Gradient students:

Answer all questions. Write your work and calculation clearly.
Total perfect score is 150 points.

Undergraduate students:

Option 1.
Answer questions 1 to 4. Your score will be multiply with a factor of 5/4.
Total perfect score is 125 points.

Option 2.
Answer all questions. Write your work and calculation clearly.
Total perfect score is 125 points.

Your decision (undergraduate student only, circle one):

Option 1 / Option 2
1. (Total: 25 points)

Consider the following circuit under normal negative feedback condition:

![Circuit Diagram]

a. (5 points)
What is the potential at point B?

Potential at B = potential at A = 2V

b. (5 points)
What is the potential at the output of the opamp, C?

Potential at C = Potential at B + 0.6V = 2.6V
c. (5 points)
What is the current through the diode?

Current through the diode = current through the 4000Ω resistor = \( \frac{2V}{4000\Omega} \)
= 0.5 mA

d. (5 points)
What is the power dissipated by the diode?

Power dissipated by the diode = \( VI = 0.6 \, V \times 0.5 \, mA = 0.3 \, mW \)

e. (5 points)
Identify A and B as the inverted (-) or non-inverted (+) input:

A: +

B: -
2. (Total: 25 points)

Consider the following circuit in this problem.

![Circuit Diagram]

a. (5 points)
Sketch the waveform of $V_{out}$ versus time.

![Waveform Graph]
b. (5 points)
Briefly explain the functions of the JFETs in the circuit. What will happen if they are removed and replaced with resistors?

The JFETs form a constant current source to charge and discharge the capacitor so the voltage across the capacitor \( V_{\text{out}} \) varies linearly with time. If they are replaced with resistors, the output voltage will not be a perfect triangular wave:

![Graph showing a triangular waveform](image)

c. (7 points)
If supply voltages of +15V and -15V are applied to pin 7 and pin 4 of the opamp respectively, what is the amplitude of the oscillation in \( V_{\text{out}} \)?

The output of the opamp will flip flop between \( \pm 15V \) as \( V_{\text{out}} \) is compared with the voltage at the non-inverted input.

\[
\therefore V_{\text{out}} \text{ oscillates between } +15 \times \frac{1500}{1500 + 1500} = +7.5V \text{ and } -15 \times \frac{1500}{1500 + 1500} = -7.5V.
\]

d. (8 points)
If the current through the JFET channel is 1mA, calculate the frequency of the oscillator.

\[
C = \frac{Q}{V}
\]

Consider time required to charge \( C \) from -7.5V to +7.5V so that \( \Delta V = 15V \)

\[
\therefore \frac{\Delta t}{\Delta V} \Rightarrow \Delta t = \frac{\text{CAV}}{I} = \frac{1 \times 10^{-6} \times 15}{1 \times 10^{-3}} = 0.015s
\]

\[
\therefore \text{Period } T = \frac{2 \times 0.015s}{0.03s} = 33.3Hz
\]
3. (Total 25 points)

a. (5 points)
Fill in the words “small” or “large” in the blanks:

An ammeter has a \text{small} input impedance.

A voltmeter has a \text{large} input impedance.

A voltage source has a \text{small} output impedance.

A current source has a \text{large} output impedance.

b. (5 points)
Calculate the impedance of a 1000\mu F capacitor at frequency \(f=60\text{Hz}\).

\[
\omega = 2\pi f = 2\pi \times 60 = 376.99 \text{ rad/s}
\]

\[
X_C = \frac{1}{i\omega C} = -i \cdot \frac{1}{376.99 \times 1000 \times 10^{-6}} = -i \times 2.65 \Omega
\]

c. (5 points)
Calculate the impedance of a 10mH coil at frequency \(f=60\text{Hz}\).

\[
\omega = 2\pi f = 2\pi \times 60 = 376.99 \text{ rad/s}
\]

\[
X_L = i\omega L = i \times 376.99 \times 10 \times 10^{-3} = i \times 3.77 \Omega
\]
d. (10 points)
Consider the following circuit at f=60Hz:

Calculate the current through the coil with amplitude and phase with respect to $V_{in}$.

$\omega = 2\pi f = 2\pi \times 60 = 376.99$ rad/s

$X_C = \frac{1}{i\omega C} = -i \cdot \frac{1}{376.99 \times 1000 \times 10^{-6}} = -i \ 2.65 \ \Omega$

$X_L = i\omega L = i \times 376.99 \times 10 \times 10^{-3} = i \ 3.77 \ \Omega$

$\therefore I = \frac{V_{in}}{X_C + X_L} = \frac{V_{in}}{-i \ 2.65 + i \ 3.77} = \frac{V_{in}}{i \ 1.12} = -i \ \frac{V_{in}}{1.12}$

$\therefore |I| = \frac{10}{1.12} = 8.9A$

Because the $-i$ factor, the current lacks voltage by 90°.
4. (25 points)

Each part of this question is independent of each other.

a. (6 points)

Fill in the truth table of an NAND gate and an XOR gate.

\[
\begin{array}{c|c|c}
\text{In1 (A)} & \text{In2 (B)} & \text{Out (F)} \\
0 & 0 & 1 \\
0 & 1 & 1 \\
1 & 0 & 1 \\
1 & 1 & 0 \\
\end{array}
\]

b. (6 points)

(i) Make an inverter (NOT gate) out of NAND gates.  (ii) Make an inverter (NOT gate) out of XOR gate.

