circuit. If the switch is opened (figure b), current exists in \( R_2 \). After the switch is opened, the reading on the ammeter (a) increases, (b) decreases, (c) does not change.
QUICK QUIZ 18.2 ANSWER

(b). When the switch is opened, resistors \( R_1 \) and \( R_2 \) are in series, so that the total circuit resistance is larger than when the switch was closed. As a result, the current decreases.

QUICK QUIZ 18.3

With the switch in this circuit (figure a) open, there is no current in \( R_4 \). There is current in \( R_1 \) and this current is measured with the ammeter at the right side of the circuit. If the switch is closed (figure b), there is current in \( R_2 \). When the switch is closed, the reading on the ammeter (a) increases, (b) decreases, or (c) remains the same.

QUICK QUIZ 18.3 ANSWER

(a). When the switch is closed, resistors \( R_1 \) and \( R_2 \) are in parallel, so that the total circuit resistance is smaller than when the switch was open. As a result, the total current increases.

Five resistors are connected as shown in the diagram. The potential difference between points A and B is 15 V.

1. What is the equivalent resistance between the points A and B?
   (a) 1.5 \( \Omega \)  
   (b) 8.7 \( \Omega \)  
   (c) 11.1 \( \Omega \)  
   (d) 4.8 \( \Omega \)  
   (e) 10.4 \( \Omega \)

2. What is the current in the 3.6-\( \Omega \) resistor?
   (a) 1.3 A       (b) 2.9 A        (c) 15 A      (d) 1.7 A (e) 3.5 A

3. What is the current in the 2.7-\( \Omega \) resistor?
   (a) 12 A         (b) 2.2 A        (c) 1.2 A (d) 0.8 A      (e) 0.4 A

4. What amount of energy is dissipated in the 2.7-\( \Omega \) resistor in 9.0 seconds?
   (a) 15 J          (b) 29 J           (c) 52 J        (d) 24 J   (e) 35 JX

Solution:

A potential difference \( V=12 \) V is applied across the set of resistors shown. Find the current that flows through each resistor.

\[
\begin{align*}
R_1 &= 3 \Omega, R_2 = 12 \Omega, \\
R_3 &= 8 \Omega, R_4 = 6 \Omega, \\
R' &= 4.8 \Omega, R'' &= 7.8 \Omega
\end{align*}
\]

\[
\begin{align*}
V &= V_{\text{upper}} = V_{\text{lower}} = 12 \text{ V} \\
I_4 &= V/R_4 = 12 \text{ V} / 6 \Omega = 2 \text{ A} \\
I_1 &= V/R_1 = 12 \text{ V} / 3 \Omega = 4 \text{ A} \\
V' &= I_1 R' = (1.54 \text{ A})(4.8 \Omega) = 7.4 \text{ V} \\
V'' &= V - V' = 12 \text{ V} - 7.4 \text{ V} = 4.6 \text{ V} \\
I_2 &= V'/R_2 = 4.6 \text{ V} / 12 \Omega = 0.39 \text{ A} \\
I_3 &= V'/R_3 = 4.6 \text{ V} / 8 \Omega = 0.58 \text{ A} \\
I_5 &= V'/R_5 = 4.6 \text{ V} / 12 \Omega = 0.39 \text{ A}
\end{align*}
\]
**Question:** Household outlets are often double outlets. Are these connected in series or parallel?

**Answer:** The double outlets are connected in parallel because they can be used independently and each provides the full voltage. If they were in series, both outlets would have to be used to complete the circuit, and the voltage would be divided between the two loads.

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**Answer:** Two lightbulbs of resistance $R_1$ and $R_2$ ($>R_1$) are connected in series. Which is brighter? What if they are connected in parallel?

**Answer:** For the connection in series, brightness depends on power transformed into radiant power: $P=I^2R$. Since $I=I_1=I_2$, $P_1=I_1^2R_1$, $P_2=I_2^2R_2$, and $R_2$ is brighter. The greater $R$ or $V$, the brighter the bulb.

If they are connected in parallel, $V=V_1=V_2$.

Since $P=V^2/R$, $P_1=V^2/R_1$, $P_2=V^2/R_2$, $R_1$ is brighter. The greater $I$, the brighter the bulb.

---

Some light bulbs are connected in parallel to a 120 V source as shown in the figure. Each bulb dissipates an average power of 60 W. The circuit has a fuse $F$ that burns out when the current in the circuit exceeds 9 A. Determine the largest number of bulbs, which can be used in this circuit without burning out the fuse.

**(a) 9  (b) 25  (c) 36  (d) 17X  (e) 34**

---

What is the total power dissipated in the two resistors in the circuit shown?

(a) 10 W  (b) 67 W  (c) 15 W  (d) 670 W  (e) 33 W

---

1. Determine the equivalent resistance for this circuit.

(a) 50  (b) 29  (c) 12X  (d) 120  (e) 5

2. The smallest current passes through which resistor(s)?

(a) the 10Ω− resistor  (d) the 60Ω− resistor  
(b) the 20Ω− resistor  (e) It is the same and the smallest in  
(c) the 30Ω− resistor  (c) the 30Ω− and 60Ω− resistors.

3. The largest potential difference is across which resistor(s)?

(a) the 10Ω− resistor  (d) the 60Ω− resistor  
(b) the 20Ω− resistor  (e) It is the same and the largest for  
(c) the 30Ω− resistor  (c) the 30Ω− and 60Ω− resistors.

---

**Electromotive Force or Emf**

**Emf,** $\mathcal{E}$, is no-load voltage of a source

The potential difference across the terminals of the source is always less than its emf due to its internal resistance, $r$.

