University of Kentucky  
Department of Physics and Astronomy  

PHY 525. Introduction to Solid State Physics II  

Test 2.  

Date:  Oct 12, 2001  
Time:  9:00-9:50  
Answer all questions.  

1. (25 points)  
The electron energy near the top of the valance band in a semiconductor is given by  
\[ E_v = -10^{-37} k^2 \]  
(E$_v$ in Joules, k in m$^{-1}$)  
where \( k \) is the wavevector.  An electron is removed from the state  
\[ k = 10^9 \hat{k}_x \text{ m}^{-1} \]  
where \( \hat{k}_x \) is a unit vector along the x axis.  Calculate the following quantities of the resulting hole:  
(i) The effective mass.  
(ii) The energy.  
(iii) The momentum.  
(iv) The velocity.  
Each quantity must include the sign (or direction).  

2. (25 points)  
Consider the close orbits of an electron in real space and k space when an external magnetic field \( B \) is applied.  Let the area be \( A \) and \( S \) respectively.  Note that the magnet flux \( BA \) is quantized in unit of \( \Phi_0 = \hbar/e \).  
(i) Write down the relationship between \( A \) and \( S \) and hence the relationship between \( S \) and \( B \).  
(ii) Calculate \( S \) for a metal X of valance 1 (i.e. one conducting electron per atom).  The atomic density of the metal is \( 8.5 \times 10^{28} \text{ m}^{-3} \).  Assume free electron model.  
(iii) In a de Haas-van Alphen experiment of metal X, the magnetic susceptibility is oscillating periodically with \( \delta(1/B) \).  Calculate the periodicity.  How many oscillations are there as \( B \) is changed from 10.70 T to 10.93 T?
Solution:

1.(i) \[ m_h = \frac{1}{\hbar^2} \frac{\partial^2 E_h}{\partial k^2} = -\frac{1}{\hbar^2} \frac{\partial^2 E_v}{\partial k^2} \]
\[ = -\frac{1}{\hbar^2} \frac{\partial^2}{\partial k^2} \left[ -10^{-37} k^2 \right] \]
\[ = -\frac{1}{(1.055 \times 10^{-34})^2} \left[ 2 \times (-10^{-37}) \right] \]
\[ = 5.57 \times 10^{-32} \text{ kg}, \text{ or } 0.061 m_{\text{free electron}} \]

Note that the mass is positive.

(ii) \[ E_h = -E_v = -\left[ -10^{-37} k^2 \right] \]
\[ = 10^{-37} \times (10^9)^2 \]
\[ = 1 \times 10^{-19} \text{ J}, \text{ or } 0.624 \text{ eV} \]

Note that the energy is positive.

(iii) \[ \bar{p}_h = \hbar \bar{k}_e = -1.055 \times 10^{-34} \times (10^9 \bar{x}) \]
\[ = -1.055 \times 10^{-25} \text{ kgm/s } \bar{x} \]

Note that it is in the - \( \bar{x} \) direction.

(iv) \[ \bar{p}_h = m_h \bar{v}_h \Rightarrow \bar{v}_h = \frac{\bar{p}_h}{m_h} = \frac{-1.055 \times 10^{-25} \bar{x}}{5.57 \times 10^{-32}} \]
\[ = -1.896 \times 10^{-6} \text{ m/s } \bar{x} \]

Note that it is in the - \( \bar{x} \) direction.
2. (i) \( \hbar k = e \mathbf{v} \times \mathbf{B} \Rightarrow \hbar \Delta k = e \Delta \mathbf{r} \mathbf{B} \Rightarrow \hbar^2 (\Delta k)^2 = e^2 (\Delta r)^2 \mathbf{B}^2 \)
\[ \Rightarrow \hbar^2 S = e^2 AB^2 \]  
--- (1)

With \( AB = \Phi \), (1) \( \Rightarrow \hbar^2 S = e^2 \Phi B \)  
--- (2)

(ii) \( 2 \times \frac{\frac{4}{3} \pi k_f^3}{(2\pi)^3} = N \Rightarrow k_f^3 = \frac{N}{V} (2\pi)^3 \frac{3}{4\pi} \cdot \frac{1}{2} \)
\[ \Rightarrow k_f^3 = 8.5 \times 10^{28} \times (2\pi)^3 \frac{3}{4\pi} \cdot \frac{1}{2} \]
\[ \Rightarrow k_f^3 = 2.5167 \times 10^{30} \]
\[ \Rightarrow k_f = 1.3602 \times 10^{10} \text{ m}^{-1} \]
\[ \therefore \ S = \pi k_f^2 = \pi (1.3602 \times 10^{10})^2 = 5.813 \times 10^{20} \text{ m}^{-2} \]

(iii) (2) \( \Rightarrow \frac{1}{B} = \frac{e^2 \Phi}{\hbar^2 S} \Rightarrow \delta \left( \frac{1}{B} \right) = \frac{e^2}{\hbar^2 S} \Phi_0 \)  
(\( \Phi_0 = \frac{\hbar}{e} \))
\[ \Rightarrow \delta \left( \frac{1}{B} \right) = \frac{\Phi e}{\hbar^2 S} \]
\[ \Rightarrow \delta \left( \frac{1}{B} \right) = \frac{\frac{e^2}{h} \frac{h}{\hbar^2 S}}{\frac{e}{h^2 S}} = \frac{\Phi e}{\hbar^2 S} \]
\[ \Rightarrow \delta \left( \frac{1}{B} \right) = \frac{6.626 \times 10^{-34} \times 1.6 \times 10^{-19}}{(1.055 \times 10^{-34})^2 \times 5.813 \times 10^{20}} \]
\[ \Rightarrow \delta \left( \frac{1}{B} \right) = 1.639 \times 10^{-5} \text{ T}^{-1} \]

For the given magnetic fields, \( B_1 = 10.7 \text{ T} \) \( \Rightarrow \frac{1}{B_1} = \frac{1}{10.7} = 0.09346 \text{ T}^{-1} \)
\[ B_2 = 10.93 \text{ T} \Rightarrow \frac{1}{B_2} = \frac{1}{10.93} = 0.09149 \text{ T}^{-1} \]
\[ \therefore A \left( \frac{1}{B} \right) = \frac{1}{B_1} - \frac{1}{B_2} = 0.09346 - 0.09149 = 1.969 \times 10^{-3} \text{ T}^{-1} \]
\[ \therefore \text{ Number of oscillations} = \frac{1.969 \times 10^{-3}}{1.639 \times 10^{-5}} = 120.2 \text{ oscillations} \]