(i)
(ii)

![Circuit Diagram]

c. (13 points)
Construct a circuit with AND, NOT, and OR gates only to produce the following truth table. You can use three inputs AND gates if necessary.

<table>
<thead>
<tr>
<th>In1 (A)</th>
<th>In2 (B)</th>
<th>In3 (C)</th>
<th>Out (F)</th>
<th>Cook up the result for each row with output 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>( \overline{A} \cdot \overline{B} \cdot \overline{C} )</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>( \overline{A} \cdot \overline{B} \cdot \overline{C} )</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>( \overline{A} \cdot \overline{B} \cdot \overline{C} )</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>( \overline{A} \cdot \overline{B} \cdot \overline{C} )</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( \overline{A} \cdot \overline{B} \cdot \overline{C} )</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>( \overline{A} \cdot \overline{B} \cdot \overline{C} )</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>( \overline{A} \cdot \overline{B} \cdot \overline{C} )</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>( A \cdot B \cdot C )</td>
</tr>
</tbody>
</table>

![Circuit Diagram]

A

B

C

\( A \cdot B \cdot C \)

\( A \cdot B \cdot C \)
5. Optional for undergraduate students (Total: 25 points)
Consider the following circuit:

\[ \begin{array}{c}
10V \\
\hline
\hline
\end{array} \]

\[ \begin{array}{c}
100k\Omega \\
\hline
B
\end{array} \]

\[ \begin{array}{c}
400k\Omega \\
A
\end{array} \]

\[ \begin{array}{c}
\hline
\hline
\end{array} \]

a. (10 points)
According to Thevenin’s theorem, above circuit between points A and B is equivalent to the following circuit:

\[ \begin{array}{c}
\hline
\hline
A
\end{array} \]

\[ \begin{array}{c}
\hline
\hline
B
\end{array} \]

\[ e \quad r \]

Calculate the values of \( e \) and \( r \) in this equivalent circuit.

\( e \) is given by the voltage across AB where there is no load across it.

\[ \therefore e = 10 \cdot \frac{400k}{400k + 100k} = 10 \cdot \frac{4}{8} = 8V \]

If we short A and B with a wire, the current in the wire will be given by

\[ I = \frac{e}{r} = \frac{8}{r} \]

On the other hand, if we short A and B in the voltage divider circuit, the current in the wire will be

\[ I = \frac{10}{100k} = 10^{-4} A \]

\[ \therefore \frac{8}{r} = 10^{-4} \Rightarrow r = 8 \times 10^4 \Omega \text{ or } 80k\Omega \]

Alternatively, \( r \) is the output impedance of the divider circuit.

\[ \therefore r = R_1 // R_2 = \frac{(400k)(100k)}{(400k + 100k)} = 80k\Omega \]
b. (5 points)
According to Norton’s theorem, above circuit between points A and B is equivalent to the following circuit:

\[ I_0 \]

\[ r' \]

Calculate the values of \( I_0 \) and \( r' \) in this equivalent circuit.

If we short A and B with a wire, the current in the wire will be given by \( I_0 \).

\[ \therefore I_0 = \frac{10}{100k} = 10^{-4} \text{ A or } 0.1 \text{ mA} \]

The voltage across AB where there is no load is calculated in part (a) as 8V, but this also equals to \( I_0r \) in this Norton's equivalent circuit.

\[ \therefore 10^{-4} r' = 8 \Rightarrow r' = \frac{8}{10^{-4}} = 8 \times 10^4 \Omega \text{ or } 80k\Omega \]

c. (5 points)
If a voltmeter of input impedance 1M\( \Omega \) is connected across points A and B, what will be the reading of the voltmeter?
Voltage across the voltmeter = 10V \cdot \frac{400k\Omega/1M\Omega}{100k\Omega + 400k\Omega/1M\Omega}

400k\Omega/1M\Omega = \frac{(400k)(1000k)}{(400k + 1000k)} = 285.71k

\therefore \text{Voltage across the voltmeter} = 10V \cdot \frac{285.71k}{100k + 285.71k} = 10V \cdot \frac{285.71}{385.71k} = 7.41V

Alternatively, we can also use the equivalent circuit of part (a):

Voltage across the voltmeter = 8V \cdot \frac{1M\Omega}{80k\Omega + 1M\Omega}

= 8V \cdot \frac{1000}{80 + 1000}

= 8V \cdot \frac{1000}{1080} = 7.41V

You can also use the equivalent circuit of part (b) and get the same answer. Try it!

d. (5 points)
If a student accidentally connects an ammeter of input impedance 100\Omega across points A and B (not the right way to use an ammeter), what will be the reading of the ammeter?
Voltage across the ammeter = $10V \cdot \frac{400k\Omega/100\Omega}{100k\Omega + 400k\Omega/1M\Omega}$

$400k\Omega/100\Omega = \frac{(400k)(0.1k)}{(400k + 0.1k)} = 0.100k$

$\therefore$ Voltage across the ammeter = $10V \cdot \frac{0.100k}{100k + 0.100k} = 0.0100V$

$\therefore$ Current through the ammeter = $\frac{0.0100V}{100\Omega} = 1.0 \times 10^{-4}A$, or 0.1mA

Alternatively, we can also use the equivalent circuit of part (a):

```
e=8   r=80k

Current through the ammeter = $\frac{8V}{80k\Omega + 100\Omega}$

= $\frac{8}{80.1k}$

= $1.0 \times 10^{-4}A$, or 0.1mA
```

You can also use the equivalent circuit of part (b) and get the same answer. Try it!