Hence the actual $V$ across a source of emf $\mathcal{E}$ and $r$ is

$V=\mathcal{E}-Ir$

**Terminal voltage=emf-potential drop within source**

$V=\mathcal{E}$ if $I=0$
Example: A “D” cell of emf 1.5 V and internal resistance 0.3 \( \Omega \) is connected to a flashlight bulb whose resistance is 3 \( \Omega \). Find the current in the circuit and the terminal voltage of the cell.

Solution: The current in the circuit is 
\[ I = \frac{E}{R+r} = \frac{1.5 \text{ V}}{3 \text{ } \Omega + 0.3 \text{ } \Omega} = 0.45 \text{ A} \]

\[ V = E - Ir = 1.5 \text{ V} - (0.45 \text{ A})(0.3 \text{ } \Omega) = 1.37 \text{ V} \]

---

When a single cell is too small for a particular application:

\[ E = E_1 + E_2 + E_3 + \ldots \]
\[ r = r_1 + r_2 + r_3 + \ldots \]

When the emf of a cell is sufficient but its capacity is too small:

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Example: A storage battery whose emf is 12 V and whose internal resistance is 0.2 \( \Omega \) is to be charged at a rate of 20 A. What applied voltage is required?

Solution: The applied voltage \( V \) must exceed the battery’s emf \( E \) by the amount \( Ir \) to provide the charging current \( I \). Hence,

\[ V = E + Ir = 12 \text{ V} + (20 \text{ A})(0.2 \text{ } \Omega) = 16 \text{ V} \]

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Kirchhoff’s Rules

First rule: The sum of the currents flowing into a junction is equal to the sum of the currents flowing out of the junction (Conservation of Charge)

\[ I_1 = I_2 + I_3 \]

Second rule: The sum of the emf’s around a loop is equal to the sum of the IR potential drops around the loop (Conservation of Energy)

Loop 1: \( E_1 = I_1R_1 + I_2R_2 \)
Loop 2: \( -E_2 = -I_2R_2 + I_3R_3 \)
Loop 3: \( -E_2 + E_1 = I_1R_1 + I_3R_3 \)

---

Example: A 5-\( \Omega \) resistor and 20-\( \Omega \) resistor are connected in series and potential difference of 100 V is applied across them by means of a generator. Find (a) the equivalent R of the circuit, (b) the current that flows in it, (c) the potential difference across each resistor, (d) the power dissipated by each resistor, (e) the power dissipated by the entire circuit.

Solution:
(a) \( R = R_1 + R_2 = 5 \text{ } \Omega + 20 \text{ } \Omega = 25 \text{ } \Omega \)
(b) \( I = \frac{V}{R} = \frac{100 \text{ V}}{25 \text{ } \Omega} = 4 \text{ A} \)
(c) \( V_1 = IR_1 = (4 \text{ A})(5 \text{ } \Omega) = 20 \text{ V} \), \( V_2 = IR_2 = (4 \text{ A})(20 \text{ } \Omega) = 80 \text{ V} \)
(d) \( P_1 = IV_1 = (4 \text{ A})(20 \text{ V}) = 80 \text{ W} \), \( P_2 = IV_2 = (4 \text{ A})(80 \text{ V}) = 320 \text{ W} \)
(e) \( P = IV = (4 \text{ A})(100 \text{ V}) = 400 \text{ W} \)

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Example: A 5-\( \Omega \) resistor and 20-\( \Omega \) resistor are connected in parallel and potential difference of 100 V is applied across them by means of a generator. Find (a) the equivalent R of the circuit, (b) the current that flows in each resistor, (c) the power dissipated by each resistor, (d) the power dissipated by the entire circuit.

Solution: (a) \( 1/R = 1/R_1 + 1/R_2 = (R_1 + R_2)/R_1R_2 \)
\( R = R_1R_2/(R_1 + R_2) = (5 \text{ } \Omega)(20 \text{ } \Omega)/(5 \text{ } \Omega + 20 \text{ } \Omega) = 4 \text{ } \Omega \)
(\( R = 25 \text{ } \Omega \) if they are connected in series)
(b) \( I_1 = V/R_1 = 100 \text{ V}/5 \text{ } \Omega = 20 \text{ A} \), \( I_2 = V/R_2 = 100 \text{ V}/25 \text{ } \Omega = 4 \text{ A} \)
(\( I = 4 \text{ A} \) if they are connected in series)
(c) \( P_1 = IV_1 = (20 \text{ A})(100 \text{ V}) = 2000 \text{ W} \), \( P_2 = IV_2 = (5 \text{ A})(100 \text{ V}) = 500 \text{ W} \)
(P1 = 80 W and P2 = 320 W for the series connection)
(e) \( P = IV = (1+1)IV = (20+5)(100) = 2500 \text{ W} \)
(P = 400 W for the series connection)
QUICK QUIZ 18.4

You have a large supply of lightbulbs and a battery. You start with one lightbulb connected to the battery and notice its brightness. You then add one lightbulb at a time, each new bulb being added in parallel to the previous bulbs. As the lightbulbs are added, what happens (a) to the brightness of the bulbs? (b) to the current in the bulbs? (c) to the power delivered by the battery? (d) to the lifetime of the battery? (e) to the terminal voltage of the battery?

*Hint:* Do not ignore the internal resistance of the battery.

QUICK QUIZ 18.4 ANSWER

(a) The brightness of the bulbs *decreases*

(b) The current in the bulbs *decreases*

(c) The power delivered by the battery *increases*

(d) The lifetime of the battery *decreases*

(e) The terminal voltage of the battery *decreases*